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Union of Fuzzy Sub - Quadradicgroups , Fuzzy Sub - Pentantgroups, and Fuzzy Sub - N groups .

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ABSTRACT

In this paper, union of Fuzzy subsets, Fuzzy sub – Quadratic group, fuzzy sub- Pentant group and Fuzzy sub–N groups are discussed. Moreover, some properties and theorems based on these have been derived and derive the definitions of Union of Fuzzy sub – Quadratic group, definitions of Union of fuzzy sub- Pentant group and soon definitions of Union of Fuzzy sub–N groups, and derive the definitions of Fuzzy Quadratic group, definitions of fuzzy Pentant group and soon definitions of Fuzzy N group and definition of Quadratic group, definitions of Pentant group and soon definitions of N group.

Keywords : Fuzzy Group**,** Fuzzy Sub-Bigroup ,Fuzzy Subgroup, Fuzzy Sub-Trigroup , Union of Fuzzy Subsets, Fuzzy Sub – Quadratic Group, Fuzzy Sub- Pentant Group and Fuzzy Sub–N Groups.

I. INTRODUCTION

The concept of fuzzy sets was initiated by Zadeh. Then it has become a vigorous area of research in engineering, medical science, social science, graph theory etc. Rosenfeld gave the idea of fuzzy subgroups Since the paper fuzzy set theory has been considerably developed by zadeh himself and some researchers. The original concept of fuzzy sets was introduced as an extension of crisps (usual) sets, by enlarging the truth value set of "grade of members"from the two value set {0,1} to unit interval {0,1} of real numbers. The study of fuzzy group was started by Rosenfeld. It was extended by Roventa who have introduced the fuzzy groups operating on fuzzy sets.

Rosenfield introduced the notion of fuzzy group and showed that many group theory results can be extended in an elementary manner to develop the theory of fuzzy group. The underlying logic of the theory of fuzzy group is to provide a strict fuzzy algebraic structure where level subset of a fuzzy group of a group G is a subgroup of the group.

The notion of bigroup was first introduced by P.L.Maggu in 1994. W.B.Vasanthakandasamy introduced fuzzy sub-bigroup with respect to +" and ."and illustrate it with example. W.B.Vasanthakandasamy was the first one to introduce the notion of bigroups in the year 1994. Several mathematicians have followed them in investigating the fuzzy group theory.

II. PRELIMINARIES

In this section contain some definitions, examples and some results **.**

2 .1. Concept of a Fuzzy set:

The concept of a fuzzy set is an extension of the concept of a crisp set. Just as a crisp set on a universal set U is defined by its characteristic function from U to {0,1} , a fuzzy set on a domain U is defined by its membership function from U to [0,1] .

Let U be a non-empty set, to be called the **Universal set** (or) **Universe of discourse or simply a domain**. Then, by a fuzzy set on U is meant a function

A: U →[0 ,1]. A is called **the membership function; A (x)** is called **the membership grade** of x in A. We also write

A= {(x, A (x)): x∈U}.

Examples:

Consider U={a ,b ,c ,d} and A:U→**1** defined by A(a)=0,A(b)=0.7,A(c)=0.4,and A(d)=1.Then A is a fuzzy set can also be written as follows:

A= {(a, 0), (b,0.7),(c,0.4),(d,1)}.

2.2. Relation between Fuzzy sets:

Let U be a domain and A, B be fuzzy sets on U.

Inclusion (or) Containment: A is said to be included (or) contained in B if and only if A(x)≤B(x) for all x in U. In symbols, we write, A⊆B. We also say that A is a subset of B.

2.3 Definition

Let S be a set. A fuzzy subset A of S is a function

A: S → [0,1].

2.4. Definition of Union of Fuzzy sets:

The union of two fuzzy subsets μ_1,μ_2 is defined by

$(\mu_1 \cup \mu_2)$ (x) = max{μ_1(x),μ_2(x)} for every x in U.

2.5. Definition of Fuzzy Subgroup:

Let G be a group. A fuzzy subset μ of a group G is called a fuzzy subgroup of the group G if

i) μ(xy) ≥ min {μ(x),μ(y)}for every x,y∈G. And

ii) $\mu(x^{-1})$ =μ(x) for every x∈G.

2.6. Definition of Fuzzy Union of the fuzzy sets $\mu_1$and μ_2:

Let $\mu_1$be a fuzzy subset of a set $x_1$and μ_2 be a fuzzy subset of a set x_2 , then the fuzzy union of the fuzzy sets $\mu_1$and μ_2 is defined as a function.

$\mu_1 \cup \mu_2 : x_1 \cup x_2 \rightarrow [0,1]$ given by

$$(\mu_1 \cup \mu_2)(x) = \begin{cases} \max(\mu_1(x),\mu_2(x)) \text{ if } x\in x_1 \cap x_2. \\ \mu_1(x) \text{ if } x\in x_1 \,\&\, x\notin x_2. \\ A\,\mu_2(x) \text{ if } x\in x_2 \,\&\, x\notin x_1. \end{cases}$$

2.7.Definition :

A set (G , + , ·) with two binary operations ' + ' and '. ' is called a **bigroup** if there exist two proper subsets G_1 and G_2 of G such that

i. G = $G_1 \cup G_2$

ii. (G_1 , +) is a group.

iii. (G_2 , .) is a group.

A non-empty subset H of a bigroup (G , + , .) is called a sub-bigroup, if H itself is a bigroup under ' + ' and '. 'operations defined on G.

2.8. Definition of fuzzy sub-bi group of the bigroup G :

Let (G,+ , .) be a bigroup with two binary operations +(addition) , . (multiplications) .Then μ : G → [0,1] is said to be a fuzzy sub-bigroup of the bigroup G under ,+ , . ,operations defined on G. if there exists two proper fuzzy subsets $\mu_1$of G_1 and μ_2 of G_2 such that

(i) (μ_1,+) is a fuzzy subgroup of (G_1,+)

(ii) (μ_2,.) is a fuzzy subgroup of (G_2, .)

(iii) μ = ($\mu_1 \cup \mu_2$).

2.9.Definition :

A set (G , + , · ,*) with three binary operations ' + ' , '. ' and * is called a **Trigroup** if there exist three proper subsets G_1 , G_2 and G_3 of G such that

i. G = $G_1 \cup G_2 \cup G_3$.

ii. (G_1 , +) is a group.

iii. (G_2 , .) is a group.

iv. (G_3 , *) is a group.

A non-empty subset H of a Trigroup (G , + , . ,*) is called a sub- trigroup, if H itself is a trigroup under ' + ' , '. ' and * operations defined on G.

2.10. Definition of the fuzzy union of the fuzzy sets $\mu_1$and μ_2 , μ_3 :

Let μ_1 be a fuzzy subset of a set X_1 and μ_2 be a fuzzy subset of a set X_2 , μ_3 be a fuzzy subset of a set X_3. then the fuzzy union of the fuzzy subsets μ_1 and μ_2 , μ_3 is defined as a function.

$\mu_1 \cup \mu_2 \cup \mu_3 : X_1 \cup X_2 \cup X_3 \rightarrow [0,1]$ given by

$$(\mu_1 \cup \mu_2 \cup \mu_3)(x) = \begin{cases} \max(\mu_1(x), \mu_2(x), \mu_3(x)) \text{ if } x \in X_1 \cap X_2 \cap X_3 . \\ \max(\mu_1(x), \mu_2(x)) \text{ if } x \in X_1 \cap X_2 \ \& \ x \notin X_3 . \\ \max(\mu_2(x), \mu_3(x)) \text{ if } x \in X_2 \cap X_3 \ \& \ x \notin X_1 . \\ \max(\mu_3(x), \mu_1(x)) \text{ if } x \in X_3 \cap X_1 \ \& \ x \notin X_2 \\ \mu_1(x) \text{ if } x \in X_1 \& \ x \notin X_2, X_3 \\ \mu_2(x) \text{ if } x \in X_2 \& x \notin X_1, X_3 . \\ \mu_3(x) \text{ if } x \in X_3 \ \& \ x \notin X_1, X_2 . \end{cases}$$

2.11. Definition of fuzzy sub-trigroup of the trigroup G :

Let (G,+ , .,*) be a trigroup with three binary operations +(addition) , . (multiplications) , * (ab/2). Then $\mu : G \rightarrow [0,1]$ is said to be a fuzzy sub-trigroup of the trigroup G under ,+ , .,* operations defined on G. if there exists three proper fuzzy subsets μ_1 of G_1 and μ_2 of G_2, μ_3 of G_3 such that

(i) $(\mu_1,+)$ is a fuzzy subgroup of $(G_1,+)$

(ii) $(\mu_2, .)$ is afuzzy subgroup of $(G_2, .)$

(iii) $(\mu_3,*)$ is a fuzzy subgroup of $(G_3,*)$

(iv) $\mu = (\mu_1 \cup \mu_2 \cup \mu_3)$.

Note:

A fuzzy subset μ of a group G is said to be a union of three fuzzy sub-groups of the group G if there exists three fuzzy subgroups μ_1 and μ_2 , μ_3 of μ (μ_1= A, μ_2= A and μ_3= A) such that $\mu =(\mu_1 \cup \mu_2 \cup \mu_3)$.

Here by the term fuzzy subgroup λ of μ says that λ is a fuzzy subgroup of the group G and $\lambda \subseteq \mu$ (where μ is also a fuzzy subgroup of G).Similarly , we define the definitions of fuzzy union of the Quadratic fuzzy sets , and soon fuzzy union of the n fuzzy sets .

2.12. Definition :

A set (G , + , · ,*,**) with four binary operations ' + ' , '. ' and *,** is called a **Quadratic group** if there exist four proper subsets G_1 , G_2 and G_3 , G_4 of G such that

i. G = $G_1 \cup G_2 \cup G_3 \cup G_4$.
ii. $(G_1 , +)$ is a group.
iii. $(G_2 , .)$ is a group.
iv. $(G_3 , *)$ is a group.
v. $(G_4 , **)$ is a group.

A non-empty subset H of a Quadratic group (G , + , . ,*,**) is called a sub- Quadratic group, if H itself is a Quadratic group under ' + ' , '. ' and *,** operations defined on G.

2.13. Definition of the fuzzy union of the fuzzy sets μ_1 and μ_2 ,μ_3 , μ_4 :

Let μ_1 be a fuzzy subset of a set X_1 and μ_2 be a fuzzy subset of a set X_2 , μ_3 be a fuzzy subset of a set X_3 , μ_4 be a fuzzy subset of a set X_4. then the fuzzy union of the fuzzy subsets μ_1 and μ_2 , μ_3, μ_4, is defined as a function

$\mu_1 \cup \mu_2 \cup \mu_3 \cup \mu_4 : X_1 \cup X_2 \cup X_3 \cup X_4 \rightarrow [0,1]$ given by

$$(\mu_1 \cup \mu_2 \cup \mu_3 \cup \mu_4)(x) = \begin{cases} \max(\mu_1(x), \mu_2(x), \mu_3(x), \mu_4(x)) \text{ if } x \in X_1 \cap X_2 \cap X_3 \cap X_4 . \\ \max(\mu_1(x), \mu_2(x), \mu_3(x)) \text{ if } x \in X_1 \cap X_2 \cap X_3 , \& \ x \notin X_4 . \\ \max(\mu_2(x), \mu_3(x), \mu_4(x)) \text{ if } x \in X_2 \cap X_3 \cap X_4 , \& \ x \notin X_1 . \\ \max(\mu_3(x), \mu_4(x), \mu_1(x)) \text{ if } x \in X_3 \cap X_4 \cap X_1 , \& \ x \notin X_2 . \\ \max(\mu_4(x), \mu_1(x), \mu_2(x)) \text{ if } x \in X_4 \cap X_1 \cap X_2 , \& \ x \notin X_3 . \\ \max(\mu_1(x), \mu_2(x)) \text{ if } x \in X_1 \cap X_2 \ \& \ x \notin X_3 , X_4 . \\ \max(\mu_2(x), \mu_3(x)) \text{ if } x \in X_2 \cap X_3 \ \& \ x \notin X_1, X_4 . \\ \max(\mu_3(x), \mu_4(x)) \text{ if } x \in X_3 \cap X_4 \ \& \ x \notin X_1, X_2 . \\ \max(\mu_4(x), \mu_1(x)) \text{ if } x \in X_4 \cap X_1 \ \& \ x \notin X_2, X_3 . \\ \mu_1(x) \text{ if } x \in X_1 \& \ x \notin X_2, X_3, X_4 . \\ \mu_2(x) \text{ if } x \in X_2 \& x \notin X_1, X_3, X_4 . \\ \mu_3(x) \text{ if } x \in X_3 \ \& \ x \notin X_1, X_2 , X_4 . \\ \mu_4(x) \text{ if } x \in X_4 \ \& \ x \notin X_1, X_2, X_3 . \end{cases}$$

2.14.Definition :

A set (G , + , · ,*,**,***) with five binary operations ' + ' , '. ' and *,**,*** is called a **Pentant group** if there exist five proper subsets G_1 , G_2 and G_3 , G_4 , G_5 of G such that

i. G = $G_1 \cup G_2 \cup G_3 \cup G_4 \cup G_5$.
ii. $(G_1 , +)$ is a group.

iii. $(G_2 , .)$ is a group.
iv. $(G_3 , *)$ is a group.
v. $(G_4 , **)$ is a group.
vi. $(G_5 , ***)$ is a group.

A non-empty subset H of a Pentant group (G , + , . ,*,**,***) is called a sub- Pentant group, if H itself is a Pentant group under ' + ' , '. ' and *,**,*** operations defined on G.

2.15. Definition of the fuzzy union of the fuzzy sets μ_1 and μ_2 , μ_3 , μ_4 , μ_5:

Let μ_1 be a fuzzy subset of a set X_1 and μ_2 be a fuzzy subset of a set X_2 , μ_3 be a fuzzy subset of a set X_3 , μ_4 be a fuzzy subset of a set X_4, μ_5 be a fuzzy subset of a set X_5. then the fuzzy union of the fuzzy subsets μ_1 and μ_2 , μ_3, μ_4, μ_5 ,is defined as a function.

$\mu_1 \cup \mu_2 \cup \mu_3 \cup \mu_4 \cup \mu_5 : X_1 \cup X_2 \cup X_3 \cup X_4 \cup X_5 \to [0,1]$ given by

$$(\mu_1 \cup \mu_2 \cup \mu_3 \cup \mu_4 \cup \mu_5)(x) = \begin{cases} \max(\mu_1(x), \mu_2(x), \mu_3(x), \mu_4(x), \mu_5(x)) \text{ if } x \in X_1 \cap X_2 \cap X_3 \cap X_4. \\ \max(\mu_1(x), \mu_2(x), \mu_3(x), \mu_4(x)) \text{ if } x \in X_1 \cap X_2 \cap X_3 \cap X_4 \,\&\, x \notin X_5 \\ \max(\mu_2(x), \mu_3(x), \mu_4(x), \mu_5(x)) \text{ if } x \in X_2 \cap X_3 \cap X_4 \cap X_5 \,\&\, x \notin X_1 \\ \max(\mu_3(x), \mu_4(x), \mu_5(x), \mu_1(x)) \text{ if } x \in X_3 \cap X_4 \cap X_5 \cap X_1 \,\&\, x \notin {}_2 . \\ \max(\mu_4(x), \mu_5(x), \mu_1(x), \mu_2(x)) \text{ if } x \in X_4 \cap X_5 \cap X_1 \cap X_2 \,\&\, x \notin X_3 . \\ \max(\mu_5(x), \mu_1(x), \mu_2(x), \mu_3(x)) \text{ if } x \in X_5 \cap X_1 \cap X_2 \cap X_3 \,\&\, x \notin X_4 . \\ \max(\mu_1(x), \mu_2(x), \mu_3(x)) \text{ if } x \in X_1 \cap X_2 \cap X_3 , \,\&\, x \notin X_4, X_5 . \\ \max(\mu_2(x), \mu_3(x), \mu_4(x)) \text{ if } x \in X_2 \cap X_3 \cap X_4 , \,\&\, x \notin X_1, X_5. \\ \max(\mu_3(x), \mu_4(x), \mu_5(x)) \text{ if } x \in X_3 \cap X_4 \cap X_5 , \,\&\, x \notin X_2, X_1 . \\ \max(\mu_4(x), \mu_5(x), \mu_1(x)) \text{ if } x \in X_4 \cap X_5 \cap X_1 , \,\&\, x \notin X_2 , X_3. \\ \max(\mu_5(x), \mu_1(x), \mu_2(x)) \text{ if } x \in X_5 \cap X_1 \cap X_2 , \,\&\, x \notin X_3 , X_4. \\ \max(\mu_1(x), \mu_2(x)) \text{ if } x \in X_1 \cap X_2 \,\&\, x \notin X_3 , X_4, X_5 . \\ \max(\mu_2(x), \mu_3(x)) \text{ if } x \in X_2 \cap X_3 \,\&\, x \notin X_1, X_4, X_5. \\ \max(\mu_3(x), \mu_4(x)) \text{ if } x \in X_3 \cap X_4 \,\&\, x \notin X_1, X_2, X_5. \\ \max(\mu_4(x), \mu_5(x)) \text{ if } x \in X_4 \cap X_5 \,\&\, x \notin X_2, X_3, X_1. \\ \max(\mu_5(x), \mu_1(x)) \text{ if } x \in X_5 \cap X_1 \,\&\, x \notin X_2, X_3, X_4. \\ \mu_1(x) \text{ if } x \in X_1 \,\&\, x \notin X_2, X_3, X_4, X_5. \\ \mu_2(x) \text{ if } x \in X_2 \,\&\, x \notin X_1, X_3, X_4 , X_5. \\ \mu_3(x) \text{ if } x \in X_3 \,\&\, x \notin X_1, X_2 , X_4 , X_5. \\ \mu_4(x) \text{ if } x \in X_4 \,\&\, x \notin X_1, X_2, X_3 , X_5. \\ \mu_5(x) \text{ if } x \in X_5 \,\&\, x \notin X_1, X_2, X , X_4. \end{cases}$$

Similarly for n

2.16. Definition of the-fuzzy union of the fuzzy sets μ_1 and μ_2 , μ_3……… ,μ_n:

Let μ_1 be a fuzzy subset of a set X_1 and μ_2 be a fuzzy subset of a set X_2 , μ_3 be a fuzzy subset of a set X_3 , and soon …..μ_n be a fuzzy subset of a set X_n. then the fuzzy union of the fuzzy subsets μ_1 and μ_2 , ……..,μ_n is defined as a function.

$\mu_1 \cup \mu_2 \cup \ldots \cup \mu_n : X_1 \cup X_2 \cup \ldots \cup X_n \to [0,1]$ given by

$$(\mu_1 \cup \mu_2 \cup \ldots \cup \mu_n)(x) = \begin{cases} \max(\mu_1(x), \mu_2(x), \ldots, \mu_n(x)) \text{ if } x \in X_1 \cap X_2 \cap \ldots \cap X_n. \\ \max(\mu_1(x), \mu_2(x), \ldots, \mu_{n-1}(x)) \text{ if } x \in X_1 \cap X_2 \cap \ldots \cap X_{n-1} \,\&\, x \notin X_n . \\ \max(\mu_2(x), \mu_3(x), \ldots, \mu_n(x)) \text{ if } x \in X_2 \cap X_3 \cap \ldots \cap X_n \,\&\, x \notin X_1 . \\ \max(\mu_3(x), \ldots, \mu_n(x), \mu_1(x)) \text{ if } x \in X_3 \cap \ldots \cap X_n \cap X_1 \,\&\, x \notin X_2 . \\ \max(\mu_4(x), \ldots \mu_n(x), \mu_1(x), \mu_2(x)) \text{ if } x \in X_4 \cap \ldots X_n \cap X_1 \cap X_2 \,\&\, x \notin X_3 \\ \cdots\cdots\cdots \\ \max(\mu_n(x), \mu_1(x), \ldots, \mu_{n-2}(x)) \text{ if } x \in X_n \cap X_1 \cap \ldots \cap X_{n-2} \,\&\, x \notin X_{n-1} . \\ \max(\mu_1(x), \ldots, \mu_{n-2}(x)) \text{ if } x \in X_1 \cap \ldots \cap X_{n-2} , \,\&\, x \notin X_{n-1}, X_n . \\ \max(\mu_2(x), \ldots, \mu_{n-1}(x)) \text{ if } x \in X_2 \cap \ldots \cap X_{n-1} , \,\&\, x \notin X_1, X_n. \\ \cdots\cdots\cdots \\ \max(\mu_1(x), \mu_2(x)) \text{ if } x \in X_1 \cap X_2 \,\&\, x \notin X_3 , X_4, \ldots, X_n . \\ \max(\mu_2(x), \mu_3(x)) \text{ if } x \in X_2 \cap X_3 \,\&\, x \notin X_1, X_4, \ldots X_n. \\ \cdots\cdots\cdots \\ \max(\mu_n(x), \mu_1(x)) \text{ if } x \in X_n \cap X_1 \,\&\, x \notin X_2, X_3, \ldots X_{n-1}. \\ \mu_1(x) \text{ if } x \in X_1 \,\&\, x \notin X_2, X_3, \ldots X_n. \\ \mu_2(x) \text{ if } x \in X_2 \,\&\, x \notin X_1, X_3, \ldots X_n. \\ \cdots\cdots\cdots \\ \mu_n(x) \text{ if } x \in X_n \,\&\, x \notin X_1, X_2, X_3 \ldots , X_{n-1}. \end{cases}$$

2.17.Definition :

A set (G , + , · ,*,**,***,……,$^{(n-2)}$*) with 'n' binary operations ' + ' , '. ' and *,**,***,……$^{(n-2)}$* is called a **"n" group** if there exist 'n' proper subsets G_1 , G_2 and G_3 ,…., G_n of G such that

i. $G = G_1 \cup G_2 \cup G_3$ ….. ∪∪∪∪ G_n.
ii. $(G_1 , +)$ is a group.
iii. $(G_2 , .)$ is a group.
iv. $(G_3 , *)$ is a group.
v. $(G_4 , **)$ is a group.
……………………………
……………………………
vi. (G_n , ***…,$^{(n-2)}$*) is a group.

A non-empty subset H of a "n" group (G , + , . ,*,**,***….. ,$^{(n-2)}$*) is called a sub- "n" group, if H itself is a "n" group under ' + ' , '. ' and *,**,*** ,….., ,$^{(n-2)}$*operations defined on G.
Similarly, we define the definitions of fuzzy union of the Quadratic fuzzy sets, and soon fuzzy union of the n fuzzy sets .

2.18. Definition of fuzzy sub-quadratic group of the Quadratic group G :

Let (G,+ , ˙,* ,**) be a Quadratic group with Four binary operations +(addition) , . (multiplications) , * ,** . Then $\mu : G \rightarrow [0,1]$ is said to be a fuzzy sub-quadratic group of the Quadratic group G under ,+ , .,*,** operations defined on G. if there exists four proper fuzzy subsets $\mu_1$of G_1 and μ_2 of G_2, μ_3 of G_3 ,μ_4 of G_4 such that

(i) (μ_1,+) is a fuzzy subgroup of (G_1,+)

(ii) (μ_2,.) is a fuzzy subgroup of (G_2, .)

(iii) (μ_3,*) is a fuzzy subgroup of (G_3 ,*)

(iv) (μ_4,**) is a fuzzy subgroup of (G_4 ,**)

(v) $\mu = (\mu_1 \cup \mu_2 \cup \mu_3 \cup \mu_4)$.

2.19. Definition of fuzzy sub- Pentant group of the pentant group G :

Let (G,+ , ˙,* ,**,***) be a Pentant group with Five binary operations + (addition),. (multiplications) , * ,**,*** . Then $\mu : G \rightarrow [0,1]$ is said to be a fuzzy sub - pentant group of the Pentant group G under ,+ , .,*,** ,***,operations defined on G. if there exists five proper fuzzy subsets $\mu_1$of G_1 and μ_2 of G_2, μ_3 of G_3 ,μ_4 of G_4 ,μ_5 of G_5 such that

(i) (μ_1,+) is a fuzzy subgroup of (G_1,+)

(ii) (μ_2,.) is a fuzzy subgroup of (G_2, .)

(iii) (μ_3,*) is a fuzzy subgroup of (G_3 ,*)

(iv) (μ_4,**) is a fuzzy subgroup of (G_4 ,**)

(v) (μ_5,***) is a fuzzy subgroup of(G_5 ,***)

(vi) $\mu = (\mu_1 \cup \mu_2 \cup \mu_3 \cup \mu_4 \cup \mu_5)$.

2.20. Definition of fuzzy sub- n group of the n group G :

Let (G,+ , ˙,* ,**,***,….,$^{(n-2)}$*) be a n group with n binary operations + (addition) , . (multiplications) , * ,**,*** ,….,$^{(n-2)}$* . Then $\mu : G \rightarrow [0,1]$ is said to be a fuzzy sub – n group of the n group G under ,+ , .,*,** ,***, ,….,$^{(n-2)}$*operations defined on G. if there exists n proper fuzzy subsets $\mu_1$of G_1 and μ_2 of G_2, μ_3 of G_3 ,μ_4 of G_4 ,μ_5 of G_5 …….,μ_n of G_n such that

(i) (μ_1,+) is a fuzzy subgroup of (G_1,+)

(ii) (μ_2,.) is a fuzzy subgroup of (G_2, .)

(iii) (μ_3,*) is a fuzzy subgroup of (G_3 ,*)

(iv) (μ_4,**) is a fuzzy subgroup of (G_4 ,**)

(v) (μ_5,***) is a fuzzy subgroup of (G_5 ,***)

(vi) (μ_n, (n-2)*) is a fuzzy subgroup of (G_n , (n-2)*)

(vii) $\mu = (\mu_1 \cup \mu_2 \cup \mu_3 \cup \mu_4 \cup \mu_5 \ldots . \cup \mu_n .)$.

III. Theorems

3.1. Main Theorem:

The union of the Four Fuzzy Sub- Quadratic groups of a group G is a Fuzzy Sub – Quadratic groups if and only if one is contained in the other.

Proof:
Necessary part:

Let μ_1 and μ_2 , μ_3 , μ_4 be Four Fuzzy Sub - Quadratic groups of G such that one is contained in the other.

Hence either $\mu_1 \subseteq \mu_2$, $\mu_2 \subseteq \mu_1$, $\mu_1 \subseteq \mu_3$, $\mu_3 \subseteq \mu_1$,

$\mu_3 \subseteq \mu_2$, $\mu_2 \subseteq \mu_3$, $\mu_3 \subseteq \mu_4$, $\mu_4 \subseteq \mu_3$ $\mu_1 \subseteq \mu_4$, $\mu_4 \subseteq \mu_1$, $\mu_2 \subseteq \mu_4$ and $\mu_4 \subseteq \mu_2$.

To prove:

$\mu = \mu_1 \cup \mu_2 \cup \mu_3 \cup \mu_4$ is a fuzzy sub- quadratic group of G.

By Definition of the fuzzy union of the fuzzy sets μ_1 and μ_2 , μ_3 , μ_4 .

$$\mu(x) = (\mu_1 \cup \mu_2 \cup \mu_3 \cup \mu_4)(x)$$
$$= \max(\mu_1(x), \mu_2(x), \mu_3(x), \mu_4(x)) .$$

which implies either

$\mu(x) = (\mu_1 \cup \mu_2 \cup \mu_3 \cup \mu_4)(x) = \mu_1(x)$ ……………..1

(or) $\mu(x) = (\mu_1 \cup \mu_2 \cup \mu_3 \cup \mu_4)(x) = \mu_2(x)$ ……….2

(or) $\mu(x) = (\mu_1 \cup \mu_2 \cup \mu_3 \cup \mu_4)(x) = \mu_3(x)$ ……….3.

(or) $\mu(x) = (\mu_1 \cup \mu_2 \cup \mu_3 \cup \mu_4)(x) = \mu_4(x)$ ……….4.

From 1,2,3 ,4

Since μ_1 and μ_2 , μ_3 μ_4 be Four Fuzzy Sub – Quadratic groups .

$\mu = (\mu_1 \cup \mu_2 \cup \mu_3 \cup \mu_4)$ is a fuzzy sub- quadratic group of G.

Sufficient part:

Let $\mu = (\mu_1 \cup \mu_2 \cup \mu_3 \cup \mu_4)$ is a fuzzy sub-quadratic group of G.

To prove :

$\mu_1 \subseteq \mu_2$, $\mu_2 \subseteq \mu_1$, $\mu_1 \subseteq \mu_3$, $\mu_3 \subseteq \mu_1$, $\mu_3 \subseteq \mu_2$, $\mu_2 \subseteq \mu_3$, $\mu_3 \subseteq \mu_4$, $\mu_4 \subseteq \mu_3$ $\mu_1 \subseteq \mu_4$, $\mu_4 \subseteq \mu_1$, $\mu_2 \subseteq \mu_4$ and $\mu_4 \subseteq \mu_2$.

By **2.14. Definition of the fuzzy sub-quadratic group of the Quadratic group G and Note**

(i) $(\mu_1,+)$ is a fuzzy subgroup of $(G_1,+)$

(ii) $(\mu_2,.)$ is a fuzzy subgroup of $(G_2, .)$

(iii) $(\mu_3,*)$ is a fuzzy subgroup of $(G_3,*)$

(iv) $(\mu_4,**)$ is a fuzzy subgroup of $(G_4,**)$

(v) $\mu = (\mu_1 \cup \mu_2 \cup \mu_3 \cup \mu_4)$.

Which implies ,

$\mu_1 \subseteq \mu = \mu_1 \cup \mu_2 \cup \mu_3 \cup \mu_4$………. 5

$\mu_2 \subseteq \mu = \mu_1 \cup \mu_2 \cup \mu_3 \cup \mu_4$……….6

$\mu_3 \subseteq \mu = \mu_1 \cup \mu_2 \cup \mu_3 \cup \mu_4$……….7

$\mu_4 \subseteq \mu = \mu_1 \cup \mu_2 \cup \mu_3 \cup \mu_4$……….8

Substitute 6,7 in 5 We get ,

$\mu_1 \subseteq \mu_2$ and $\mu_1 \subseteq \mu_3$,……………I.

Similarly Substitute 5 and 7 in 6 , We get ,

$\mu_2 \subseteq \mu_1$ and $\mu_2 \subseteq \mu_3$,…………II

Substitute 5 and 6 in 7, we get ,

$\mu_3 \subseteq \mu_1$ and $\mu_3 \subseteq \mu_2$, …………III

Similarly we get ,

$\mu_3 \subseteq \mu_4$ and $\mu_4 \subseteq \mu_3$………..IV

$\mu_1 \subseteq \mu_4$ and $\mu_4 \subseteq \mu_1$…………V

$\mu_2 \subseteq \mu_4$ and $\mu_4 \subseteq \mu_2$…………VI

From I ,II, and III ,IV,V,VI We get,

$\mu_1 \subseteq \mu_2$, $\mu_2 \subseteq \mu_1$, $\mu_1 \subseteq \mu_3$, $\mu_3 \subseteq \mu_1$, $\mu_3 \subseteq \mu_2$, $\mu_2 \subseteq \mu_3$, $\mu_3 \subseteq \mu_4$, $\mu_4 \subseteq \mu_3$ $\mu_1 \subseteq \mu_4$, $\mu_4 \subseteq \mu_1$, $\mu_2 \subseteq \mu_4$ and $\mu_4 \subseteq \mu_2$.

Hence the union of the four fuzzy sub- Quadratic groups of a group G is a fuzzy sub-Quadratic groups if and only if one is contained in the other.

3.2. Main Theorem:

The union of the Five Fuzzy Sub- Pentant groups of a group G is a Fuzzy Sub – Pentant groups if and only if one is contained in the other.

Proof:

Necessary part:

Let μ_1 and μ_2 , μ_3 , μ_4 , μ_5 be Five Fuzzy Sub - Pentant groups of G such that one is contained in the other.

Hence either $\mu_1 \subseteq \mu_2$, $\mu_2 \subseteq \mu_1$, $\mu_1 \subseteq \mu_3$, $\mu_3 \subseteq \mu_1$, $\mu_3 \subseteq \mu_2$, $\mu_2 \subseteq \mu_3$, $\mu_3 \subseteq \mu_4$, $\mu_4 \subseteq \mu_3$, $\mu_1 \subseteq \mu_4$ $\mu_4 \subseteq \mu_1$, $\mu_2 \subseteq \mu_4$, $\mu_4 \subseteq \mu_2$, $\mu_1 \subseteq \mu_5$, $\mu_5 \subseteq \mu_1$, $\mu_5 \subseteq \mu_2$, $\mu_2 \subseteq \mu_5$, $\mu_5 \subseteq \mu_3$, $\mu_3 \subseteq \mu_5$, $\mu_5 \subseteq \mu_4$, $\mu_4 \subseteq \mu_5$.

To prove:
$\mu = \mu_1 \cup \mu_2 \cup \mu_3 \cup \mu_4$ is a fuzzy sub- pentant group of G.
By Definition of the fuzzy union of the fuzzy sets μ_1**and** μ_2 **,** μ_3 **,** μ_4 **,**

$\mu(x) = (\mu_1 \cup \mu_2 \cup \mu_3 \cup \mu_4 \cup \mu_5)(x)$
$= \max(\mu_1(x), \mu_2(x), \mu_3(x), \mu_4(x), \mu_5(x))$.

which implies either
$\mu(x) = (\mu_1 \cup \mu_2 \cup \mu_3 \cup \mu_4 \cup \mu_5)(x)$
$= \mu_1(x)$ ……………..1.(or)
$\mu(x) = (\mu_1 \cup \mu_2 \cup \mu_3 \cup \mu_4 \cup \mu_5)(x)$
$= \mu_2(x)$ …………….2.
(or) $\mu(x) = (\mu_1 \cup \mu_2 \cup \mu_3 \cup \mu_4 \cup \mu_5)(x)$
$= \mu_3(x)$ …………….3.
(or) $\mu(x) = (\mu_1 \cup \mu_2 \cup \mu_3 \cup \mu_4 \cup \mu_5)(x)$
$= \mu_4(x)$ …………….4.
(or) $\mu(x) = (\mu_1 \cup \mu_2 \cup \mu_3 \cup \mu_4 \cup \mu_5)(x)$
$= \mu_5(x)$ …………….5.
From 1,2,3 ,4 ,5.
Since μ_1 and μ_2 , μ_3 , μ_4, μ_5 be Five Fuzzy Sub – Pentant groups
$\mu = (\mu_1 \cup \mu_2 \cup \mu_3 \cup \mu_4 \cup \mu_5)$ is a fuzzy sub- Pentant group of G.

Sufficient part:

Let $\mu = (\mu_1 \cup \mu_2 \cup \mu_3 \cup \mu_4 \cup \mu_5)$ is a fuzzy sub- Pentant group of G.

To prove :

$\mu_1 \subseteq \mu_2, \mu_2 \subseteq \mu_1, \mu_1 \subseteq \mu_3, \mu_3 \subseteq \mu_1, \mu_3 \subseteq \mu_2,$
$\mu_2 \subseteq \mu_3, \mu_3 \subseteq \mu_4, \mu_4 \subseteq \mu_3, \mu_1 \subseteq \mu_4, \mu_4 \subseteq \mu_1,$
$\mu_2 \subseteq \mu_4, \mu_4 \subseteq \mu_2, \mu_1 \subseteq \mu_5, \mu_5 \subseteq \mu_1, \mu_5 \subseteq \mu_2,$
$\mu_2 \subseteq \mu_5, \mu_5 \subseteq \mu_3, \mu_3 \subseteq \mu_5, \mu_5 \subseteq \mu_4, \mu_4 \subseteq \mu_5.$

By **2.15. Definition of fuzzy sub- Pentant group of the Pentant group G and Note**

(i) $(\mu_1,+)$ is a fuzzy subgroup of $(G_1,+)$

(ii) $(\mu_2,.)$ is a fuzzy subgroup of $(G_2, .)$

(iii) $(\mu_3,*)$ is a fuzzy subgroup of $(G_3,*)$

(iv) $(\mu_4,**)$ is a fuzzy subgroup of $(G_4,**)$

(v) $(\mu_5,***)$ is a fuzzy subgroup of $(G_5,***)$

(vi) $\mu = (\mu_1 \cup \mu_2 \cup \mu_3 \cup \mu_4 \cup \mu_5)$.

Which implies ,

$\mu_1 \subseteq \mu = \mu_1 \cup \mu_2 \cup \mu_3 \cup \mu_4 \cup \mu_5$ ……….5.
$\mu_2 \subseteq \mu = \mu_1 \cup \mu_2 \cup \mu_3 \cup \mu_4 \cup \mu_5$ ……….6
$\mu_3 \subseteq \mu$ …………………,……..….7
$\mu_4 \subseteq \mu$…………..,…,……………….8.
$\mu_5 \subseteq \mu$……..,……………………..9.

Substitute 5,6 We get ,
$\mu_1 \subseteq \mu_2$ and $\mu_1 \subseteq \mu_2$,…………….I.
Similarly Substitute 5,7 We get ,
$\mu_3 \subseteq \mu_1$ and $\mu_1 \subseteq \mu_3$,…………II
Substitute 6,7 we get ,
$\mu_2 \subseteq \mu_3$ and $\mu_3 \subseteq \mu_2$,…………III .
Similarly we get ,
$\mu_3 \subseteq \mu_4$ and $\mu_4 \subseteq \mu_3$……….IV
$\mu_1 \subseteq \mu_4$ and $\mu_4 \subseteq \mu_1$…………V
$\mu_2 \subseteq \mu_4$ and $\mu_4 \subseteq \mu_2$…………VI
$\mu_1 \subseteq \mu_5$ and $\mu_5 \subseteq \mu_1$………….VII
$\mu_5 \subseteq \mu_2$ and $\mu_2 \subseteq \mu_5$………….VIII
$\mu_5 \subseteq \mu_3$ and $\mu_3 \subseteq \mu_5$……………IX
$\mu_5 \subseteq \mu_4$ and $\mu_4 \subseteq \mu_5$……………X

From I ,II, and III ,IV,V,VI,VII,VIII,IX,X, We get, $\mu_1 \subseteq \mu_2$, $\mu_2 \subseteq \mu_1$, $\mu_1 \subseteq \mu_3$, $\mu_3 \subseteq \mu_1$, $\mu_3 \subseteq \mu_2$, $\mu_2 \subseteq \mu_3$, $\mu_3 \subseteq \mu_4$, $\mu_4 \subseteq \mu_3$, $\mu_1 \subseteq \mu_4$ $\mu_4 \subseteq \mu_1$, $\mu_2 \subseteq \mu_4$, $\mu_4 \subseteq \mu_2$, $\mu_1 \subseteq \mu_5$, $\mu_5 \subseteq \mu_1$, $\mu_5 \subseteq \mu_2$, $\mu_2 \subseteq \mu_5$, $\mu_5 \subseteq \mu_3$, $\mu_3 \subseteq \mu_5$, $\mu_5 \subseteq \mu_4$, $\mu_4 \subseteq \mu_5$.

Hence the union of the five fuzzy sub- Pendant groups of a group G is a fuzzy sub- Pendant groups if and only if one is contained in the other.
Similarly,

3.3. Main Theorem:

The union of the “n” Fuzzy Sub- “n” groups of a group G is an Fuzzy Sub – “n” groups if and only if one is contained in the other.
Proof:
Proof is Similar like above Theorems.

3.4 . Main Theorem:

Every fuzzy sub –Tri group of a group G is a fuzzy subgroup of the group G but not conversely .

proof:

It follows from the definition of a fuzzy sub – Tri group of a group G that every a fuzzy sub –Tri group of a group G is a fuzzy subgroup of the group G . . Converse part is not true .

3.5 . Main Theorem:

Every fuzzy sub – Quadratic group of a group G is a fuzzy subgroup of the group G but not conversely

proof:

It follows from the definition of a fuzzy sub – Quadratic group of a group G that every an fuzzy sub – Quadratic group of a group G is a fuzzy subgroup of the group G .Converse part is not true .

3.6 . Main Theorem:

Every fuzzy sub – Pentant group of a group G is a fuzzy subgroup of the group G but not conversely .

proof:

It follows from the definition of a fuzzy sub – Pentant group of a group G that every an fuzzy sub – Pentant group of a group G is a fuzzy subgroup of the group G . Converse part is not true .

3.7 . Main Theorem:

Every fuzzy sub – "n" group of a group G is a fuzzy subgroup of the group G but not conversely .

proof:

It follows from the definition of a fuzzy sub – "n" group of a group G that every fuzzy sub – "n" group of a group G is a fuzzy subgroup of the group G . . Converse part is not true .

IV. CONCLUSION

In this paper, We derive the union of the fuzzy sub- "n"groups of a group G for n=1,2,3,…..n .and Definitions of Fuzzy Sub- Quadratic groups and Fuzzy Sub- pendant groups and soon up to definitions of Sub" n" groups and derive the definitions of Fuzzy Quadratic group, definitions of fuzzy Pendant group and soon definitions of Fuzzy N group . Shows that Every Fuzzy Sub Bi- groups ,Fuzzy Sub Tri- groups and soon Fuzzy Sub "n" groups are Fuzzy Sub group .

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