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Connectedness and Compactness of Fuzzy (Anti- Fuzzy) Subgroups and Fuzzy (Anti- Fuzzy) Normal Subgroups

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ABSTRACT

In this paper, We discuss the definition of fuzzy subgroup, definition of anti- fuzzy subgroup,the definition of fuzzy normal subgroup, definition of anti- fuzzy normal subgroup and show the relation between fuzzy normal subgroups and an Anti- fuzzy normal subgroups and derive the definitions of Connectedness and also the definitions of Seperation and derive some theorems on Compactness and Connectedness and define some results on compactness in Fuzzy (anti- fuzzy) subgroups and Fuzzy (anti- fuzzy) normal subgroups.

Keywords : Fuzzy Subgroup, Anti- Fuzzy Subgroup, Fuzzy Normal Subgroup, Anti- Fuzzy Normal Subgroup, Closed Sets, Compositions, Compactness And Connected.

I. INTRODUCTION

The concept of fuzzy sets was initiated by Zadeh. Then it has become a vigorous area of research in engineering, medical science, social science, graph theory etc. Since the paper fuzzy set theory has been considerably developed by zadeh himself and some researchers. The original concept of fuzzy sets was introduced as an extension of crisps (usual) sets, by enlarging the truth value set of "grade of members"from the two value set {0,1} to unit interval {0,1} of real numbers. The study of fuzzy group was started by Rosenfeld. It was extended by Roventa who have introduced the fuzzy groups operating on fuzzy sets.

II. PRELIMINARIES

2.1 Concept of a Fuzzy set:

The concept of a fuzzy set is an extension of the concept of a crisp set. Just as a crisp set on a universal set U is defined by its characteristic function from U to {0,1}, a fuzzy set on a domain U is defined by its membership function from U to [0,1].

Let U be a non-empty set, to be called the **Universal set** (or) **Universe of discourse or simply a domain**. Then, by a fuzzy set on U is meant a function **A: U →[0,1]. A** is called **the membership function; A (x)** is called **the membership grade** of x in A. We also write
A= {(x, A (x)): x∈U}.

2.2 Definition

Let S be a set. A fuzzy subset A of S is a function A: S → [0,1].

2.3. Definition of Fuzzy Subgroup:

Let G be a group. A fuzzy subset μ of a group G is called a fuzzy subgroup of the group G if
i) μ(xy) ≥min {μ(x),μ(y)}for every x,y ∈ G. And ii) $\mu(x^{-1})$ =μ(x) for every x∈ G.

2.4. Definition of Anti Fuzzy Subgroup:

Let G be a group. A fuzzy subset μ of a group G is called an anti-fuzzy subgroup of the group G if

i) μ(xy) ≤ max {μ(x),μ(y)}for every x,y ∈G and
ii)$\mu(x^{-1})$ =μ(x) for every x ∈G.

From the definitions 2.3. and 2.4. we have
min {μ(x),μ(y)} ≤ μ(xy)
≤ max {μ(x),μ(y)}…………..1.

2.5. Definition of Fuzzy normal subgroup (Anti-Fuzzy):

Let G be a Group. A fuzzy subgroup μ of a group G is called a **Fuzzy (Anti-Fuzzy) normal subgroup** of G. if for all x, y ∈ G,
μ(xyx $^{-1}$) = μ(y) (or) μ (xy) = μ(yx).

2.6. Definition of Anti- Fuzzy normal subgroup :

Let G be a Group. An Anti -fuzzy subgroup A of a group G is called an **Anti-Fuzzy normal subgroup** of G. if for all x, y ∈ G,
A(xyx $^{-1}$) = A(y) (or) A(xy) = A(yx).

2.7. Definition of Separation of Fuzzy (Anti-Fuzzy) Subgroup :

Let G be a group and μ be a fuzzy subgroup of a group G. A **Separation** of μ of G is a pair μ_1 and μ_2 are non empty disjoint open subsets of μ of G whose union is μ. Then the fuzzy subgroup μ of a group G is said to be separated if $\mu_1, \mu_2 \in \mu$.
$\mu_1 \cup \mu_2 = \mu$ and $\mu_1 \neq \mu_2$

2.8. Definition of Connectedness of Fuzzy (Anti-Fuzzy) Subgroup :

Let G be a group and μ be a fuzzy subgroup of a group G. A **Connectedness** of μ of G is a pair μ_1 and μ_2 are non empty (or) empty open subsets of μ of G whose union is μ. Then the fuzzy subgroup μ of a group G is said to be connected if $\mu_1, \mu_2 \in \mu$.
$\mu_1 \cup \mu_2 = \mu$, μ_1 and μ_2 is either equal (or) one contained in other.
i.e $\mu_1 = \mu_2$ (or) $\mu_1 \subseteq \mu$ (or) $\mu_2 \subseteq \mu$
(or) μ is said to be connected if there does not exist a Seperation of μ.

NOTE : A Fuzzy (Anti- Fuzzy) Subgroup μ is connected if and only if the only subsets of μ that are both open and closed in μ are the empty sets and μ itself.

2.9. Definition of Totally disconnected :

A Group is Totally disconnected if its only connected subgroup are one-point sets.

Example:

The Rational Q are **not Connected**. Indeed, the only connected subgroups of Q are the One- point sets. So Rational Q is **Totally disconnected**

2.10. Definitions Related to Compactness:

(i) **Limit point Compactness:** A Set X is said to be limit point compact if every infinite subset of X has a limit point.
(ii) **Local compactness:** A Set X is said to be locally compact at x if there is some compact subset C of X that contains a neighbourhood of x. If X is locally compact at each of its points, X is said to be l**ocally compact.**
(iii) If Y is a Compact Hausdorff set and X is a proper subset of Y whose closure equals Y, then Y is said to be a **Compactification of X.** If Y-X equals a single point, then Y is called the **one-point Compactification.**

Note Related to Compactness :

1. Compactness implies limit point compactness, but not conversely.
2. A compact set is automatically locally compact.
3. X has a one-point Compactification Y if and only if X is locally compact Hausdorff set that is not itself compact.
4. A Set X is called a **Hausdorff set** if for each pair x_1 and x_2 of distinct points of X, there exist neighbourhoods of U_1 and U_2 of x_1 and x_2, respectively, that are disjoint.

Examples of Compact :

1. The Real line R is Compact. and Every closed interval in R is also Compact.
2. but Q is not Compact.
3. The Real line R is locally Compact and Every closed interval in R is also locally Compact but Q is not locally Compact. (since by Note 2).

III. THEOREMS

Theorem: 3.1.

If the sets μ_1 and μ_2 forms a Seperation of μ, and if μ_3 is a connected Fuzzy (Anti- Fuzzy) Subgroup of, Then μ_3 lies entirely within either μ_1 (or) μ_2.

Proof:

Let the sets μ_1 and μ_2 forms a Seperation of μ,

By the definitions of Seperation of μ,

Since μ_1 and μ_2 are both non empty disjoint open subsets of μ of G whose union is μ. (i.e) $\mu_1, \mu_2 \in \mu$.

$\mu_1 \cup \mu_2 = \mu$ and $\mu_1 \neq \mu_2$

and μ_3 is a connected Fuzzy (Anti- Fuzzy) Subgroup of μ it is also open.

The sets $\mu_1 \cap \mu_3$ and $\mu_2 \cap \mu_3$ are also open inμ_3.

These two sets $\mu_1 \cap \mu_3$ and $\mu_2 \cap \mu_3$ are disjoint and their union is μ_3.

$\mu_3 = (\mu_1 \cap \mu_3) \cup (\mu_2 \cap \mu_3)$.

If they were both non empty, they constitute a separation of μ_3.

Which Contradicts the hypothesis that μ_3 is a connected.

Therefore, one of them is empty.

Hence μ_3 must lies entirely in μ_1 (or) *in* μ_2.

Thus proved.

Theorem: 3.2.

The Union of a Collection of connected Fuzzy (Anti- Fuzzy) Subgroup of μ that have a point in common is connected.

Proof:

Let $\{ \mu_n \}$ be the collection of connected Fuzzy (Anti-Fuzzy) Subgroup of μ ;

Let x be a point of $\cap\, \mu_n$,

i.e., $x \in \mu_1 \cap \mu_2 \cap \mu_3 \ldots\ldots\cap \mu_n$.

To Prove :

$\vartheta = \cup\, \mu_n = \mu_1 \cup \mu_2 \cup \mu_3 \ldots\ldots\cup \mu_n$ is connected.

Suppose that $\vartheta = \alpha \cup \beta$ is a separation of ϑ, since x be a point of $\cap\, \mu_n$. The point x is in one of sets α (or) β.

Suppose $x \in \alpha$, since μ_n is a connected Fuzzy (Anti-Fuzzy) Subgroup of μ, (By using the theorem:3.1.). Then μ_n must lies entirely within either α (or) β, and it cannot lie in, because it contains the point x of α.

$x \in \mu_n$ for each n.

Hence $\mu_n \subset \alpha$ for every n, so that $\cup\, \mu_n \subset \alpha$, which implies β is empty.

Which contradicts the fact that β is nonempty and $\alpha \cup \beta$ is a separation of $\vartheta = \cup\, \mu_n$.

there is no separation of $\vartheta = \cup\, \mu_n$.

Therefore $\vartheta = \cup\, \mu_n$ is connected.

Hence The Union of a Collection of connected Fuzzy (Anti- Fuzzy) Subgroup of μ that have a point in common is connected.

Theorem: 3.3.

The union of two fuzzy (Anti- Fuzzy) subgroups of a group G is a fuzzy (Anti- Fuzzy) subgroup is Connected.

Proof :

By using the following theorem:1

The union of two fuzzy (Anti- Fuzzy) subgroups of a group G is a fuzzy (Anti- Fuzzy) subgroup if and only if one is contained in the other.

We get, $\mu_1 \subseteq \mu_2$ and $\mu_2 \subseteq \mu_1$.

And by the definition of Connectedness of a fuzzy (Anti- Fuzzy) subgroups,

We get, The union of two fuzzy (Anti- Fuzzy) subgroups of a group G is a fuzzy (Anti- Fuzzy) subgroup is Connected.

Thus Proved.

Corollary :

The theorem is true for n=2,3,4,....,n.

Theorem: 3.4.

The union of "n" fuzzy (Anti- Fuzzy) subgroups of a group G is a fuzzy (Anti- Fuzzy) sub-"n" group is Connected.

Proof:

By using the following theorem:1

The union of "n" fuzzy (Anti- Fuzzy) subgroups of a group G is a fuzzy (Anti- Fuzzy) sub "n" group if and only if one is contained in the other.

We get, $\mu_1 \subseteq \mu_2$ and $\mu_2 \subseteq \mu_1$ ………..$\mu_1 \subseteq \mu_n$ and $\mu_n \subseteq \mu_1$.

And by the definition of Connectedness of a fuzzy (Anti- Fuzzy) subgroups,

We get, The union of fuzzy (Anti- Fuzzy) sub "n"groups of a group G is a fuzzy (Anti- Fuzzy) sub"n" group is Connected.

Thus Proved.

Theorem: 3.5.

Let μ_1 be a connected fuzzy (Anti- Fuzzy) subgroups of μ. if $\mu_1 \subset \mu_2 \subset \bar{\mu}_1$. Then μ_2 is also connected.

Proof:

Let μ_1 be a connected fuzzy (Anti- Fuzzy) subgroups of μ and let $\mu_1 \subset \mu_2 \subset \bar{\mu}_1$.

Suppose that $\mu_2 = \mu_3 \cup \mu_4$ is a separation of μ_2.

by theorem 3.1.

The set μ_1 must lies entirely in μ_3 (or) $in\, \mu_4$; suppose that $\mu_1 \subset \mu_3$.Then $\bar{\mu}_1 \subset \bar{\mu}_3$; since $\bar{\mu}_3$ and μ_4 are disjoint, μ_2 cannot intersect μ_4.

By the hypothesis, $\mu_2 \subset \bar{\mu}_1$.This contradicts the fact that μ_4 is a nonempty subset of μ_2

Which implies μ_4 is a empty.

Hence μ_2 is also connected.

Results:

1.μ is connected which implies $\bar{\mu}$.

1. 2.A finite Cartesian product of connected fuzzy subgroups is connected.

Theorem: 3.6.

By condition 1, prove that A Fuzzy (Anti-Fuzzy) subgroup $\mu$of a group G is compact.

Proof :

Let μ is a Fuzzy (Anti-Fuzzy) subgroup of a group G.

By Heine –Borel Theorem : 1

"The set is compact if and only if it is closed and bounded".

It is enough to prove that μ is closed and bounded.

The equation 1 is compared with

a≤x≤b, it is closed.

Obviously, μ is closed,

by 1

μ(xy) ≤ max {μ(x),μ(y)} =M,

Obviously μ is bounded.

Thus μ is closed and bounded.

Hence μ is a Fuzzy (Anti-Fuzzy) subgroup of a group G is compact.

Theorem 3.7:

A Closed subset of a compact set of a fuzzy (Anti-fuzzy) subgroups is compact.

proof

Let μ_1 and μ_2 are closed subsets of a compact set of a fuzzy (Anti-fuzzy) subgroups μ,

$\mu_1 \subseteq \mu$ and $\mu_2 \subseteq \mu$

by 1,

min {μ(x),μ(y)} ≤ $\mu_1(xy)$ ≤ μ(xy)
≤ max {μ(x),μ(y)}……….2.

min {μ(x),μ(y)} ≤ $\mu_2(xy)$ ≤ μ(xy)
≤ max {μ(x),μ(y)}……….3.

from 2 and 3,

Thus μ_1 and μ_2 are closed and bounded.

By Theorem 1, “The set is compact if and only if it is closed and bounded”.

Hence μ_1 and μ_2 are compact.

Therefore, A Closed subset of a compact set of a fuzzy (Anti-fuzzy) subgroups is compact.

Note:

Every Closed subset of a compact set of a fuzzy (Anti-fuzzy) subgroups is compact.

Theorem 3.8:

A Finite union of compact sets of a fuzzy (Anti-fuzzy) subgroups is compact.

proof

Let μ_1 and μ_2 are compact subsets of a compact set of a fuzzy (Anti-fuzzy) subgroups μ,

$\mu_1 \subseteq \mu_1 \cup \mu_2 \subseteq \mu$ and $\mu_2 \subseteq \mu_1 \cup \mu_2 \subseteq \mu$.

by theorem 1 (or) by theorem 3.5,

Thus $\mu_1 \cup \mu_2$ is closed and bounded.

Therefore $\mu_1 \cup \mu_2$ is compact.

Hence a Finite union of compact sets of a fuzzy (Anti-fuzzy) subgroups is compact.

Theorem 3.9 :

The Product of finitely many compact sets of a fuzzy (Anti-fuzzy) subgroups is compact.

Proof:

Let μ_1 and μ_2 are compact subsets of a compact set μ,

By above theorems,

$\mu_1 \circ \mu_2$ is also compact.

Hence The Product of finitely many compact sets of a fuzzy (Anti-fuzzy) subgroups is compact.

Note:

The Product of infinitely many compact sets of a fuzzy (Anti-fuzzy) subgroups is compact.

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Theorem 3.10:

By condition 1, prove that A Fuzzy (Anti-Fuzzy) normal subgroup μ of a group G is compact.

Proof:

Let μ is a Fuzzy (Anti-Fuzzy) normal subgroup.

Since, A Fuzzy (Anti-Fuzzy) normal subgroup $\mu$of a group G is obviously a Fuzzy subgroup of G.

By using the Theorem:

A Fuzzy (Anti-Fuzzy) subgroup $\mu$of a group G is compact.

Hence a Fuzzy (Anti-Fuzzy) normal subgroup $\mu$of a group G is compact.

Theorem 3.11:

A Closed subset of a Fuzzy (Anti-Fuzzy) normal subgroup μ of a group G is compact.

Proof:

Let μ_1 and μ_2 are Closed subsets of a Fuzzy (Anti-Fuzzy) normal subgroup μ of a group G,

$\mu_1 \subseteq \mu$ and $\mu_2 \subseteq \mu$

by 1,

$\min\{\mu(x),\mu(y)\} \leq \mu_1(xy) \leq \mu(xy)$

$\leq \max\{\mu(x),\mu(y)\}$……….2.

$\min\{\mu(x),\mu(y)\} \leq \mu_2(xy) \leq \mu(xy)$

$\leq \max\{\mu(x),\mu(y)\}$……….3.

from 2 and 3, Thus μ_1 and μ_2 are closed and bounded.

By Heine Borel Theorem 1,

“The set is compact if and only if it is closed and bounded”.

Hence μ_1 and μ_2 are compact.

Hence μ_1 and μ_2 are Closed subsets of a Fuzzy (Anti-Fuzzy) normal subgroup μ of a group G is compact.

Note: Every Closed subset of a compact set of a fuzzy (Anti-fuzzy) normal subgroups is compact.

Theorem 3.12:

A Finite union of compact sets of a fuzzy (Anti-fuzzy) normal subgroups is compact.

proof

Let μ_1 and μ_2 are compact subsets of a compact set of a fuzzy (Anti-fuzzy) normal subgroups μ,

$\mu_1 \subseteq \mu_1 \cup \mu_2 \subseteq \mu$ and $\mu_2 \subseteq \mu_1 \cup \mu_2 \subseteq \mu$.
by theorem 1 (or) by theorem 3.5,
Thus $\mu_1 \cup \mu_2$ is closed and bounded.
Therefore $\mu_1 \cup \mu_2$ is compact.
Hence a Finite union of compact sets of a fuzzy (Anti-fuzzy) normal subgroups is compact.

Theorem 3.13:
The Product of finitely many compact sets of a fuzzy (Anti-fuzzy) normal subgroups is compact.
Proof:

Let μ_1 and μ_2 are compact subsets of a compact set μ,
By above theorems,
$\mu_1 \circ \mu_2$ is also compact.
Hence The Product of finitely many compact sets of a fuzzy (Anti-fuzzy) normal subgroups is compact.

Note:
The Product of infinitely many compact sets of a fuzzy (Anti-fuzzy) normal subgroups is compact.
Theorem 3.14:
Let μ_1 *and* μ_2 are fuzzy normal subgroup of a group G. If $\mu_1 \circ \mu_2$ is a fuzzy normal subgroup of a group G if and only if $\mu_1 \circ \mu_2 = \mu_2 \circ \mu_1$.
proof

Necessary part :
Let $\mu_1 \circ \mu_2 = \mu_2 \circ \mu_1$.
To prove :
$\mu_1 \circ \mu_2$ is a fuzzy normal subgroup of a group G.

Let μ_1 *and* μ_2 are fuzzy normal subgroup of a group G are also a fuzzy subgroups.
By using the following theorem :1
Let μ_1 *and* μ_2 are fuzzy subgroup of a group G. If $\mu_1 \circ \mu_2$ is a fuzzy subgroup of a group G if and only if $\mu_1 \circ \mu_2 = \mu_2 \circ \mu_1$.

Since, $\mu_1 \circ \mu_2 = \mu_2 \circ \mu_1$ which implies, $\mu_1 \circ \mu_2$ is a fuzzy subgroup of a group G is also a fuzzy normal subgroup of a group G.
Hence $\mu_1 \circ \mu_2$ is a fuzzy normal subgroup of a group G.

Sufficient part :
Let $\mu_1 \circ \mu_2$ is a fuzzy normal subgroup of a group G is also a fuzzy subgroup.
To prove :
$\mu_1 \circ \mu_2 = \mu_2 \circ \mu_1$.
Again By using the following theorem :1
Let μ_1 *and* μ_2 are fuzzy subgroup of a group G. If $\mu_1 \circ \mu_2$ is a fuzzy subgroup of a group G if and only if $\mu_1 \circ \mu_2 = \mu_2 \circ \mu_1$.

We get, $\mu_1 \circ \mu_2 = \mu_2 \circ \mu_1$.
Hence, If $\mu_1 \circ \mu_2$ is a fuzzy normal subgroup of a group G if and only if $\mu_1 \circ \mu_2 = \mu_2 \circ \mu_1$.

Thus proved.

For an Anti fuzzy subgroups, we use A instead of μ.

Theorem 3.15:
If $A_1 \circ A_2$ is an Anti- fuzzy normal subgroup of a group G if and only if $A_1 \circ A_2 = A_2 \circ A_1$.
proof:
Necessary part :
Let $A_1 \circ A_2 = A_2 \circ A_1$.
To prove :
$A_1 \circ A_2$ is an Anti-fuzzy normal subgroup of a group G.
Let A_1 *and* A_2 are an Anti- fuzzy normal subgroup of a group G is an Anti- fuzzy subgroup.
By using the following theorem :1
Let A_1 *and* A_2 are Anti -fuzzy subgroup of a group G. If $A_1 \circ A_2$ is a Anti- fuzzy subgroup of a group G if and only if $A_1 \circ A_2 = A_2 \circ A_1$.
Since, $A_1 \circ A_2 = A_2 \circ A_1$ which implies, $A_1 \circ A_2$ is a Anti- fuzzy subgroup of a group G is also an Anti-fuzzy normal subgroup of a group G

Hence, $A_1 \circ A_2$ is an Anti- fuzzy normal subgroup of a group G.

Sufficient part :

Let $A_1 \circ A_2$ is an Anti- fuzzy normal subgroup of a group G.
To prove :
$A_1 \circ A_2 = A_2 \circ A_1$.

By using the following theorem :1
Let $A_1 and\ A_2$ are Anti -fuzzy subgroup of a group G. If $A_1 \circ A_2$ is a Anti- fuzzy subgroup of a group G if and only if $A_1 \circ A_2 = A_2 \circ A_1$.
We get, $A_1 \circ A_2 = A_2 \circ A_1$.
Hence, If $A_1 \circ A_2$ is a fuzzy normal subgroup of a group G if and only if $A_1 \circ A_2 = A_2 \circ A_1$.
Thus proved.

Theorem: 3.16.

If the sets μ_1 and μ_2 forms a Seperation of μ, and if μ_3 is a connected Fuzzy (Anti- Fuzzy) normal Subgroup of μ, Then μ_3 lies entirely within either μ_1 (or) μ_2.

Proof:
Let the sets μ_1 and μ_2 forms a Seperation of μ,
By the definitions of Seperation of μ,
Since μ_1 and μ_2 are both non empty disjoint open subsets of μ of G whose union is μ. (i.e) $\mu_1, \mu_2 \in \mu$.
$\mu_1 \cup \mu_2 = \mu$ and $\mu_1 \neq \mu_2$
$and\ \mu_3$ is a connected Fuzzy (Anti- Fuzzy) normal Subgroup of μ is also a connected Fuzzy (Anti- Fuzzy) Subgroup of μ.
By using the theorem :1
If the sets μ_1 and μ_2 forms a Seperation of μ, and if μ_3 is a connected Fuzzy (Anti- Fuzzy) Subgroup of μ, Then μ_3 lies entirely within either μ_1 (or) μ_2.
Hence, μ_3 lies entirely within either μ_1 (or) μ_2.
Thus proved.

Theorem: 3.17.

The Union of a Collection of connected Fuzzy (Anti- Fuzzy) normal Subgroup of μ that have a point in common is connected.

Proof:
Let { μ_n } be the collection of connected Fuzzy normal (Anti- Fuzzy) normal Subgroup of μ is also a connected Fuzzy normal (Anti- Fuzzy) Subgroup of μ.
By using theorem:2
The Union of a Collection of connected Fuzzy (Anti- Fuzzy) Subgroup of μ that have a point in common is connected.
Hence The Union of a Collection of connected Fuzzy (Anti- Fuzzy) normal Subgroup of μ that have a point in common is connected.

Theorem: 3.18.

The union of two fuzzy (Anti- Fuzzy) normal subgroups of a group G is a fuzzy (Anti- Fuzzy) normal subgroup is Connected.

Proof :
By using the following theorem:3
The union of two fuzzy (Anti- Fuzzy) normal subgroups of a group G is a fuzzy (Anti- Fuzzy) normal subgroup is connected.
We get, $\mu_1 \subseteq \mu_2$ and $\mu_2 \subseteq \mu_1$.
We get, The union of two fuzzy normal (Anti- Fuzzy) subgroups of a group G is a fuzzy normal (Anti- Fuzzy) subgroup is Connected.
Thus Proved.
Corollary :
The theorem is true for n =2,3,4,....n.

Theorem: 3.19.

The union of "n" fuzzy (Anti- Fuzzy) normal subgroups of a group G is a fuzzy (Anti- Fuzzy) normal sub-"n" group is Connected.

Proof:
By using the following theorem:4
The union of "n" fuzzy (Anti- Fuzzy) normal subgroups of a group G is a fuzzy (Anti- Fuzzy) normal sub "n" group is connected.

We get, $\mu_1 \subseteq \mu_2$ and $\mu_2 \subseteq \mu_1$..........$\mu_1 \subseteq \mu_n$ and $\mu_n \subseteq \mu_1$.

We get, The union of fuzzy (Anti- Fuzzy) normal sub "n"groups of a group G is a fuzzy (Anti- Fuzzy) normal sub"n" group is Connected.

Thus Proved.

Theorem: 3.20.

Let μ_1 be a connected fuzzy (Anti- Fuzzy) normal subgroups of μ. if $\mu_1 \subset \mu_2 \subset \bar{\mu}_1$. Then μ_2 is also connected fuzzy (Anti- Fuzzy) normal subgroups of μ.

Proof:

Let μ_1 be a connected fuzzy (Anti- Fuzzy) normal subgroups of μ is also a fuzzy (Anti- fuzzy) subgroup and let $\mu_1 \subset \mu_2 \subset \bar{\mu}_1$.

By using the theorem :5

Let μ_1 be a connected fuzzy (Anti- Fuzzy) normal subgroups of μ. if $\mu_1 \subset \mu_2 \subset \bar{\mu}_1$. Then μ_2 is also connected fuzzy (Anti- Fuzzy) normal subgroups of μ.

Hence μ_2 is also connected fuzzy (Anti- Fuzzy) normal subgroups of μ.

Results:

1. μ is connected which implies $\bar{\mu}$.
2. A finite Cartesian product of connected fuzzy normal subgroups is connected.

IV. CONCLUSION

I have derived the definitions of Connectedness and seperations of fuzzy (Anti-fuzzy) Subgroups and derive some theorems on Compactness and Connectedness and shows some results on Compactness and Connectedness and define some Definitions related to compactness, and shows that the set is compact and their union is also compact , compact properties and shown the composition properties and note Related to Compactness in Fuzzy (Anti-Fuzzy) subgroups and Fuzzy (Anti-Fuzzy) normal subgroups

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