

Passive Dynamic Bounding Control using Symmetry Condition Control Laws

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ABSTRACT

Legged locomotion is preferred over the wheeled locomotion as it can be used both for flat and rough terrains. Quadruped robots are preferred since they can offer better stability with considerable reliability. In recent years, passive dynamics has been used to obtain near zero-energy bounding gaits. Although theoretically such gaits consume no energy, in practice some additional energy is required to overcome losses. Existence and stability of such gaits have been thoroughly studied in literature for quadruped models with the assumption that the mass distribution and stiffness in the front and back legs are symmetric. Fixed points found using Poincare map indicate touchdown angle-liftoff angle symmetry between front and back legs. This property can be used to search for fixed points with ease. However, the range of initial conditions where the bounding gait is stable is highly limited. Control laws based on symmetry conditions observed are proposed in this paper to improve the stability region. One such control law based on body-fixed touchdown angles theoretically allows redesign of quadruped robot with physical cross coupling between legs to achieve inherent stability without leg actuation.

Keywords : Dynamic bounding, Robot, Stability.

I. INTRODUCTION

Passive dynamics means dynamical behavior of actuators, robots, or when there is no active supply of energy to achieve the motion. In legged robots design and more relaxed control of passive dynamics has become a complementary (or even alternative) approach to joint-positioning control methods. In terms of Ioannis Poulakakis, passive dynamics means the unforced response of a system under a set of initial conditions. In general, characterizing the properties and conditions of the passive behavior and identifying regions of the model parameters where the system can passively stabilize itself, can lead to designing controllers, which are not entirely based on continuous state-feedback like computed-torque

controllers [1,2]. Simulations and analysis suggest that suitably designed legged machines will be able to run passively i.e. without actuation and control. However, due to practical limitations, there are no legged robots which operate completely passively, except McGeer's passive dynamic walker bipeds [3, 4].

Smith and Berkemeier extended McGeer's work from bipedal to quadrupedal locomotion [5]. While running, the leg acts as a spring compressing during contact with ground phase and decompressing during the reactive phase. The Spring Loaded Inverted Pendulum (SLIP) system has been reported in [6, 7, 8, 9, 10]. In the SLIP concept the authors explained, the kinetic and gravitational potential energies are stored as elastic energy in the spring at the contact phase and

recovered in the reactive phase. Higher speeds can be achieved because of the compression of the spring, so that the leg remains in contact with the ground. Raibert used the SLIP model to develop controllers to stabilize the legged robots. An analytical study of the SLIP model can be found in [11].

Passive dynamic bounding gaits are periodic gaits that begin at stable or unstable initial conditions called fixed points. Such a gait when started with some initial state at the beginning of a gait cycle will end at state which is identical to the initial state (except for the horizontal distance). These gaits over a flat and level surface do not consume any additional energy for locomotion if the gait is self-stabilizing [12, 13]. This means, the Cost of Transport is theoretically zero.

Stable gaits do not require any control input and can tolerate disturbances (i.e., are self-stabilizing). Unstable gaits can be stabilized by the application of appropriate control inputs. Whether a periodic gait is stable or unstable is determined by the eigenvalues of Poincare map. While self-stabilizing gaits are quite attractive to implement, the region of initial conditions (fixed points) where they exist is limited. Controllers for stabilizing gaits starting from unstable fixed points is an active area of research [14].

Passive dynamic bounding gaits with either stable or unstable fixed points, show certain symmetry properties. In this paper, control laws for stabilizing the passive dynamic bounding gaits based on the symmetry of fixed points are introduced and studied. Implementation of control law in body fixed touchdown angles by means of physical cross-coupling without using a controller is also discussed. With the addition of feedback of pitch angle in the control law, it is shown that the stability region is considerably increased. These control laws require that the gait does not have a double support phase.

II. QUADRUPEL ROBOT MODEL FOR PASSIVE DYNAMIC BOUNDING

Since bounding gait is a planar gait, the model of quadruped robot considered is planar with body and two mass-less telescopic legs with identical springs on them. The mass less legs are connected to the robot at the hip through revolute joints. The distribution of mass in the robot body is assumed to be uniform so that the center of mass is the geometric center. Figure 1 shows the schematic along with notation.

Each gait cycle of bounding can consist of four phases: flight phase, back-leg support phase, double support phase, and front-leg support phase. In this work, we do consider gaits that do not have double support phase. Various phases of the bounding gait are shown in Figure 2. In the flight phase 1 prior to the back-leg support phase, the back leg is controlled such that at the time of touching the ground it makes a back-leg touch down angle γ_b^{td} with the vertical.

During back-leg support phase the back-leg spring compresses and decompresses. As soon as the length of the leg equals to the free length l_0 , back support phase ends and the robot lifts

off the ground at lift-off angle γ_b^{lo} to flight phase. Similarly, during the flight phase 2 prior to the front-leg support phase, the front leg is controlled such that at the time of touching the ground, it makes a front leg touchdown angle of γ_f^{td} with the vertical. Again when it lifts off the ground, it does so at a liftoff angle γ_f^{lo} . Since the legs are assumed to be massless, control action for touchdown does not influence the robot dynamics.

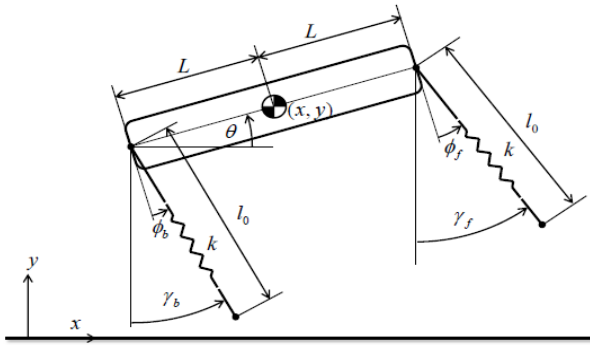


Figure 1: Schematic of the quadruped robot

where F_x and F_y are the forces exerted by the back-leg or front-leg on the robot body at the hip joint, and r_x and r_y are the coordinates of the back or front hip joint with respect to the body center of mass. The forces F_x and F_y are calculated from the compression of the spring. If l is the length of the leg, then the spring force is given by $k(l_0 - l)$. The direction of this force is along the leg where F_x and F_y are the components of this force along x and y -axes respectively. For double support phase, forces and moments on the right hand side of the equations of motion are the sum of components of front and back leg spring forces and moments.

While the stiffness k and free length l_0 are constants, the actual length l is calculated as follows:

$$l = \sqrt{(x_{tip} - x + L \cos \theta)^2 + (y - L \sin \theta)^2} \quad (7)$$

where x_{tip} is point on the ground where the tip of the back or front leg is in contact.

B. Touchdown and Liftoff Events

The transition between phases occur at the touchdown and the liftoff events. There are two touchdown events (back leg touchdown and front leg touchdown) and two liftoff events (back leg liftoff and front leg liftoff). Conditions for event detection of back and front leg touchdown events respectively are given below:

$$y - L \sin \theta - l_0 \cos \gamma_b^{td} = 0 \quad (8)$$

$$y + L \sin \theta - l_0 \cos \gamma_f^{td} = 0 \quad (9)$$

Similarly, the conditions for event detection of back and front leg liftoff events respectively are given below:

$$l_0 - \sqrt{(x_{tip} - x + L \cos \theta)^2 + (y - L \sin \theta)^2} = 0 \quad (10)$$

$$l_0 - \sqrt{(x_{tip} - x - L \cos \theta)^2 + (y + L \sin \theta)^2} = 0 \quad (11)$$

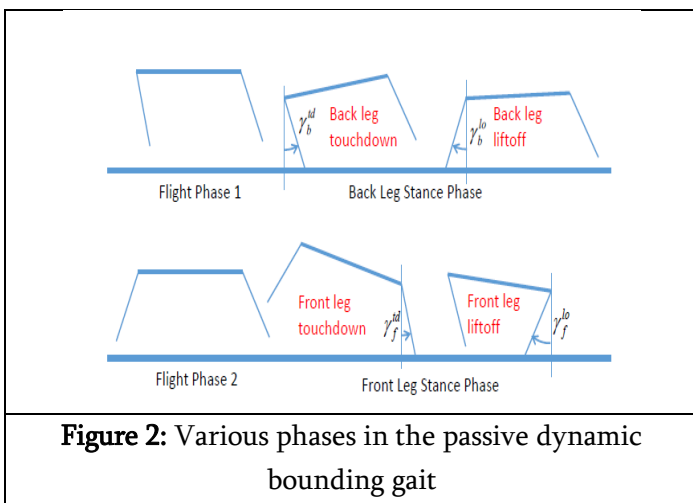


Figure 2: Various phases in the passive dynamic bounding gait

A. Equations of Motion

During flight phase, the equations of motions are

$$m\ddot{x} = 0; \quad (1)$$

$$m\ddot{y} = -mg; \quad (2)$$

$$I_G \ddot{\theta} = 0; \quad (3)$$

where x and y are the coordinates of the center of mass of the robot body, and θ is the angle made by the longitudinal axis of the body with the horizontal.

During back-leg or front-leg support phase, the equations of motion are

$$m\ddot{x} = F_x \quad (4)$$

$$m\ddot{y} = -mg + F_y; \quad (5)$$

$$I_G \ddot{\theta} = r_x F_y - r_y F_x; \quad (6)$$

where x_{btip} and x_{ftip} are the back and front tip contact points during the back and front leg support phases.

C. Finding Fixed Points and Stability

Legged robots are hybrid systems with discrete transformations governing transitions from one phase to another phase of motion [13,15]. Hence, a Poincare return map is used to determine orbital stability of the trajectory. If apex height during flight phase, where $\dot{y} = 0$, is chosen as the initial condition, dimension reduction of Poincare section is possible. Further reduction is obtained by removing horizontal coordinate x of the center of mass since it increases monotonically and is not relevant to a periodic trajectory. We are left with four variables at apex height: $y, \theta, \dot{x}, \text{ and } \dot{\theta}$

If apex event during flight phase is taken as the initial condition for a gait cycle, the final state after one gait cycle at apex event should be identical (except for the horizontal displacement x) to the initial state if the gait cycle is periodic. A Poincare return map can be defined mapping initial and final states:

$$X_{n+1} = P(X_n) \quad (12)$$

Equation (12) can be rearranged to define a function whose roots satisfy the periodicity condition.

$$X - P(X) = 0 \quad (13)$$

Roots of (13) are called the fixed points. For the given back and front touchdown angles, Newton-Raphson method can be used to search for the roots of (13), provided searching starts at a good initial guess. There are two different ways of finding fixed points using Newton-Raphson method. For a detailed description of these ways, refer to [13].

Stability of fixed points so found can be determined from the eigen values of Jacobian matrix of return map P . One of the eigen values is always unity,

indicating the conservative nature of the system [13]. Stability of a fixed point depends on whether the remaining eigen v values are within unit circle (stable) or outside the unit circle (unstable).

All the fixed points, stable or unstable, share two common properties: pitch angle at apex is zero, and touchdown liftoff angle symmetry. This latter condition of symmetry can be described as follows:

$$\gamma_b^{td} = -\gamma_f^{lo}, \quad \gamma_f^{td} = -\gamma_b^{lo} \quad (14)$$

D. Symmetry Condition Control Law with Absolute Touch-down Angles

Corresponding to every fixed point $y, \theta, \dot{x}, \text{ and } \dot{\theta}$, there exists at least a pair of touchdown angles which allow the gait cycle to be periodic. When a bounding gait starts from a stable fixed point, maintaining the touchdown angles corresponding to the fixed point, every gait cycle allows the gait to continue indefinitely. The same is not true with unstable fixed points because any small error in the fixed point grows rapidly till the gait fails. It is possible to stabilize unstable fixed points by using control law based on known fixed point and the error in liftoff angle [16].

Another way of stabilizing a fixed point is reported in [17], where the control law is based on touchdown angle liftoff angle symmetry condition in (14). The advantage of this method is that it does not require the use of known fixed point in the control law. Algorithm for the control law is as follows:

1. Start with apex initial conditions for $y, \theta, \dot{x}, \text{ and } \dot{\theta}$
 - θ Should be zero as this is the property of fixed points.
 - $\dot{\theta}$ Is positive so that back leg touchdown happens first.

2. End the flight phase with some back leg touchdown angle if this is the first gait cycle or with the negative of front leg liftoff angle of the previous iteration if this is not the first gait cycle.
3. Measure and store the back leg liftoff angle after the back leg stance phase.
4. End the flight phase after the back leg stance phase with front leg touchdown angle taken as the negative of back leg liftoff angle measured in 3.
5. Measure and store the front leg liftoff angle after the front leg stance phase.
6. Go to 2.

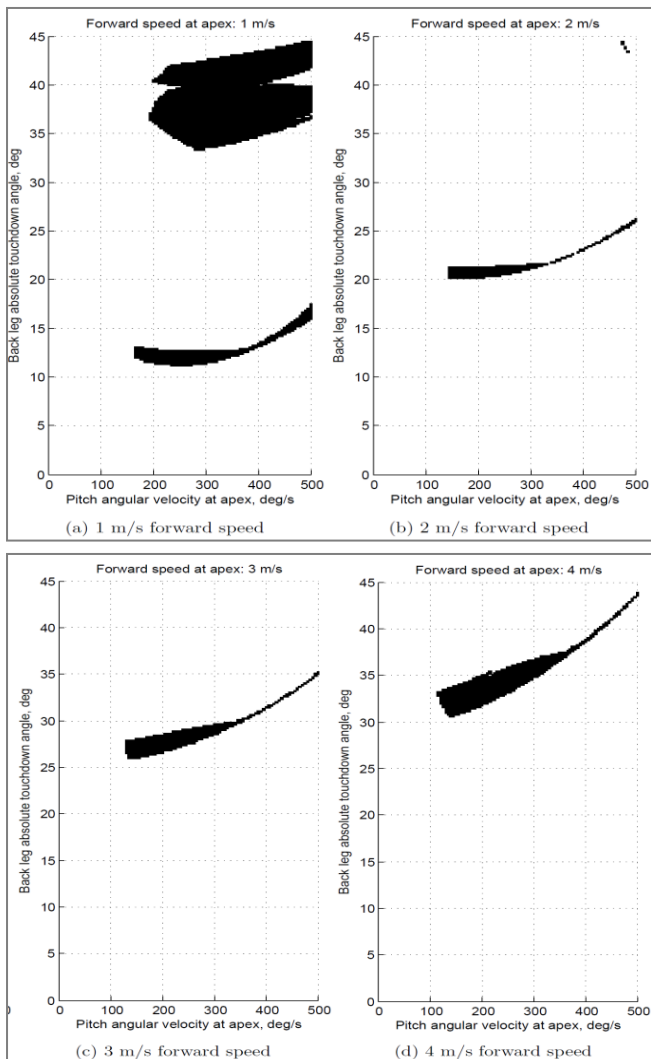


Figure 3: Stability region with back leg absolute touchdown angle vs pitch angular velocity at apex for apex height of 0.35 m

The passive dynamic bounding is considered failed if the liftoff does not happen within a reasonable time or double support occurs. Figure 3 shows the stability region for back leg touchdown angles at various pitch angular velocities and forward speeds when the apex height is 0.35 m. An initial condition is considered stable if the bounding does not fail for 200 gait cycles.

E. Symmetry Condition Control Law with Body-Fixed Touchdown Angles

An additional property that has been observed is that the touchdown liftoff angles at fixed points show symmetry not only in terms of absolute angles measured with respect to the vertical, but also with local or relative angles measured with respect to the robot body.

$$\phi_b^{td} = \phi_f^{lo}, \quad \phi_f^{td} = -\phi_b^{lo} \quad (15)$$

Using (15) in (14),

$$\gamma_b^{tf} = -\gamma_f^{lo}, \quad \gamma_f^{td} = -\gamma_b^{lo} \quad (16)$$

$$\phi_b^{td} + \theta_b^{td} = -(\phi_f^{lo} + \theta_f^{lo}), \quad \phi_f^{td} + \theta_f^{td} = -(\phi_b^{lo} + \theta_b^{lo}) \quad (17)$$

$$\theta_b^{td} = -\theta_f^{lo}, \quad \theta_f^{td} = -\theta_b^{lo} \quad (18)$$

From (18), it is clear that symmetry condition exists even for the body pitch angle.

Instead of using touchdown angles measured with respect to absolute vertical, touchdown angles measured with respect to body can also be used for control. Body-fixed touchdown angles have several advantages compared to absolute touchdown angles as follows [18]:

1. No need to measure body pitch angle in order to maintain touchdown angle.
2. No active control is required during the flight phase in order to obtain the desired leg angle at touchdown.

Figure 4 shows the stability region with back leg relative touchdown angle versus pitch angular velocity at apex for various forward speeds. No stability region could be found at higher forward speeds of 3 and 4 m/s. Comparing Figure 3 and Figure 4; it is clear that use of absolute touchdown angles with symmetry condition control law gives larger stability region. However, the advantage of easy implementation of body fixed touchdown angles is attractive when we consider controller-less system discussed in the next section.

F. Inherent Stability with Physical Cross Coupling

The idea of control using symmetry condition directly as proposed in [17] is more useful if body-fixed touchdown angles are used instead of absolute touchdown angles. In addition to the advantages of body-fixed touchdown angles, there is an additional advantage of physical cross coupling (shown in Figure 5) in implementing the symmetry control law. Touchdown angle once set need not be changed for a stable gait. If the back leg touchdown happens first, the front leg will be set to proper front leg touchdown angle when the back leg lifts off. During the flight phase, the legs should be locked from changing the angle by using a brake.

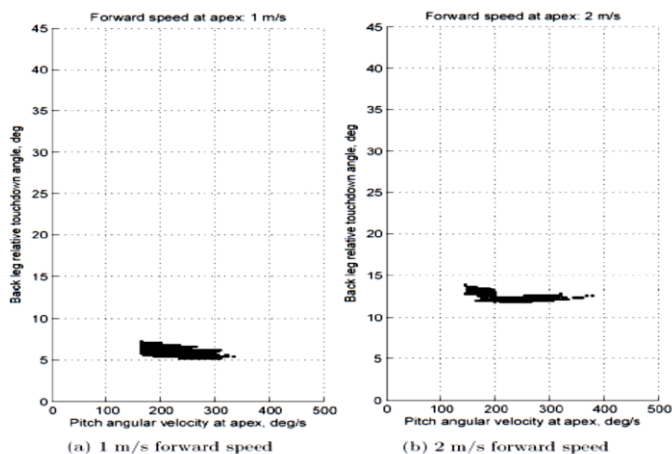


Figure 4: Stability region with back leg relative touchdown angle vs pitch angular velocity at apex for apex height of 0.35 m

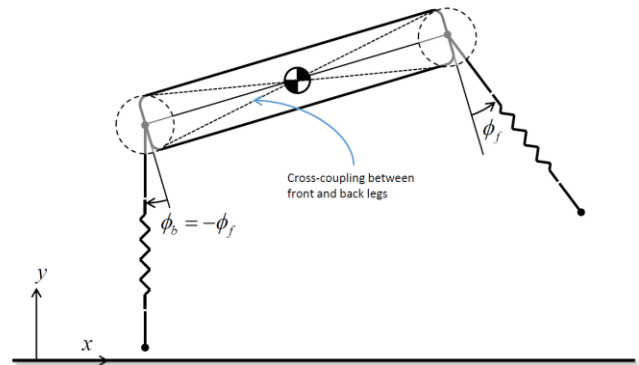


Figure 5: Quadruped robot with front and back leg coupled

The brake is released when the front leg touchdown happens. Similarly, when the front leg lifts off, the back leg will be in correct back leg touchdown angle. There is a limitation introduced by the cross coupling of the legs. When both the legs are in contact with the ground, the robot body and the two legs form a four bar mechanism with the ground as a fixed link. The motion of the robot requires both the legs to rotate in the same direction about their respective contact points. This does not satisfy the symmetry condition. Hence, double support phase is not allowed when legs are physically cross coupled.

III. CONCLUSION

Although a conclusion may review the main points of the paper, do not replicate the abstract as the conclusion. A conclusion might elaborate on the importance of the work or suggest applications and extensions. Authors are strongly encouraged not to call out multiple figures or tables in the conclusion—these should be referenced in the body of the paper.

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