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Binary Regular ^ Generalized Closed Sets In Binary Topological Spaces

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ABSTRACT

In this article, we introduce binary regular ^ generalized closed sets **(shortly μ_b r^g-closed sets)** in binary topological spaces. Also we examine some of its properties. Furthermore we study the relationship between these binary r^g-closed sets with other binary closed sets in binary topological spaces.

Keywords : Binary generalized topology, binary r^g-closed sets, binary r^g-open sets

I. INTRODUCTION

In 2011, the authors [3], introduced the notion of binary topological spaces and studied some basic characteristics. In prolongation, some researchers have found some other binary closed sets and linked fundamental properties which are defined over two master sets with appropriate parameters [3].

If A is a subset of X and B is a subset of Y , then the topological structures on X and Y provide a little information about the ordered pair (A, B). In 2011, S. Jothi S.N. [3] introduced a single structure which carries the subsets of X as well as the subsets of Y for studying the information about the ordered pair (A, B) of subsets of X and Y. Such a structure is called a binary structure from X to Y . Mathematically a binary structure from X to Y is defined as a set of ordered pairs (A, B) where $A \subseteq X$ and $B \subseteq Y$.

In continuation, in this article, we have defined and explored several properties of binary regular ^ generalized closed sets.

II. PRELIMINARIES

Definition 2.1[2]: Let X and Y be any two non empty sets. A binary generalized topology from X to Y is a binary structure $\mu_b \subseteq P(X) \times P(Y)$ that satisfies the following axioms:

(i) $(\phi,\phi) \in \mu_b$ and $(X,Y) \in \mu_b$.

(ii) $(A_1 \cap A_2, B_1 \cap B_2) \in \mu_b$ whenever (A_1,B_1) and $(A_2,B_2) \in \mu_b$

(iii) If $\{(A_\alpha,B_\alpha) : \alpha \in \Delta\}$ is a family of members of μ_b, then $(\cup A_\alpha, \cup B_\alpha) \in \mu_b$.

If μ_b is a binary generalized topology from X to Y then the triplet (X, Y, μ_b) is called a binary generalized topological space and the members of μ_b are called binary generalized open sets.

The compliment of an element of $P(X) \times P(Y)$ is defined component wise. That is the binary compliment of (A, B) is (X – A, Y – B).

Definition 2.2[3]: Let (X, Y, μ_b) be a binary generalized topological space and $A \subseteq X$, $B \subseteq Y$. Then (A, B) is called binary generalized closed if (X – A, Y – B) is binary generalized open.

Definition 2.3[2]: Let (A,B), (C,D) $\in P(X) \times P(Y)$. Then

(i) (A, B) $\subseteq$ (C, D) if A $\subseteq$ C and B $\subseteq$ D.
(ii) (A, B) $\cup$ (C, D) =(A $\cup$ C, B $\cup$ D).
(iii) (A, B) $\cap$ (C, D) =(A $\cap$ C, B $\cap$ D).

Definition 2.4[2]: Let (X, Y, μ_b) be a binary generalized topological space and *(x, y)* $\in$ X $\times$ Y, then a subset (A, B) of (X, Y) is called a binary generalized neighbourhood of *(x, y)* if there exists a binary generalized open set (U, V) such that *(x, y)* $\in$ (U, V) $\subseteq$ (A, B).

Definition 2.5[2]: Let (X, Y, μ_b) be a binary generalized topological space and A $\subseteq$ X, B $\subseteq$ Y. Let $(A, B)^{1^*} = \cap\{A_\alpha$:(A_α, B_α)is binary generalized closed and (A, B) $\subseteq$ (A_α, B_α)} and $(A, B)^{2^*} = \cap\{B_\alpha$:(A_α, B_α)is binary generalized closed and (A, B) $\subseteq$ (A_α, B_α)}. Then the pair ($(A, B)^{1^*}$,$(A, B)^{2^*}$) is called the binary generalized closure of (A,B) and denoted by μ_bCl(A,B).

Remark 2.6[2]: The binary generalized closure μ_bCl(A,B) is binary generalized closed such that (A,B)$\subseteq\mu_b$Cl(A,B).

Definition 2.7[2]: Let (X, Y, μ_b) be a binary generalized topological space and A $\subseteq$ X,B $\subseteq$ Y. Let $(A, B)^{1^\circ} = \cup\{A_\alpha$: (A_α, B_α) is binary generalized open and (A_α, B_α) $\subseteq$ (A, B)} and $(A, B)^{2^\circ} = \cup\{B_\alpha$: (A_α, B_α)is binary generalized open and (A_α, B_α) $\subseteq$ (A, B)}. Then the pair ($(A, B)^{1^\circ}$,$(A, B)^{2^\circ}$) is called the binary generalized interior of (A,B) and denoted by μ_bInt(A,B).

Remark 2.8[2]: The binary generalized interior μ_bInt(A,B) is binary generalized open such that μ_bInt(A,B) $\subseteq$ (A,B).

Definition 2.9[2]: A subset (A,B) of topological space (X, Y, μ_b) is called a binary regular open set **(shortly μ_b regular open set)** if (μ_bInt($\mu_b$cl A,B)) $\subseteq$ (A,B).

Definition 2.10[2]: A subset (A,B) of topological space (X, Y, μ_b) is called a binary pre closed set **(shortly μ_b preclosed set)** if $\mu_b$cl (μ_bInt(A,B) $\subseteq$ (A,B).

Definition 2.11[5]: A subset (A,B) of topological space (X, Y, μ_b) is called a binary semi closed set **(shortly μ_b semi closed set)** if μ_bInt ($\mu_b$cl (A,B) $\subseteq$ (A,B).

Definition 2.12[5]: A subset (A,B) of topological space (X, Y, μ_b) is called a binary semi preclosed set **(shortly μ_b semi preclosed set)** if $\mu_b$cl (μ_bInt ($\mu_b$cl (A,B) $\subseteq$ (A,B)).

Definition 2.13[5]: A subset (A,B) of topological space (X, Y, μ_b) is called a
(i) $\mu_b$gclosed set if $\mu_b$gcl(A,B) $\subseteq$ (U,V) whenever (A,B) $\subseteq$ (U,V) and (U,V) is open in (X, Y, μ_b)
(ii) μ_bg*closed set $\mu_b$cl(A,B) $\subseteq$ (U,V) whenever (A,B) $\subseteq$ (U,V) and (U,V) is μ_b g-open in (X, Y, μ_b)
(iii) $\mu_b$rgclosed set if $\mu_b$cl(A,B) $\subseteq$ (U,V) whenever (A,B) $\subseteq$ (U,V) and (U,V) is μ_b regular open in (X, Y, μ_b).

Definition 2.14[5]: Let (X, Y, μ_b) be a binary topological space. Let (A,B) $\subseteq$ (X,Y). Define (A,B,$\mu_{A,B}$) = {(A$\cap$U, B$\cap$V) : (U,V) $\in\mu_b$}. Then $\mu_{A,B}$ is a binary topology from A to B. The binary topological space (A,B,$\mu_{A,B}$) is called a binary subspace of (X, Y, μ_b).

III. BINARY REGULAR ^ GENERALIZED CLOSED SETS IN BINARY TOPOLOGICAL SPACES

In this section, we begin with the definition of binary regular ^ generalized closed set in binary topological spaces.

Definition 3.1: Let (X,Y,μ_b) be a binary topological space. Let (A,B) $\subseteq$ (X,Y). Then (A,B) is called a binary regular ^ generalized closed set **(shortly μ_br^g-closed set)** if there exists a binary regular open set (U,V) such that $\mu_b$gcl(A,B) $\subseteq$ (U,V) whenever (A,B) $\subseteq$ (U,V).

Example 3.2: Consider X = {a,b} , Y = {1,2}. Clearly μ_b = {(ϕ,ϕ),(ϕ,{1}),({a},{1}),({b},ϕ), ({b},{1}),(X,{1}},(X,ϕ),(X,Y)} is a binary topology from X to Y. μ_b^c = {(ϕ,ϕ),(ϕ,Y), (ϕ,{2}),({a},{2}),({a},Y)({b},{2}),(X,{2}),(X,Y). Let (A,B) = ({b},{1}), (U,V)= (X,Y) is a binary regular open set, then gCl(A,B) $\subseteq$ (X,Y). Therefore (A,B) is a μ_br^g-closed set.

Definition 3.3: Let (X,Y,μ_b) be a binary topological space. Let (A,B) $\subseteq$ (X,Y). Then (A,B) is called a binary regular ^ generalized open set **(shortly μ_br^g open set)** if the compliment of (A,B) is μ_br^g closed set.

Theorem 3.4: The union of two μ_br^g-closed sets is also a μ_br^g-closed set.

Proof: Assume that (A,B) and (C,D) are two μ_br^g-closed sets. Let (A,B) $\cup$ (C,D) $\subseteq$ (U,V) where (U,V) is μ_b regular open. Then (A,B) $\subseteq$ (U,V) and (C,D) $\subseteq$ (U,V). Since (A,B) and (C,D) are μ_br^g-closed sets,

$\mu_b$gcl(A,B) ⊆ (U,V) and $\mu_b$gcl(C,D) ⊆ (U,V). Then $\mu_b$gcl (A,B) ∪ (C,D) ⊆ (U,V). Hence (A,B) ∪ (C,D) is also an μ_br^g-closed set.

Remark 3.5: The intersection of two μ_br^g-closed sets need not be an μ_br^g-closed set as seen in the following example.

Example 3.6: Let X = {a,b,c}, Y = {1,2}, μ_b = {(ϕ,ϕ),(ϕ,{1}),({a},{1}),({b},ϕ),({a,b},{1}), ({b,c},{2}),(X,Y)}.In the binary topological space (X,Y, μ_b), let A = ({a},{1}), B = ({b},{1}), then A and B are μ_br^g closed sets but A∩B = (ϕ,{1}) is not an μ_br^g closed set.

Theorem 3.7: Let (X,Y,μ_b) be a binary topological space and A ⊆ X, B ⊆ Y. If (A,B) is binary open in (X,Y,μ_b) then A^c is μ_br^g closed set in X and B^c is an μ_br^g closed set in Y.

Proof: By definition, we have μ_X = {A ⊆ X; (A,B) ∈ μ_b for some B ⊆ Y}is a topology on X and μ_Y = {A ⊆ X; (A,B) ∈ μ_b for some A ⊆ X}is a topology on Y. Since (A,B) is binary open in (X,Y,μ_b), we have A ∈ μ_X and B ∈ μ_Y. That is A is open in (X, μ_X) implies A^c is closed in (X, μ_X). Similarly B^c is closed in (Y, μ_Y). Every closed set is r^g closed set. Therefore A^c and B^c are r^g closed sets in (X, μ_X) and (Y, μ_Y) respectively.

Remark 3.8: The converse of the above theorem need not be true from the fact that every r^g closed sets need not be a closed set which is shown in the following example.

Example 3.9: Let X = {a,b}, Y = {1,2} μ_X = {ϕ, X, {a}} and μ_Y = {ϕ, Y, {2}). In (X, μ_X), {b} is an r^g closed set and ϕ is an r^gclosed set in (Y,μ_Y) but ({a}, ϕ) is not a binary open set in (X, Y, μ_b).

Theorem 3.10: Every (i) closed set
(ii) μ_bg closed set
(iii) μ_bg*closed set is μ_br^g closed set.

Proof: Proof is straight forward from the fact that every $\mu_b$closed , $\mu_b$gclosed and μ_bg*closed sets are μ_br^g closed sets.

Remark 3.11: The converse of the above theorem need not be true as seen in the following example.

Example 3.12: Let X = {a,b,c}, Y = {1,2} μ_b = {(ϕ,ϕ), (ϕ,{1}), ({a},{1}), ({b},ϕ), ({b},{1}), ({a,b},{1}),({b,c},{2}), (X,Y)). In binary topological space (X,Y, μ_b), let A = (ϕ,{1}), A is μ_br^g closed set but it is not $\mu_b$closed set. Let B = ({b,c},ϕ) then B is μ_br^g closed but it is not both $\mu_b$glosed and μ_bg*closed set.

Theorem 3.13: Let (A,B) be a binary r^g closed set in a binary topological space (X,Y, μ_b) and suppose (A,B) ⊆ (C,D) ⊆ $\mu_b$gcl(A,B). Then (C,D) is a binary r^g closed set.

Proof: Since (A,B) is a binary r^g closed set, there exists a binary open set (U,V) such that $\mu_b$gcl(A,B) ⊆ (U,V). Since (C,D) ⊆ $\mu_b$gcl(A,B), $\mu_b$gcl(C,D) ⊆ $\mu_b$gcl($\mu_b$gcl(A,B)) i.e., $\mu_b$gcl(C,D) ⊆ $\mu_b$gcl(A,B) ⊆ (U,V). Therefore (C,D) is also a binary r^g closed set.

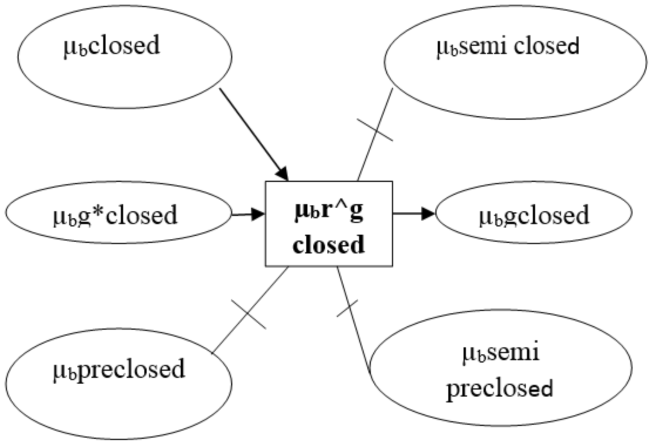
Remark 3.14: The concept of μ_br^g closed sets and $\mu_b$preclosed sets are independent to each other as shown in the following example.

Example 3.15: In example 3.2, in the binary topological space (X,Y,μ_b), let A = ({a}, ϕ) then A is binary preclosed but it is not binary r^g closed set. Similarly B = ({b},{1}), then B is μ_br^g closed but it is not $\mu_b$preclosed set.

Remark 3.16: The concept of $\mu_b$semiclosed sets and $\mu_b$semipreclosed sets are independent to the concept of μ_br^g closed sets as seen in the following example.

Example 3.17: Let X = {a,b,c}, Y = {1,2}, μ_b= {(ϕ,ϕ),({a},ϕ),({b},ϕ),({a,b},ϕ),(X,ϕ), (X,{1}), (X,{2}),(X,Y)}. In the binary topological space (X,Y,μ_b), let A = ({a},ϕ) then A is both $\mu_b$semiclosed and $\mu_b$semipreclosed set but it is not μ_br^g closed set. Let B = ({a,b},ϕ), then B is μ_br^g closed set but it is not both $\mu_b$semiclosed and $\mu_b$semipreclosed sets.

The above discussions are implemented in the following diagram.



where A → B represents A implies B but not conversely and A ⟷ B represents A and B are independent.

Theorem 3.18: Let (X,Y,μ_b) be a binary topological space and $(A,B,\mu_{A,B})$ is a binary subspace of (X,Y,μ_b). Let (C,D) be a $\mu_b r$^g closed set in (X,Y,μ_b) and $(C,D) \subseteq (A,B)$. Then (C,D) is $\mu_b r$^gclosed set in $(A,B,\mu_{A,B})$.

Proof: Since (C,D) is a $\mu_b r$^g closed set in (X,Y,μ_b), we have $\mu_b gcl(C,D) \subseteq (U,V)$ where (U,V) is binary regular open in (X,Y, μ_b) and hence it should be in μ_b. By definition of a binary subspace, $(U\cap A, V\cap B) \in \mu_{A,B}$. Let (U,V) is a $\mu_{A,B}$regular open set, $(C,D) \subseteq (A,B)$, and $(C,D) \subseteq (U,V)$, $\mu_{A,B}gcl(C,D) = \mu_{A,B}gcl(C\cap A,D\cap B) \subseteq \mu_{A,B}gcl(U\cap C,V\cap D) \subseteq (U,V)$. Thus (C,D) is a $\mu_b r$^gclosed set in $(A,B,\mu_{A,B})$.

Remark 3.19: The converse of the above theorem need not be true as shown in the following example.

Example 3.20: Let X = {a,b,c}, Y = {1,2} Clearly μ_b = {(ϕ,ϕ),(ϕ,{2}), ({b},{1}),({b},Y) ,({c},{2}),({b,c},Y),(X,{2}),(X,Y)} is a binary topology from X to Y. Also {(ϕ,ϕ),(ϕ,{1}), ({a},ϕ),({a,b},{1}),({a,c},ϕ),({a,c},{2}),(X,{1}(X,Y)} are closed sets of (X,Y,μ_b). Let (A,B) = ({a,b},{1,2}) which is a subset of (X,Y). Clearly $\mu_{A,B}$ = {(ϕ,ϕ),(ϕ,{2}),({b},{1}),({b},B), (A,{2}), (A,B)} is a binary topology from A to B. The binary closed sets of $(A,B,\mu_{A,B})$ are {(ϕ,ϕ),(ϕ,{1}),({a},ϕ),({a},{2}),(A,{1}),(A,B)}Consider (C,D) = ({b},{1}) ⊆ (A,B) . Clearly ({b},{1}) ⊆ ({a,b},{1,2}) is a $\mu_{A,B}r$^g closed set in $(A,B,\mu_{A,B})$ but (C,D) is not a $\mu_b r$^g closed set in (X,Y,μ_b).

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