

## FE Based Vibration and Stability Analysis of Functionally Graded Rotating Shaft System Under Thermal Environment

Gavvala Somanjineyulu<sup>1</sup>, I. Balasubramanyam<sup>2</sup>

<sup>1</sup>M.Tech Student, CAD/CAM, Department of Mechanical Engineering, PVKK Institute of Technology, Ananthapuramu, Andhra Pradesh, India

<sup>2</sup> HOD & Associate Professor, Department of Mechanical Engineering, PVKK Institute of Technology, Ananthapuramu, Andhra Pradesh, India

### ABSTRACT

The present work deals with the study of vibration and stability analyses of functionally graded (FG) spinning shaft system under thermal environment using three noded beam element based on Timoshenko beam theory (TBT). Temperature field is assumed to be a uniform distribution over the shaft surface and varied in radial direction only. Material properties are assumed to be temperature dependent and graded in radial direction according to power law gradation and exponential law gradation respectively. In the present analysis, the mixture of Aluminum Oxide (Al<sub>2</sub>O<sub>3</sub>) and Stainless Steel (SUS304) is considered as FG material where metal contain (SUS304) is decreasing towards the outer diameter of shaft. In this work the effects of both internal viscous and hysteretic damping have also been incorporated in the finite element model. The analysis of numerical results reveals that temperature field and power law gradient index have a significance role on the materials properties (such as Young modulus, Poisson ratio, modulus of rigidity, coefficient of thermal expansion etc.) of FG shaft. Various results have also been obtained such as Campbell diagram, stability speed limit (SLS), damping ratio and time responses for FG shaft due unbalance masses and also compared with conventional steel shaft.

**Keywords :** Power law gradient index; Functionally graded shaft; Temperature dependent material properties; Viscous and hysteretic damping; Rotor-Bearing-shaft system; Finite element method; Campbell diagram; Damping ratio; stability speed limit (SLS)

### I. INTRODUCTION

Composite materials and structures are more and more frequently used in advanced engineering fields mainly because of their high stiffness-to-weight ratio that is particularly favorable. However the main downside of composite materials is represented by the weakness of interfaces between adjacent layers known as delimitation phenomena that may lead to structural failure. To partially overcome these

problems, a new class of materials named Functionally Graded Materials (FGMs) has recently been proposed whose various material properties vary through the radial and thickness direction in a continuous manner and thus free from interface weakness. The gradation of material properties reduces thermal stresses, residual stresses, and stress concentrations. A functionally graded structure is defined as, those in which the volume fractions of two or more materials are varied continuously as a

function of position along certain dimension (typically the radius and thickness) of the structure to achieve a require function. FGMs can provide designers with tailored material response and exceptional performance in thermal environments. For example, the Space Shuttle utilizes ceramic tiles as thermal protection from heat generated during re-entry into the Earth's atmosphere. An FGM composed of ceramic on the outside surface and metal on the inside surface.

Due to high strength, high stiffness, and low density characteristics, FGMs rotor shafts have been sought as new potential candidates for replacement of the conventional metallic shafts in many application areas for the design of rotating mechanical components such as, driveshaft for helicopters and cars and jet engine, commercial and military rotating machines, aerospace and space vehicles etc. In Rotor-dynamic applications, composites have been demonstrated both numerically and experimentally. Accompanied by the development of many new advance composite materials and various mathematical models of rotor-shafts were also developed by researchers.

## II. METHODS AND MATERIAL

As FGMs are heterogeneous materials so there is need for the determination of effective material properties. To achieve best performance, accurate material property estimation is essential for analysis and design of FG structures/system. There are various models developed to determine the material properties of FGM such as Rules of mixtures employ bulk constituent properties assuming no interaction between phases. This approach derived from continuum mechanics and is free from empirical considerations. In variational approach, variational principles of thermo-mechanics used to derive the bounds for effective thermo-physical properties. Micromechanical approaches include information about spatial distribution of the constituent materials. Standard micromechanical approach is based on

concept of unit cell or representative volume element (RVE) to represent the microstructure of composite.

### 2.1 Modeling for Material Properties of FG Circular Cross-section

A FGM shaft is considered with finite length  $L$ , inner radius ( $r_i$ ) and outer radius ( $r_o$ ).

Material of the shaft is considered in top surface ( $z = r_i / 2$ ) ( $z = r_o / 2$ ) as metal as ceramic and in bottom surface.

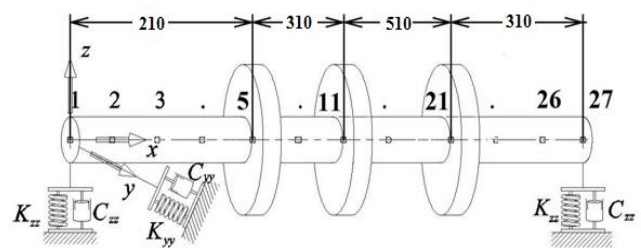


Fig. 1 Volume fraction of ceramic and metal throughout the FGM layer

### 2.2 Temperature Distribution in a FG Shaft

The temperature distribution in FG layer is nonlinear which is clearly shown in Fig. 1.

This is due to that of thermal conductivity, coefficient of thermal expansion, modulus of elasticity and density which are the function of the radius of shaft only. For  $k = 0$  and  $k = \infty$ , the temperature distribution is a straight line and does not depend upon the material properties of the shaft. For other values of  $k$ , the temperature distribution depends upon radial positions and material properties and also the law of gradation and it is presented in Table 1. From the Table 5.4, it is clear that from zero to one and one to ten values of  $k$ , temperature gradually decreases and increases respectively.

Table 1. Temperature variation of FG shaft for different radial position and power law gradient index (k )

r (m)	k	T <sub>1</sub> (K)	T <sub>2</sub> (K)	T <sub>3</sub> (K)	T <sub>4</sub> (K)	T <sub>5</sub> (K)	T <sub>6</sub> (K)
0.05	0.0	300	360	420	480	540	600
0.06	0.5	300	352.5886	409.6354	470.0942	533.6039	600
0.07	1.0	300	352.4281	407.8254	467.2561	531.0844	600
0.08	3.0	300	355.1254	410.7097	468.0016	529.4601	600
0.09	5.0	300	356.6653	413.3858	470.6490	530.5007	600
0.1	10	300	358.1465	416.2933	474.4676	533.2644	600

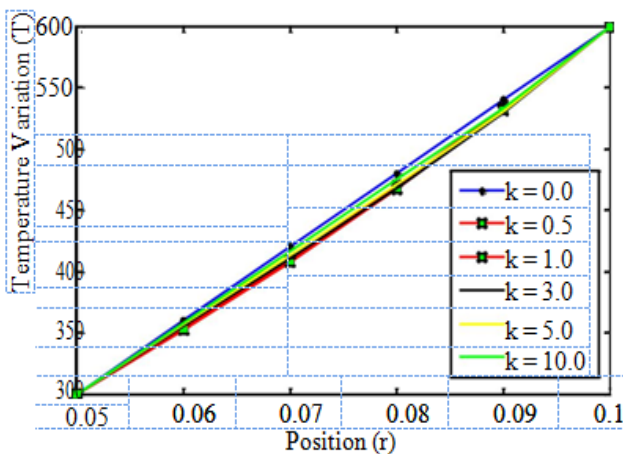


Fig 2. Temp. variation vs Position

### 2.3 Time Responses for FG Shaft System due to Unbalance Masses

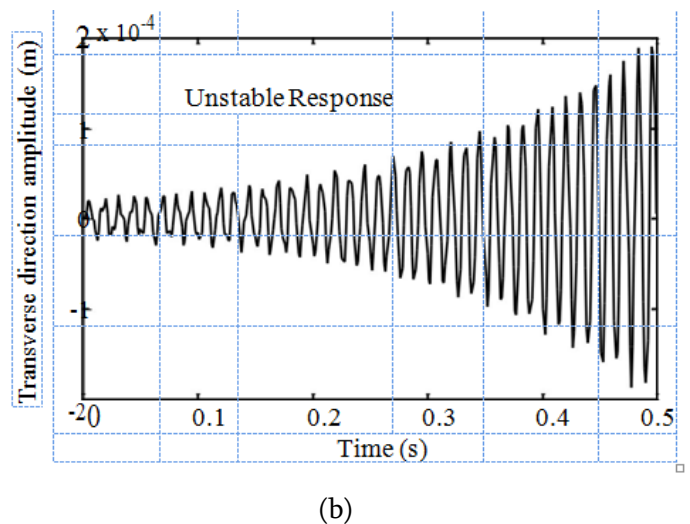
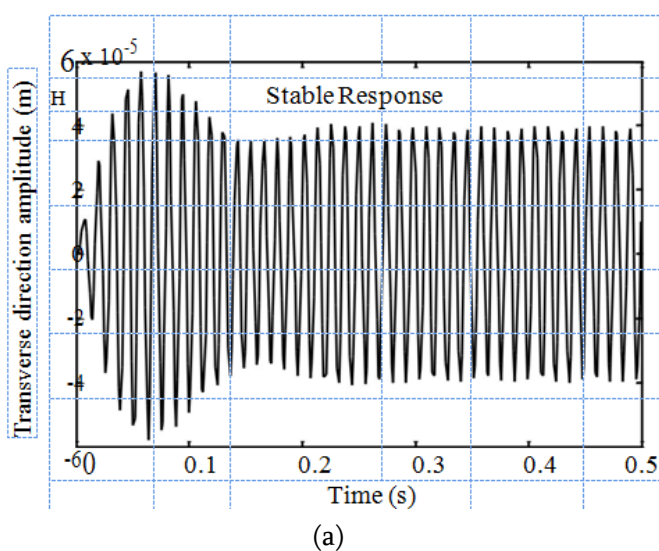
The Fig. 3 (a) and (b) show the displacement histories (stable and unstable responses) due to unbalance

masses in transverse directions for the index of  $k = 0.5$  and  $T = 300K$ . Here, the time responses in the transverse directions of this shaft have also been obtained considering the unbalance mass of the disk 2 and the responses of this system have been calculated with a time step of  $\tau/10$  s (where  $\tau$  is the time period corresponding to the first natural frequency of the system). Finally from the Fig. 3 (a) and (b), it is clear that for stable response the maximum amplitude is  $5.676 \cdot 10^{-5}$  m and for unstable response the maximum amplitude is  $1.897 \cdot 10^{-4}$  m and also from Table 5.9, it is found that for both the less values of the power law gradient index as well as the temperature variations, the maximum amplitude is less, thus it promotes more stable system than that of higher values of power law gradient and temperature variations.

Table 2. Maximum amplitudes for different temperatures and power law gradient index ( k )

		Stable response		Unstable response	
		Max. Amplitude (10 <sup>-5</sup> m)		Max. Amplitude (10 <sup>-4</sup> m)	
k	T(K)	V- direction	W- direction	V- direction	W- direction

0.0	300	8.762	6.935	2.406	2.004
	420	8.330	6.593	2.896	2.359
	600	7.980	6.351	3.801	3.205
0.5	300	7.164	5.676	2.207	1.897
	420	6.883	5.556	2.651	2.418
	600	6.371	5.291	3.297	2.806
1.0	300	6.188	5.005	2.530	2.229
	420	5.959	4.868	3.139	2.716
	600	5.391	4.729	3.832	3.168
3.0	300	4.230	3.844	5.327	5.023
	420	4.084	3.795	6.520	5.975
	600	3.950	3.629	7.137	6.727
5.0	300	3.579	3.268	10.06	9.428
	420	3.539	3.092	11.75	9.800
	600	3.376	3.023	13.68	12.67
10.0	300	2.806	2.729	23.55	21.92
	420	2.678	2.413	26.97	24.91
	600	2.602	2.202	31.43	29.41



**Fig. 3.** Displacement histories due to unbalance masses along the transverse direction of FG shaft: (a) stable response and (b) unstable response

### III. RESULTS AND DISCUSSION

From the comparison of various responses of the FG shaft without and with temperatures consideration, it has been noticed that the FG shaft is more stable in case of without temperature consideration than that of with temperature consideration.

The power law gradient index plays an important role in the responses (viz. Campbell diagram, damping ratio, critical speed, stability limit speed and time responses) of the FG shaft system.

It is observed that the less value of temperature and the power law gradient index promotes more stable system than that of higher values of temperature and power law gradient index

### IV. REFERENCES

- [1]. Kapuria, S., Ahmed, A., Dumir, P.C., 2004, "Static and dynamic thermo electromechanical analysis of angle ply hybrid piezoelectric beams using an efficient coupled zigzag theory," *Composites Science and Technology*, 64, pp. 2463-2475.
- [2]. Gubran, H.B.H., Gupta, K., 2005, "The effect of stacking sequence and coupling mechanisms on the natural frequencies of composite shafts," *Journal of Sound and Vibration*, 282, pp. 231-248.
- [3]. Wang, B.L., Mai, Y.W., 2005, "Transient one dimensional heat conduction problems solved by finite element," *International Journal of Mechanical Sciences*, 47, pp. 303-317.
- [4]. Syed, K.A., Su, C.W., Chan, W.S., 2007, "Analysis of Fiber Reinforced Composite Beams under Temperature Environment," *Proceedings of the Seventh International Congress on Thermal Stresses*, Taipei, Taiwan.
- [5]. Sino, R., Baranger, T.N., Chatelet, E., Jacquet, G., 2008, "Dynamic analysis of a rotating composite shaft," *Journal of Composites Science and Technology*, 68, pp. 337-345.
- [6]. Feldman, E., Aboudi, J. 1997, "Buckling analysis of functionally graded plates subjected to uniaxial loading," *Composite Structures*, 38, pp. 29-36.
- [7]. Aboudi, J., Pindera, M.J., Arnold, S.M., 1999, "Higher-order theory for functionally graded materials," *Composites, Part B: Engineering*, 30 (8), pp.777-832.
- [8]. Praveen, G.N.; Reddy, J. N., 1998, "Nonlinear transient thermo elastic analysis of functionally graded ceramic metal plates," *International Journal of Solids and Structures*, 35(33), pp. 4457-4476.
- [9]. Gasik, M.M., 1998, "Micromechanical modelling of functionally graded materials," *Computational Materials Science*, 13 (1), pp. 42-55.
- [10]. Kunuthur M.R., Reddy B.C. (2019) Investigation of Moisture Absorption in Jute Fiber Polymer Matrix Composites. In: Vasudevan H., Kottur V., Raina A. (eds) *Proceedings of International Conference on Intelligent Manufacturing and Automation. Lecture Notes in Mechanical Engineering*. Springer, Singapore DOI[https://doi.org/10.1007/978-981-13-2490-1\\_34](https://doi.org/10.1007/978-981-13-2490-1_34)
- [11]. Suresh, S., Mortensen, A., 1998, "Fundamentals of functionally graded materials", London, UK: IOM Communications Limited.
- [12]. Aboudi, J., Pindera, M.J., Arnold, S.M., 1999, "Higher-order theory for functionally graded materials," *Composites, Part B: Engineering*, 30 (8), pp.777-832.
- [13]. Mekala P., Kunuthur M.R., Chandramohana Reddy B. (2019) Evaluation of the Mechanical Properties of Recycled Jute Fiber-Reinforced Polymer Matrix Composites. In: Vasudevan H., Kottur V., Raina A. (eds) *Proceedings of International Conference on Intelligent Manufacturing and Automation. Lecture Notes*

in Mechanical Engineering. Springer, Singapore  
DOI [https://doi.org/10.1007/978-981-13-2490-1\\_26](https://doi.org/10.1007/978-981-13-2490-1_26)

- [14]. Wang, B.L., Han, J.C., Du, S.Y., 2000, "Crack problems for Functionally Graded Materials under transient thermal loading," *Journal of Thermal Stresses*, 23 (2), pp. 143-168.

**Cite this article as :**

Gavvala Somanjineyulu, I. Balasubramanyam, "FE Based Vibration and Stability Analysis of Functionally Graded Rotating Shaft System Under Thermal Environment", *International Journal of Scientific Research in Science, Engineering and Technology (IJSRSET)*, Online ISSN : 2394-4099, Print ISSN : 2395-1990, Volume 6 Issue 4, pp. 301-306, July-August 2019.

Journal URL : <http://ijsrset.com/IJSRSET196439>