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# Solving Fuzzy Sequencing Problem Using Triangular Fuzzy Numbers

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## ABSTRACT

In this paper, we proposed to solve the total time for the Fuzzy Sequencing Problem using Triangular Fuzzy Numbers. Here machines are represented as Fuzzy Numbers. In this Paper a Ranking function for Solving Fuzzy Sequencing Problem are represented where all the machines are in the form of Triangular Fuzzy numbers. Numerical examples show that the Fuzzy Ranking Method offers an effective tool for handling the Fuzzy Sequencing Problem

**Keywords :** Triangular fuzzy numbers, Membership function, Fuzzy Ranking, Fuzzy Sequencing.

## I. INTRODUCTION

Sequencing problem is considered to be one of the classic and important applications of operations research. The main role of the classical sequencing problem is to find the optimal sequence of the jobs on machines so as to minimize the total amount of time required to complete the process of all the jobs. The simplest pure sequencing problem is one in which there is a single resource, or machine, and all processing times are deterministic. The goal of the sequencing problem consists of determining the order or sequence in which the machines will process the jobs so as to optimize some measure of performance (i.e. cost, time or mileage, weight etc.) to complete the process. The effectiveness of the sequencing problem can be measured in terms of minimized costs, maximized profits, minimized elapsed time and meeting due dates etc. In the past, because of its practical and significant use in production field many researchers have shown their interest in sequencing problems. One of the renowned work in the field of sequencing considered till date is by Johnson's, who gave the algorithm in 1954 for production scheduling in which he had minimized the total idle time of machines and the total production times of the jobs. Later in 1967 Smith and Dudek developed a general algorithm for the solution of the n- job on m-machine sequencing problem of the flow shop when no passing is allowed. Similarly, Maggu and Das(1977), Maggu (2002), Rao *et.al.* (2013) and many others gave the technique to minimize the total ideal time of machines or the total production time of the jobs on the two machines production scheduling problems. A heuristic algorithm for solving general sequencing or flow shop scheduling problem was given by Nawaz *et.al.* (1983), Johnny and Chang(1991), Koulamas (1998), Laha and Chakraborty (2009) for minimizing elapsed time in no-wait flow-shop scheduling. Cai*et.al.*(1997) in their work have concerned the problem of scheduling *n* jobs with a common due date on a single machine so as to minimize the total cost arising from earliness and tardiness.

### 1.1. FUZZY NUMBERS

A fuzzy set $\tilde{A}$ defined on the set of real numbers $\mathbb{R}$ is said to be a fuzzy number if its membership function $\mu_A : \mathbb{R} \rightarrow [0,1]$ has the following characteristics

(i) $A$ is normal. It means that there exists an $x \in \mathbb{R}$ such that $\mu_A(x) = 1$

(ii) $A$ is convex. It means that for every $x_1, x_2 \in \mathbb{R}$,

$\mu_A(\lambda x_1 + (1-\lambda)x_2) \geq \min\{(\mu_A(x_1)), (\mu_A(x_2))\}, \lambda \in [0,1]$

(iii) $\mu_A$ is upper semi-continuous.

(iv) supp ($\tilde{A}$) is bounded in $\mathbb{R}$.

### 1.2 TRIANGULAR FUZZY NUMBER

A fuzzy number $\tilde{A}$ in R is said to be a triangular fuzzy number if its membership function $\mu_A : \mathbb{R} \rightarrow [0,1]$ has the following characteristics.

$$\mu_{\tilde{A}} = \begin{cases} \frac{x-a1}{a2-a1}, & a1 \leq x \leq a2 \\ 1, & x = a2 \\ \frac{a3-x}{a3-a2}, & a2 \leq x \leq a3 \\ 0, & otherwise \end{cases}$$

It is denoted by $\tilde{A} = (a_1; a_2; a_3)$, where $a_2$ is Core (A), a1 is left width and $a_3$ is right width. The geometric representation of triangular fuzzy number is shown in figure. The shape of the triangular fuzzy number e A is usually in the form of triangle and hence it is called so.

The parametric form of a triangular fuzzy number is represented by

$$\tilde{A} = [a_1 - a_2(1 - r); a1 + a_3(1 - r)].$$

**Definition 1.3** A positive triangular fuzzy number e A is denoted as

$$\tilde{A} = (a_1; a_2; a_3) \text{ where all } a1 > 0.$$

**Definition 1.4** A negative triangular fuzzy number e A is denoted as

$$\tilde{A} = (a_1; a_2; a_3) \text{ where all } a3 < 0.$$

**Definition 1.5** Two triangular fuzzy numbers $\tilde{A} = (a_1; a_2; a_3)$ and $\tilde{B} = (b_1; b_2; b_3)$ are said to be equal iff $a_1 = b_1$, $a_2 = b_2$, $a_3 = b_3$.

**Definition 1.6** Let $A = (a_1; a_2; a_3)$ and $B = (b_1; b_2; b_3)$ be two triangular fuzzy numbers then

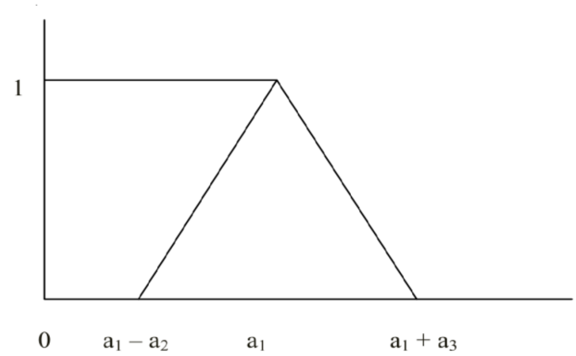


**Figure 1.** Membership function of triangular fuzzy number

(1). $\tilde{A} \oplus \tilde{B} = (a_1 + b_1; a_2 + b_2; a_3 + b_3)$.

(2). $\tilde{A} \ominus \tilde{B} = (a_1 + b_3; a_2 + b_2; a_3 + b_1)$.

(3). If $\tilde{B} = (b1; b2; b3)$ be a non-negative triangular fuzzy number then

$$\tilde{A} \otimes \tilde{B} = \begin{cases} a1b1, a2b2, a3b3, & if\ a1 \geq 0. \\ a1b3, a2b2, a3b3, & if\ a1 < 0, a3 \geq 0. \\ a1b3, a2b2, a3b1, & if\ a3 < 0. \end{cases}$$

(4). $k(a_1; a_2; a_3) = (ka_1; ka_2; ka_3)$ if $k \geq 0$.

(5). $k(a_1; a_2; a_3) = (ka_3; ka_2; ka_1)$ if $k \leq 0$

.

### 1.7 RANKING OF TRIANGULAR FUZZY NUMBER

Several approach for the ranking of fuzzy numbers have been proposed in the literature. An efficient approach for computing the fuzzy numbers is by the use of a ranking function. That is, for every

$\tilde{A} = (a_1; a_2; a_3) \in \mathbb{F}(\mathbb{R})$, the ranking function $\mathfrak{R} : \mathbb{F}(\mathbb{R}) \rightarrow \mathbb{R}$ by graded mean is defined as

$$R(\tilde{A}) = \left(\frac{a1 + 4a2 + a3}{6}\right)$$

For any two triangular fuzzy numbers ).

$\tilde{A} = (a_1; a_2; a_3)$ and $\tilde{B} = (b_1; b_2; b_3)$ in ($\mathbb{R}$), we have the following comparison.

(1). $\tilde{A} < \tilde{B}$ if and only iff $R(\tilde{A}) < R(\tilde{B})$.

(2). $\tilde{A} > \tilde{B}$ if and only iff $R(\tilde{A}) > R(\tilde{B})$.
(3) $\tilde{A} \approx \tilde{B}$ if and only iff $R(\tilde{A}) = R(\tilde{B})$.
(4) $\tilde{A} \ominus \tilde{B}$ if and only iff $R(\tilde{A}) \ominus R(\tilde{B})$.
(5). If $R(\tilde{A}) > 0$ then e $A > 0$.
(6). If $R(\tilde{A}) = 0$ then e $A \approx 0$.

## II. SEQUENCING PROBLEM

In production scheduling, when we have $m$ number of facility or machine and $n$ number of jobs or tasks, then we have $(n!)^m$ of possible sequence but the most optimal sequence is one which minimizes the idle time or elapsed time (i.e. the time from the start of the 1st job to the completion of last job) by satisfying the order in which each job must be performed through $m$ machines one at a time. Sequencing problems are concerned with an appropriate selection of a sequence of jobs to be given a finite number of service facilities.

**Sequencing problem constitutes of various terminology:**

- ✓ Processing order means the order in which various machines are required for completing the job.
- ✓ Processing time means the time each job on each machine.
- ✓ Idle time on a machine is the time for which machine remains idle (not working) during the total elapsed time
- ✓ Total elapsed time is the time between starting the first job and completing the last job. This also includes the idle time.
- ✓ No passing rule means the passing is not allowed if each of the n job is to be processed through two machines say M1 and M2 in order M1M2 then the rule means that each job will go to machine M1 first & then to machine M2.

**Table 1** : Processing n jobs through K

| jobs / Machines | 1 | 2 | 3 | ….. | k | ….. | n |
|---|---|---|---|---|---|---|---|
| 1 | $t_{11}$ | $t_{12}$ | $t_{13}$ | ….. | $t_{1k}$ | ….. | $t_{1n}$ |
| 2 | $t_{21}$ | $t_{22}$ | $t_{23}$ | ….. | $t_{2k}$ | ….. | $t_{2n}$ |
| 3 | $t_{31}$ | $t_{11}$ | $t_{33}$ | ….. | $t_{3k}$ | ….. | $t_{3n}$ |
| . . | . . | . . | . . | . . | . . | . . | . . |
| . . | . . | . . | . . | . . | . . | . . | . . |
| i . . | $t_{i1}$ . . | $t_{i2}$ . . | $t_{i3}$ . . | | $t_{ik}$ . . | | $t_{in}$ . . |
| m | $t_{m1}$ | $t_{m2}$ | $t_{m3}$ | ….. | $t_{mk}$ | ….. | $t_{mn}$ |

**Following are the assumption related to sequencing problem:**

- ✓ Only one operation is carried out on a machine at a time.
- ✓ Processing times are known and do not change.
- ✓ The processing times of machines are independent of the order of processing the jobs.
- ✓ The time involved in moving jobs from one machine to another is negligible.
- ✓ Each operation once started, must be completed.
- ✓ A job is processed as soon as possible but only in the order specified.

## III. NUMERICAL EXAMPLE

**Example 3.1:** Determine the Fuzzy sequencing of jobs that minimize the fuzzy total elapsed time. Based on the following information processing time on machines is given in hours and passing is not allowed. The problem is then solved by processing n jobs through k machines.

**Table 2 :** Processing 6 jobs through 3 machines

| Jobs | A | B | C | D | E | F |
|---|---|---|---|---|---|---|
| Machine 1 | (2,3,4) | (3,5,7) | (8,9,10) | (11,13,15) | (6,7,8) | (3,5,7) |
| Machine 2 | (6,8,10) | (0,1,2) | (3,4,5) | (3,5,7) | (8,9,10) | (10,12,14) |
| Machine 3 | (3,6,9) | (1,2,3) | (6,8,10) | (10,11,12) | (9,10,11) | (6,8,10) |

**Solution:**

Now, we conclude R( 2,3,4 ) by applying Ranking function convert into initial processing time.

| Jobs | A | B | C | D | E | F |
|---|---|---|---|---|---|---|
| Machine 1 | **3** | **5** | **9** | **13** | **7** | **5** |
| Machine 2 | **8** | **1** | **4** | **5** | **9** | **12** |
| Machine 3 | **6** | **2** | **8** | **11** | **10** | **8** |

If G and H are two machines such that,

$G = t_{1j} + t_{2j}$ , $H = t_{2j} + t_{3j}$ for j = 1,2… 6 and then the problem can be rewritten as the following 6 jobs and 2 machines problem:

| **Jobs** / **Machines** | A | B | C | D | E | F |
|---|---|---|---|---|---|---|
| G | 11 | 6 | 13 | 18 | 16 | 17 |
| H | 14 | 3 | 12 | 16 | 19 | 20 |

$$\mathbf{R}(\widetilde{A}) = \frac{\mathbf{a_1+4a_2+a_3}}{\mathbf{6}} = \frac{2+4(3)+4}{6} = 3$$

Similarly we get the initial processing time are given below in table 3

Job sequence can easily be obtained as follows:

| | | | | | |
|---|---|---|---|---|---|
| **A** | **E** | **F** | **D** | **C** | **B** |

For total elapsed time we have,

Table 3

| JOBS | MACHINE 1 | | MACHINE 2 | | MACHINE 3 | | IDEALTIME | | |
|---|---|---|---|---|---|---|---|---|---|
| | IN | OUT | IN | OUT | IN | OUT | A | B | C |
| **A** | 0 | 3 | 3 | 11 | 11 | 17 | 0 | 3 | 11 |
| **E** | 3 | 10 | 11 | 20 | 20 | 30 | 0 | 0 | 3 |
| **F** | 10 | 15 | 20 | 32 | 32 | 40 | 0 | 0 | 2 |
| **D** | 15 | 28 | 32 | 37 | 40 | 51 | 0 | 0 | 0 |
| **C** | 28 | 37 | 37 | 41 | 51 | 59 | 0 | 0 | 0 |
| **B** | 37 | 42 | 42 | 43 | 59 | 61 | 0 | 1 | 0 |

Total elapsed time = 61 hours
Idle time for Machine 3 =11+2+3= 16 hours
Idle time for Machine 2 = 22 hours
Idle time for Machine 1 = 61 – 42= 19 hours

## IV.CONCLUSION

In this paper, the duration hours are considered as imprecise numbers described by triangular fuzzy numbers which are more realistic and general in nature. Processing n job through k machines for optimal sequence of algorithm to solve a fuzzy sequencing problem. Numerical example discussed by the fuzzy sequencing problem converted into classical sequencing problem using Ranking function to solve the total elapsed time.

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