

# A Survey on Shape Representations

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## ABSTRACT

Geometric structures have an important role in shape analysis. The reconstruction problem is an active and challenging problem due to its ill-posed nature. It has various applications in the fields of computational geometry, computer vision, computer graphics, image processing, medical fields, and pattern recognition. There exist a few challenges in approximating the shape of a point set. First, it is unclear that which geometric shape approximates the optimal shape due to mathematical inconvenience. Second, the point set shapes are highly subjective and often depend on a specific application context or other human cognitive factors. As a consequence, the shapes perceived by humans for a majority of point sets vary and reaching a conclusion on the optimum shape is an extremely difficult task. The rich variety of shapes available in nature and the heterogeneity of point sets further weaken a well-defined formulation of the shape approximation problem.

**Keywords :** Shape Analysis, Voronoi Diagram, Delaunay Triangulation, Computational Geometry, Medial axis, 2D point sets, Shape Reconstruction, Modelling, Curves and Surfaces, Dominant Points, Curve Reconstruction

## I. INTRODUCTION

Visual information has an important role in our life. It is typically represented in the form of an image. In the modern era, a large number of images can be generated in digital form. An image can be described or represented by certain features. Shape is an important visual feature used to describe image content. Recovering shape representation of an object from its boundary is a challenging problem in several fields like computer graphics, computational geometry, computer vision. Representations like contours and skeletons, derived from the point set provide valuable information about the geometry of the corresponding object. Different types of geometric structures like Voronoi diagram, Delaunay triangulation, skeletons,

etc play an important role in shape analysis by improving the computational performance and reducing the storage requirements.

There exist a different number of reconstruction like shape reconstruction, surface reconstruction, curve reconstruction, and manifold reconstructions. The simplest reconstruction is curve reconstruction. The curve reconstruction algorithm computes a polygonal approximation of the curve. Manifold reconstruction involves constructing some topological space in Geometric space. Surface reconstruction requires more computation. Shape reconstruction is the most challenging and crucial problems in several computer science field. The purpose of this problem is reconstructing a well-defined structure (e. g. graphs, polygons, parametric curves, etc) from some sample

points which are taken from an initial shape. Here we will mainly focus on different shape reconstruction techniques based on curve reconstruction, dominant points, and medial axis.

The rest of the paper is organized as follows: Definitions in section II. In section III, the survey on the various shape representation methods. Section IV describes our conclusion.

## II. DEFINITIONS

Given a set of points  $S$ , sampled from a curve  $C$ , computes a polygonal (piecewise linear) approximation of the curve. Given below are the major terms that are useful for capturing the concepts of reconstruction algorithms.

### A. Voronoi Diagram

Input: Set of points (generating points)

Output: The segmentation of the space into cells so that each cell contains exactly one generating point and the locus of all points which are closer to this generating point than to others.

### Properties of Voronoi diagram

#### a. Voronoi complex:

The diagram is a cell complex, whose faces are convex polygons. Each point on an edge of the Voronoi diagram is equidistant from its two nearest neighbors  $i$  and  $j$ . Thus, there is a circle centered at such a point such that  $i$  and  $j$  lie on this circle, and no other site is interior to the circle.

#### b. Voronoi vertices :

It follows that the vertex at which three Voronoi cells  $V(i)$ ,  $V(j)$ , and  $V(k)$  intersect, called a Voronoi vertex is equidistant from all sites. Thus it is the center of the circle passing through these sites, and this circle contains no other sites in its interior.

#### c. Degree :

If we make the general position assumption that no four sites are circular, then the vertices of the Voronoi diagram all have degree three.

#### d. Convex hull :

A cell of the Voronoi diagram is unbounded if and only if the corresponding site lies on the convex hull. Thus, given a Voronoi diagram, it is easy to extract the convex hull in linear time.

#### e. Size :

If  $n$  denotes the number of sites, then the Voronoi diagram is a planar graph with exactly  $n$  faces. It follows from Euler's formula that the number of Voronoi vertices is at most  $2n - 5$  and the number of edges is at most  $3n - 6$ .

### B. Delaunay triangulation

For a given number of points, a Delaunay triangulation is the circumcircle associated with each triangle contains no other point in its interior.

### Properties of Delaunay triangulation

#### a. Convex hull :

The boundary of the exterior face of the Delaunay triangulation is the boundary of the convex hull of the point set.

#### b. Circumcircle property :

The circumcircle of any triangle in the Delaunay triangulation is empty.

#### c. Empty circle property :

Two sites  $I, J$  are connected by an edge in the Delaunay triangulation, if and only if there is an empty circle passing through  $I$  and  $J$ .

#### d. Closest pair property :

The closest pair of sites in  $P$  are neighbors in the delaunay triangulation.

#### C. Medial axis

The medial axis of a shape is the closure of the set of points that have more than one closest point in  $\Sigma$ .

#### D. Reconstruction

The reconstruction of  $C$  from its sample  $P$  is a geometric graph  $G = (P, E)$  where an edge  $PQ$  belongs to  $E$  if and only if  $p$  and  $q$  are adjacent sample points on  $C$ .

#### E. $\epsilon$ -uniform sample:

A sample  $P$  of a shape  $\Sigma$  is  $\epsilon$ -uniform if for each  $x \in \Sigma$  there is a sample point  $p \in P$  so that  $d(p, x) \leq \epsilon f_{\min}$  where  $f_{\min} = \min\{f(x), x \in \Sigma\}$  and  $\epsilon > 0$  is a constant.

#### F. Dominant Points

A dominant point(corner point)on a curve is a point of local maximum curvature.

### III. RELATED WORKS AND SURVEY

Surveying different types of shape representation methods.

1. Curve reconstruction
2. Medial axis
3. Dominant points

#### Curve Reconstruction

Curve reconstruction is the problem of computing a linear approximation to a curve from a set of input point sets. Applications include detecting boundaries in an image processing, computing patterns in computer vision and intelligent systems, extracting pieces of information for geographic information systems(GIS), and fitting a spline through a set of points in mathematical modeling. Curve reconstruction can be applied for both smooth curves

and non uniformly sampled curves.

There are mainly two types of input in reconstruction algorithms.

#### A. DOT PATTERN

A Set of input is a dot pattern, then whose elements are visible as well as fairly densely and more or less evenly distributed in the planar point set.

#### B. BOUNDARY SAMPLE

The set of input is a boundary sample, then it only contains some sample points of the boundary of the original shape which fairly represent the perimeter of the shape.

Fernando de Goes et al[1] introduced a robust 2D shape reconstruction and simplification algorithm. Here the input is a defect-laden point set with noise and outliers. They introduced an optimal-transport driven approach where the input point set is considered as the sum of uniform measures on 0- and 1-simplices. A fine to coarse scheme is developed to construct the resulting structure through the greedy decimation of a Delaunay triangulation of the point set. The proposed method performs well with line drawings to grayscale images, with or without noise, features, and boundaries.

Jun Wang et al [2] introduced a robust algorithm for reconstructing the 2D curve from unorganized point data with a high level of noise and outliers. By constructing the quadtree of the input point data, grid-like boundaries of the quadtree can be extracted, and smoothen the boundaries using a modified Laplacian method. Then the skeleton of the smoothed boundaries is computed and hence the initial curve is generated by circular neighboring projection. After that, a normal-based processing method is applied to the initial curve to get smooth jagged features at low curvatures areas and recover

sharp features at high curvature areas. As a result, the curve is reconstructed accurately with small details and sharp features can also be preserved.

Subhasree Methirumangalath et al [3] introduced a reconstruction problem to compute a polygon which best approximates the geometric shape induced by a given point set,  $S$ . The input point set can either be a boundary sample or a dot pattern. A Delaunay-based, unified method for reconstruction is proposed irrespective of the type of the input point set. From the Delaunay Triangulation (DT) of  $S$ , exterior edges are iteratively removed subject to circle and regularity constraints to obtain a resultant boundary which is termed as ec-shape and it is similar to a simple closed curve. The time and space complexities of the algorithm is  $O(n \log n)$  and  $O(n)$  respectively, where  $n$  is the number of points in Set  $S$ .

Jiju Peethambaran et al [4] introduced a fully automatic Delaunay based sculpting algorithm for approximating the shape of a finite point of set  $S$ . The algorithm outputs a relaxed Gabriel graph (RGG) that consist of most of the Gabriel edges and a few non-Gabriel edges induced by the Delaunay triangulation. Holes are characterized through a structural pattern called body-arm formed by the Delaunay triangles in the void regions. RGG is created through an iterative removal of Delaunay triangles subjected to circumcenter (of triangle) and topological regularity constraints. This algorithm doesn't require tuning of any external parameter to approximate the geometric shape of the point set and hence human intervention is eliminated.

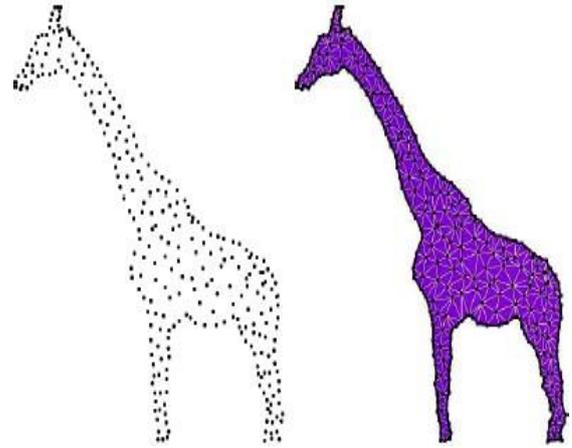


Fig 1: Shape Hull (Image Courtesy: [4])

Jiju Peethambaran et al [5] proposed a Voronoi based algorithm for closed curve reconstruction. The algorithm computes one of the poles (farthest Voronoi vertices of a Voronoi cell) and hence the normals at each sample point by drawing an analogy between a residential water distribution system and Voronoi diagram of the given input samples. The algorithm then categorize Voronoi vertices as either inner or outer concerning the original curve and subsequently construct a piece-wise linear approximation to the boundary and the interior medial axis of the original curve for a class of curves having bi-tangent neighborhood convergence (BNC). The algorithm can reconstruct sparsely and non-uniformly sampled curves with sharp corners, outliers, and a collection of curves.

S. Ohrhallinger et al [6] proposed a reconstruction algorithm. This algorithm manages the distance between geometrically consecutive points. The algorithm is based on local feature size and developed as a minimum local feature size along intervals between samples. HNN crust algorithm is used. The reconstruction algorithm works by connecting the nearby points. The new sampling condition, smooth curves can be reconstructed from even fewer points, typically half of the state-of-the-art bound, in the limit roughly one third.

Amal Dev Parakkat et al [7] proposed an algorithm for curve reconstruction. Here a novel non-parametric curve reconstruction algorithm based on Delaunay triangulation has been introduced and it has been theoretically proved that the proposed method reconstructs the original curve under  $\epsilon$ -sampling. Starting from an initial Delaunay seed edge, the algorithm starts by finding an appropriate neighboring point and adding an edge between them. The proposed algorithm is capable of reconstructing curves with different features like sharp corners, outliers, multiple objects, objects with holes, open curves, etc.

Subhasree Methirumangalath et al [8] proposed a hole detection technique in a planar point set. Outer boundary detection (shape reconstruction) is an extensively studied problem whereas, inner boundary (hole) detection is not a well-studied problem because detecting the presence of a hole is itself a difficult task. The proposed method uses a Delaunay triangulation based strategy to detect the presence of holes and an algorithm to reconstruct them. This algorithm is a unified one that reconstructs holes, both for a boundary sample (points sampled only from the boundary of the object) as well as for a dot pattern (points sampled from the entire object). Also, the method is a non-parametric one that detects holes irrespective of its shape.

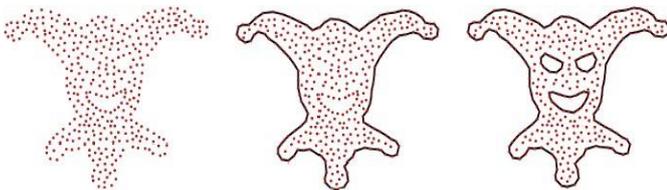


Fig 2: ec-shape (Image Courtesy [8])

Amal Dev Parakkat et al [9] proposed a Delaunay triangulation based algorithm to compute a polygonal approximation of the curve. The process starts by removing the longest edge of each triangle into a resulting graph. Further, each vertex of the graph is

checked for a degree constraint to compute simple closed/open curves. Assuming S-sampling, we provide a theoretical guarantee which ensures that a simple closed or open curve is a linear approximation of the original curve. The proposed method handles the input point set without outliers, identifies the presence of noise in a noisy point set. The algorithm can also be able to detect closed or open curves, disconnected components, multiple holes, and sharp corners.

S. Ohrhallinger et al [10] proposed a "fit connect" which is a reconstructing algorithm used for manifold reconstruction. FITCONNECT algorithm is based on the HNN crust algorithm. Extending the HNN crust algorithm by identifying the nearest neighbor to handle samples polluted by high noise extents. Additionally, it simplifies the output curve without losing features and denoises it. The fit connect algorithm extends HNN-CRUST seamlessly to connect both samples with and without noise, performs as local as the recovered features and can output multiple open or closed piece-wise curves.

### Medial Axis

The medial axis is (the closure of) the set of points in  $O$  that have at least two closest points on the object's boundary  $\partial O$ . In 2D, the MA of a plane curve is the locus of the centers of circles that are tangent to curve in two or more points, where all such circles are contained in  $O$ . This structure is defined for objects of 2D and higher dimensions. The skeleton is a concept closely related to the MA. MA is a powerful shape descriptor used in shape analysis and for feature extraction. Shape approximations can also be done using the Medial axis.

Joachim Giesen et al [11] proposed a Voronoi edge algorithm. The algorithm starts by labeling the vertices of the Voronoi diagram of the set of sample point  $S$  of the smooth boundary of a planar shape  $O$ .

For the classification we use a curve reconstruction algorithm that picks Delaunay edges from the Delaunay triangulation of  $S$  that connect the points in  $S$  in the same as they are connected along  $\partial O$ . The algorithm proves that the medial axis of the union of Voronoi balls centered at Voronoi vertices inside  $O$  has a simple structure than a general union of balls.

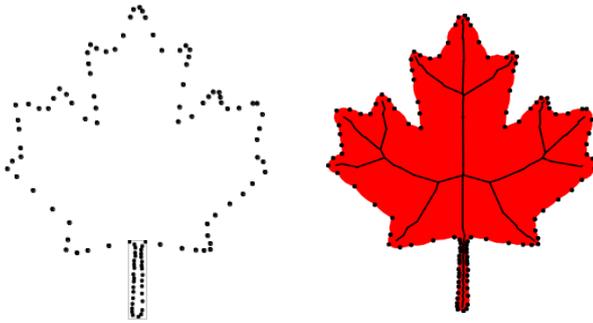


Fig 3(a) Input Samples (b) output (Image Courtesy[11])

Yanshu Zhu et al [12] proposed a system for computing the medial axis transform of an arbitrary 2D shape. The instability of the medial axis transform can be overcome by using a pruning algorithm guided by a user-defined Hausdorff distance threshold. The stable medial axis transform can be approximated by using spline curves in 3D to produce a smooth and compact representation. These spline curves are computed by minimizing the approximation error between the input shape and the shape represented by the medial axis transform. A noise pruning algorithm for the medial axis transform can also be proposed and integrated into the framework. This noise pruning algorithm filters noise in the medial axis transform robustly and provides a good initial medial axis transform for optimization in our framework.

Pan Li et al [13] proposed a method to compute a medial axis transform. They introduced an efficient method, called Q-MAT, that uses quadratic error minimization to compute a structurally simple, geometrically accurate, and compact representation of the MAT. They introduced a new error metric for

approximation and a new quantitative characterization of unstable branches of the MAT, and integrate them in an extension of the well-known quadric error metric (QEM) framework for mesh decimation. Q-MAT is fast, removes insignificant unstable branches effectively, and produces a simple and accurate linear approximation of the MAT.

### Dominant Points

The dominant point or corner point is a popular notion in pattern recognition, especially in shape representation. The most critical role of dominant points is that they are partitioning points for the decomposition of a curve into meaningful parts. Dominant Points (DP) focussed on the highly compressed linear approximation that best approximates the input shape. A dominant point on a curve has local maximum curvature. Detection of the dominant point results in better shape approximation.

Wen-Yen Wu [14] proposed the system for dominant point detection. Here an improved method is used for determining the region of support in the dominant point detection. Instead of setting the regions of support for points independently, the region of support dynamically depending on the previous region of support. The breakpoints are considered as the candidates of dominant points, it will reduce the computation time on both the determination of supporting region and curvature estimation. The method is insensitive to noise and it is robust on the scale and orientation changes.

Majed Marji et al [15] proposed a new algorithm for detecting dominant points and polygonal approximation of digitized closed curves is presented. It uses an optimal criterion for determining the region of support of each boundary point, and a mechanism for selecting the dominant points. The algorithm does not require an input parameter and handle shapes that contain features of multiple sizes efficiently. Also, the approximating polygon gives the symmetry

of the shape.

D. S Guru et al [16] proposed a novel boundary-based corner detection algorithm. The proposed method computes an expected point for every point on a boundary curve. A new 'cornerity' index for a point on the boundary curve is the distance between the point and its corresponding expected point. The larger the cornerity index, the stronger to prove that that the boundary point is a corner point. A set of rules is worked out to guide the process of locating true corner points. The proposed approach is invariant to image transformations like rotation, translation, and scaling.

Asif Masood [17] proposed an algorithm for polygonal approximation based on dominant point (DP) deletion. The proposed algorithm initially takes all the breakpoints and starts eliminating one by one until the required level of approximation is obtained. First of all, a local optimization of a few neighboring points is performed after each deletion. Although the algorithm does not guarantee to get an optimal solution, the combination of local and global optimization is expected to produce optimal results. The algorithm is extensively tested on various shapes with varying numbers of DPs and error thresholds. It incorporates both local and global optimization which results in a very high quality of results.

J peethambaran et al [18] introduced an Incremental labeling of Voronoi vertices for shape reconstruction. It is a unified framework capable of extracting curves, medial axes and dominant points (DPs) from non-uniform and possibly sparse data, sampled from the boundaries of geometric objects. The algorithm computes the normals at each sample point through poles and use the estimated normals and the corresponding tangents to determine the inner and outer location of the Voronoi vertices concerning the original curve. The vertex classification helps to construct a linear approximation to the object boundary and the interior medial axis of the original

curve. Extreme curvature portions induce specific labeling patterns of the Voronoi diagram and these labeling patterns are utilized to identify dominant points on the input curve. It can handle a collection of curves, outliers and shape corners.

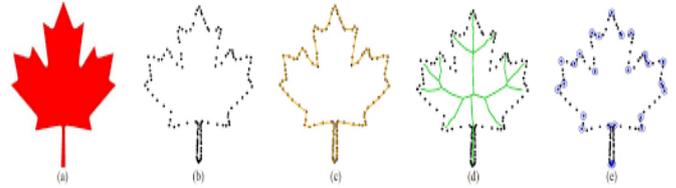


Fig 5: (a). Representative image (b). point set (c). reconstructed curve (d). medial axis (interior) (e). dominant points (Image courtesy [18])

#### IV. CONCLUSION

There are different ways to shape representation. Here we will discuss the different methods in shape approximation using curve reconstruction, medial axis, and dominant points. First of all, we discussed different curve reconstruction algorithms. A curve reconstruction simply deals with the task of constructing a polygonal chain concerning an original curve from its sample data. There are different types of curve reconstruction algorithms like crust, nn crust, HNN crust, ec shape, etc. Among these algorithms, many of them working in both boundary samples and dot patterns. Many of these algorithms deal with sharp corners, outliers, noisy point sets, multiple holes, object with holes, etc.

The medial axis is a shape descriptor used in shape analysis and feature extraction. It has different applications in character recognition, Voronoi diagrams, skeleton pruning, skeleton extraction, etc. The medial axis can also be used in shape approximation. We discussed different methods in the medial axis for shape reconstruction.

Dominant Points uses highly compressed linear polygons that approximate the input shape. dominant points have a different application in curve approximation, image recognition, and matching. We discussed several methods of polygonal approximation using dominant points.

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