

Bivariate Interpolation in Rectangular Form

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ABSTRACT

The purpose of this paper is to derive lagrange interpolation formula for a single variable and two independent variables. Firstly, single variable interpolation is derived and then two independent variable interpolation derived. Some practical problems are computed by virtue of these interpolation formula.

Keywords : Four-Points-of-Fit, Six-Points-of-Fit And Mn-Points-of-Fit, Interpolation, Approximation.

I. INTRODUCTION

Interpolation is a process of predicting or estimating values at unmeasured points using sample points with known values. Purpose perform a bivariate interpolation of series of gridded data points. Bivariate interpolation based an univariate subdivision by N Sharon. The bivariate subdivision scheme interpolating data consisting of univariate functions along equidistant parallel lines by repeated refinements solving the bivariate interpolation problem. The speed of our algorithm is determined by the speed of the method to solve the different univariate interpolation.[2]

Two dimensional interpolation takes a series of (x,y,z) points and generates estimated values for z 's at new (x,y) points. Interpolation is used when the function that generated the original (x,y,z) points is unknown. Interpolation is related to, but distinct from, fitting a function to a series of points. In particular, an interpolated function goes through all the original points while a fitted function may not.

There are two distinct types of 2d interpolation. In the first, data is available for a rectangular grid of points and interpolation is performed for points off the grid. In the second, data is available for a random set of points and the interpolation is generated on a rectangular grid. This second form can be used to generate a contour or surface plot when the data do not form a grid.

The Bilinear Interpolation and Bivariate Interpolation commands are used for the first type. The bilinear interpolation is analogous to linear interpolation. The bivariate interpolation uses an interpolating function that is a piecewise polynomial function that is represented as a tensor product of one-dimensional B-splines. That is, where $U(i)$ and $V(j)$ are one-dimensional B-spline basis functions and the coefficients $a(i,j)$ are chosen so that the interpolating function equals the z axis input values at the grid points. The main point of this paper to perform a bivariate interpolation of a series of gridded data points.[2]

II. An Alternative Way to Bivariate Approximation in Rectangular Forms

$$y = \frac{(x-x_1)f_0}{x_0-x_1} + \frac{(x-x_0)f_1}{x_1-x_0}$$

For the same input dataset, in order to avoid having to calculate the natural neighbor relationships every time interpolation is done on a single point, interpolation at single points is implemented as a three step process. Do the interpolation at the desired points.[2]

First of all, we will review the following result. Let $f(x)$ be a function of a variable x with known values $f(x_0) = f_0$ and $f(x_1) = f_1$, where $x_0 \neq x_1$. Suppose x is a point so that

$$\min\{x_0, x_1\} \leq x \leq \max\{x_0, x_1\}.$$

$$f(x) \approx y(x) = Ax + B$$

with $y(x_0) = f_0, y(x_1) = f_1, x_0 \leq x \leq x_1$

$$y(x_0) = Ax_0 + B = f_0 \dots \dots \dots (1)$$

$$y(x_1) = Ax_1 + B = f_1 \dots \dots \dots (2)$$

$$A(x_0 - x_1) = f_0 - f_1$$

$$A = \frac{f_0 - f_1}{x_0 - x_1}$$

$$(1) \Rightarrow \frac{f_0 x_0 - f_1 x_0}{x_0 - x_1} + B = f_0$$

$$B = f_0 - \frac{f_0 x_0 - f_1 x_0}{x_0 - x_1}$$

$$B = \frac{f_0 x_0 - f_0 x_1 - f_0 x_0 + f_1 x_0}{x_0 - x_1}$$

$$B = \frac{f_1 x_0 - f_0 x_1}{x_0 - x_1}$$

$$y(x) = Ax + B$$

$$y = \frac{f_0 x - f_1 x}{x_0 - x_1} + \frac{f_1 x_0 - f_0 x_1}{x_0 - x_1}$$

$$y = \frac{f_0 x - f_1 x + f_1 x_0 - f_0 x_1}{x_0 - x_1}$$

$$y = \frac{(x-x_1)f_0}{x_0-x_1} + \frac{(x_0-x)f_1}{x_0-x_1}$$

$$y = \frac{(x-x_1)f_0}{x_0-x_1} - \frac{(x-x_0)f_1}{x_0-x_1}$$

A. Four-Points-of-Fit

Let $f(x, y)$ be a function of two independent variables x and y with known values $f(x_0, y_0) = f_{00}, f(x_0, y_1) = f_{01}, f(x_1, y_0) = f_{10}$ and $f(x_1, y_1) = f_{11}$, where x_0 and x_1 are distinct.

Let (x, y) be a point such that

$$\min_{0 \leq i \leq 1} x_i \leq x \leq \max_{0 \leq i \leq 1} x_i \text{ and } \min_{0 \leq j \leq 1} y_j \leq y \leq \max_{0 \leq j \leq 1} y_j.$$

Then

$$f(x, y) \approx + \frac{(x-x_1)(y-y_0)}{(x_0-x_1)(y_1-y_0)} f_{01}$$

$$+ \frac{(x-x_0)(y-y_1)}{(x_1-x_0)(y_0-y_1)} f_{10}$$

$$+ \frac{(x-x_0)(y-y_0)}{(x_1-x_0)(y_1-y_0)} f_{11}.$$

$$f(x_0, y) \approx \frac{y-y_1}{y_0-y_1} f_{00} + \frac{y-y_0}{y_1-y_0} f_{01}$$

$$f(x_1, y) \approx \frac{y-y_1}{y_0-y_1} f_{10} + \frac{y-y_0}{y_1-y_0} f_{11}$$

$$f(x, y) \approx \frac{x-x_1}{x_0-x_1} \left(\frac{y-y_1}{y_0-y_1} f_{00} + \frac{y-y_0}{y_1-y_0} f_{01} \right)$$

$$+ \frac{x-x_0}{x_1-x_0} \left(\frac{y-y_1}{y_0-y_1} f_{10} + \frac{y-y_0}{y_1-y_0} f_{11} \right)$$

$$= \frac{(x-x_1)(y-y_1)}{(x_0-x_1)(y_0-y_1)} f_{00}$$

$$+ \frac{(x-x_1)(y-y_0)}{(x_0-x_1)(y_1-y_0)} f_{01}$$

$$+ \frac{(x-x_0)(y-y_1)}{(x_1-x_0)(y_0-y_1)} f_{10}$$

$$+ \frac{(x-x_0)(y-y_0)}{(x_1-x_0)(y_1-y_0)} f_{11}$$

1) Example

Let $x_0 = 0.4, x_1 = 0.7, y_0 = 0$ and $y_1 = 0.05$ with respective known values

$$f_{00} = 2.5, f_{01} = 2.487, f_{10} = 1.429 \text{ and } f_{11} = 1.419.$$

We will approximate $f(0.5, 0.03)$. Then we compare with its true value 2.074.

We use Bivariate Approximation with Four-points-of-fit.[1]

Now, $x = 0.5$ and $y = 0.03$.

$$\begin{aligned}
 f(x, y) &\approx \frac{(x-x_1)(y-y_1)}{(x_0-x_1)(y_0-y_1)} f_{00} \\
 &+ \frac{(x-x_1)(y-y_0)}{(x_0-x_1)(y_1-y_0)} f_{01} \\
 &+ \frac{(x-x_0)(y-y_1)}{(x_1-x_0)(y_0-y_1)} f_{10} \\
 &+ \frac{(x-x_0)(y-y_0)}{(x_1-x_0)(y_1-y_0)} f_{11} \\
 &= \frac{(0.5-0.7)(0.03-0.05)}{(0.4-0.7)(0-0.05)} (2.5) \\
 &+ \frac{(0.5-0.7)(0.03-0)}{(0.4-0.7)(0.05-0)} (2.487) \\
 &+ \frac{(0.5-0.4)(0.03-0.05)}{(0.7-0.4)(0-0.05)} (1.429) \\
 &+ \frac{(0.5-0.4)(0.03-0)}{(0.7-0.4)(0.05-0)} (1.419) \\
 &= \frac{(-0.2)(-0.02)}{(-0.3)(-0.05)} (2.5) + \frac{(-0.2)(0.03)}{(-0.3)(0.05)} (2.487) \\
 &+ \frac{(0.1)(-0.02)}{(0.3)(-0.05)} (1.429) + \frac{(0.1)(0.03)}{(0.3)(0.05)} (1.419) \\
 &\approx \frac{0.01}{0.015} + \frac{(-0.0149)}{(-0.015)} + \frac{(-0.0029)}{(-0.015)} + \frac{0.0043}{0.015} \\
 &= 0.6667 + 0.9933 + 0.1933 + 0.2867 \\
 &= 2.14.
 \end{aligned}$$

Since $2.074 - 2.14 = -0.0618$, it is true for one decimal place.

B. Six-Points-of-Fit ($n = 2, m = 3$)

$$f(x_0, y) \approx \frac{(y-y_1)(y-y_2)}{(y_0-y_1)(y_0-y_2)} f_{00}$$

$$\begin{aligned}
 &+ \frac{(y-y_0)(y-y_2)}{(y_1-y_0)(y_1-y_2)} f_{01} \\
 &+ \frac{(y-y_0)(y-y_1)}{(y_2-y_0)(y_2-y_1)} f_{02} \\
 f(x_1, y) &\approx \frac{(y-y_1)(y-y_2)}{(y_0-y_1)(y_0-y_2)} f_{10} \\
 &+ \frac{(y-y_0)(y-y_2)}{(y_1-y_0)(y_1-y_2)} f_{11} \\
 &+ \frac{(y-y_0)(y-y_1)}{(y_2-y_0)(y_2-y_1)} f_{12} \\
 f(x, y) &\approx \frac{x-x_1}{x_0-x_1} f(x_0, y) + \frac{x-x_0}{x_1-x_0} f(x_1, y) \\
 &\approx \frac{x-x_1}{x_0-x_1} \left(\frac{(y-y_1)(y-y_2)}{(y_0-y_1)(y_0-y_2)} f_{00} \right. \\
 &+ \frac{(y-y_0)(y-y_2)}{(y_1-y_0)(y_1-y_2)} f_{01} \\
 &+ \left. \frac{(y-y_0)(y-y_1)}{(y_2-y_0)(y_2-y_1)} f_{02} \right) \\
 &+ \frac{x-x_0}{x_1-x_0} \left(\frac{(y-y_1)(y-y_2)}{(y_0-y_1)(y_0-y_2)} f_{10} \right. \\
 &+ \frac{(y-y_0)(y-y_2)}{(y_1-y_0)(y_1-y_2)} f_{11} \\
 &+ \left. \frac{(y-y_0)(y-y_1)}{(y_2-y_0)(y_2-y_1)} f_{12} \right) \\
 &= \frac{(x-x_1)(y-y_1)(y-y_2)}{(x_0-x_1)(y_0-y_1)(y_0-y_2)} f_{00} \\
 &+ \frac{(x-x_0)(y-y_0)(y-y_2)}{(x_0-x_1)(y_1-y_0)(y_1-y_2)} f_{01} \\
 &+ \frac{(x-x_1)(y-y_0)(y-y_1)}{(x_0-x_1)(y_2-y_0)(y_2-y_1)} f_{02} \\
 &+ \frac{(x-x_0)(y-y_1)(y-y_2)}{(x_1-x_0)(y_0-y_1)(y_0-y_2)} f_{10} \\
 &+ \frac{(x-x_0)(y-y_0)(y-y_2)}{(x_1-x_0)(y_1-y_0)(y_1-y_2)} f_{11} \\
 &+ \frac{(x-x_0)(y-y_0)(y-y_1)}{(x_1-x_0)(y_2-y_0)(y_2-y_1)} f_{12}.
 \end{aligned}$$

1) Example

Let $x_0 = 0.4$, $x_1 = 0.7$, $y_0 = 0$, $y_1 = 0.05$ and $y_2 = 0.1$ with respective known values $f_{00} = 2.5$, $f_{01} = 2.487$, $f_{02} = 2.456$, $f_{10} = 1.429$, $f_{11} = 1.419$ and $f_{12} = 1.4$. We will approximate $f(0.5, 0.03)$. Then we compare with its true value 2.074. We use Bivariate Approximation with Six-points-of-fit. Here $n = 2$ and $m = 3$. [1]

Now, $x = 0.5$ and $y = 0.03$.

$$\begin{aligned}
 f(x, y) &\approx \frac{(x-x_1)(y-y_1)(y-y_2)}{(x_0-x_1)(y_0-y_1)(y_0-y_2)} f_{00} \\
 &+ \frac{(x-x_1)(y-y_0)(y-y_2)}{(x_0-x_1)(y_1-y_0)(y_1-y_2)} f_{01} \\
 &+ \frac{(x-x_1)(y-y_0)(y-y_1)}{(x_0-x_1)(y_2-y_0)(y_2-y_1)} f_{02} \\
 &+ \frac{(x-x_0)(y-y_1)(y-y_2)}{(x_1-x_0)(y_0-y_1)(y_0-y_2)} f_{10} \\
 &+ \frac{(x-x_0)(y-y_0)(y-y_2)}{(x_1-x_0)(y_1-y_0)(y_1-y_2)} f_{11} \\
 &+ \frac{(x-x_0)(y-y_0)(y-y_1)}{(x_1-x_0)(y_2-y_0)(y_2-y_1)} f_{12} \\
 &= \frac{(0.5-0.7)(0.03-0.05)(0.03-0.1)}{(0.4-0.7)(0-0.05)(0-0.1)} (2.5) \\
 &+ \frac{(0.5-0.7)(0.03-0)(0.03-0.1)}{(0.4-0.7)(0.05-0)(0.05-0.1)} (2.487) \\
 &+ \frac{(0.5-0.7)(0.03-0)(0.03-0.05)}{(0.4-0.7)(0.1-0)(0.1-0.05)} (2.456) \\
 &+ \frac{(0.5-0.4)(0.03-0.05)(0.03-0.1)}{(0.7-0.4)(0-0.05)(0-0.1)} (1.429) \\
 &+ \frac{(0.5-0.4)(0.03-0)(0.03-0.1)}{(0.7-0.4)(0.05-0)(0.05-0.1)} (1.419) \\
 &+ \frac{(0.5-0.4)(0.03-0)(0.03-0.05)}{(0.7-0.4)(0.1-0)(0.1-0.05)} (1.4) \\
 &= \frac{(-0.2)(-0.02)(-0.07)}{(-0.3)(-0.05)(-0.1)} (2.5)
 \end{aligned}$$

$$\begin{aligned}
 &+ \frac{(-0.2)(0.03)(-0.07)}{(-0.3)(0.05)(-0.05)} (2.487) \\
 &+ \frac{(-0.2)(0.03)(-0.02)}{(-0.3)(0.1)(0.05)} (2.456) \\
 &+ \frac{(0.1)(-0.02)(-0.07)}{(0.3)(-0.05)(-0.1)} (1.429) \\
 &+ \frac{(0.1)(0.03)(-0.07)}{(0.3)(0.05)(-0.05)} (1.419) \\
 &+ \frac{(0.1)(0.03)(-0.02)}{(0.3)(0.1)(0.05)} (1.4) \\
 &\approx \frac{(-0.0007)}{(-0.0015)} + \frac{0.0010445}{0.00075} + \frac{0.00029472}{(-0.0015)} + \frac{0.00020006}{0.0015} \\
 &+ \frac{(-0.00029799)}{(-0.00075)} + \frac{(-0.000084)}{0.0015} \\
 &= 0.4667 + 1.3927 - 0.19648 \\
 &\quad + 0.13337 - 0.39732 - 0.056 \\
 &= 2.1376.
 \end{aligned}$$

Since $2.074 - 2.1376 = -0.0636$, it is true for one decimal place.

C. Six-Points of Fit ($n = 3, m = 2$)

Let $f(x, y)$ be a function of two independent variables x and y with known values $f(x_0, y_0) = f_{00}$, $f(x_0, y_1) = f_{01}$, $f(x_1, y_0) = f_{10}$, $f(x_1, y_1) = f_{11}$, $f(x_2, y_0) = f_{20}$ and $f(x_2, y_1) = f_{21}$, where x_0, x_1 and x_2 are distinct; y_0 and y_1 are distinct.

Let (x, y) be a point such that

$$\min_{0 \leq i \leq 1} x_i \leq x \leq \max_{0 \leq i \leq 1} x_i \text{ and } \min_{0 \leq j \leq 1} y_j \leq y \leq \max_{0 \leq j \leq 1} y_j.$$

$$f(x_0, y) \approx \frac{y-y_1}{y_0-y_1} f_{00} + \frac{y-y_0}{y_1-y_0} f_{01}$$

$$f(x_1, y) \approx \frac{y-y_1}{y_0-y_1} f_{10} + \frac{y-y_0}{y_1-y_0} f_{11}$$

$$f(x_2, y) \approx \frac{y-y_1}{y_0-y_1} f_{20} + \frac{y-y_0}{y_1-y_0} f_{21}$$

$$f(x, y) \approx \frac{(x-x_1)(x-x_2)}{(x_0-x_1)(x_0-x_2)} f(x_0, y)$$

$$\begin{aligned}
 & + \frac{(x-x_0)(x-x_2)}{(x_1-x_0)(x_1-x_2)} f(x_1, y) \\
 & + \frac{(x-x_0)(x-x_1)}{(x_2-x_0)(x_2-x_1)} f(x_2, y) \\
 = & \frac{(x-x_1)(x-x_2)}{(x_0-x_1)(x_0-x_2)} \left(\frac{y-y_1}{y_0-y_1} f_{00} + \frac{y-y_0}{y_1-y_0} f_{01} \right) \\
 & + \frac{(x-x_0)(x-x_2)}{(x_1-x_0)(x_1-x_2)} \left(\frac{y-y_1}{y_0-y_1} f_{10} + \frac{y-y_0}{y_1-y_0} f_{11} \right) \\
 & + \frac{(x-x_0)(x-x_1)}{(x_2-x_0)(x_2-x_1)} \left(\frac{y-y_1}{y_0-y_1} f_{20} + \frac{y-y_0}{y_1-y_0} f_{21} \right) \\
 = & \frac{(x-x_1)(x-x_2)(y-y_1)}{(x_0-x_1)(x_0-x_2)(y_0-y_1)} f_{00} \\
 & + \frac{(x-x_1)(x-x_2)(y-y_0)}{(x_0-x_1)(x_0-x_2)(y_1-y_0)} f_{01} \\
 & + \frac{(x-x_0)(x-x_2)(y-y_1)}{(x_1-x_0)(x_1-x_2)(y_0-y_1)} f_{10} \\
 & + \frac{(x-x_0)(x-x_2)(y-y_0)}{(x_1-x_0)(x_1-x_2)(y_1-y_0)} f_{11} \\
 & + \frac{(x-x_0)(x-x_1)(y-y_1)}{(x_2-x_0)(x_2-x_1)(y_0-y_1)} f_{20} \\
 & + \frac{(x-x_0)(x-x_1)(y-y_0)}{(x_2-x_0)(x_2-x_1)(y_1-y_0)} f_{21} \cdot
 \end{aligned}$$

1) Example

Let $x_0 = 0.4, x_1 = 0.7, x_2 = 1, y_0 = 0$ and $y_1 = 0.05$ with respective known values $f_{00} = 2.5, f_{01} = 2.487, f_{10} = 1.429, f_{11} = 1.419$ and $f_{20} = 0.995$.

We will approximate $f(0.5, 0.03)$. Then we compare with its true value 2.074.

We use Bivariate Approximation with Six-points-of-fit. Here $n = 3$ and $m = 2$.

Now, $x = 0.5$ and $y = 0.03$.

$$\begin{aligned}
 f(x, y) \approx & \frac{(x-x_1)(x-x_2)(y-y_1)}{(x_0-x_1)(x_0-x_2)(y_0-y_1)} f_{00} \\
 & + \frac{(x-x_1)(x-x_2)(y-y_0)}{(x_0-x_1)(x_0-x_2)(y_1-y_0)} f_{01} \\
 & + \frac{(x-x_0)(x-x_2)(y-y_1)}{(x_1-x_0)(x_1-x_2)(y_0-y_1)} f_{10}
 \end{aligned}$$

$$\begin{aligned}
 & + \frac{(x-x_0)(x-x_2)(y-y_0)}{(x_1-x_0)(x_1-x_2)(y_1-y_0)} f_{11} \\
 & + \frac{(x-x_0)(x-x_2)(y-y_1)}{(x_2-x_0)(x_2-x_1)(y_0-y_1)} f_{20} \\
 & + \frac{(x-x_0)(x-x_2)(y-y_0)}{(x_2-x_0)(x_2-x_1)(y_1-y_0)} f_{21} \\
 = & \frac{(0.5-0.7)(0.5-1)(0.03-0.05)}{(0.4-0.7)(0.4-1)(0-0.05)} (2.5) \\
 & + \frac{(0.5-0.7)(0.5-1)(0.03-0)}{(0.4-0.7)(0.4-1)(0.05-0)} (2.487) \\
 & + \frac{(0.5-0.4)(0.5-1)(0.03-0.05)}{(0.7-0.4)(0.7-1)(0-0.05)} (1.429) \\
 & + \frac{(0.5-0.4)(0.5-1)(0.03-0)}{(0.7-0.4)(0.7-1)(0.05-0)} (1.419) \\
 & + \frac{(0.5-0.4)(0.5-0.7)(0.03-0.05)}{(1-0.4)(1-0.7)(0-0.05)} (1) \\
 & + \frac{(0.5-0.4)(0.5-0.7)(0.03-0)}{(1-0.4)(1-0.7)(0.05-0)} (0.995) \\
 = & \frac{(-0.2)(-0.5)(-0.02)}{(-0.3)(-0.6)(-0.05)} (2.5) \\
 & + \frac{(-0.2)(-0.5)(0.03)}{(-0.3)(-0.6)(0.05)} (2.487) \\
 & + \frac{(0.1)(-0.5)(-0.02)}{(0.3)(-0.3)(-0.05)} (1.429) \\
 & + \frac{(0.1)(-0.5)(0.03)}{(0.3)(-0.3)(0.05)} (1.419) \\
 & + \frac{(0.1)(-0.2)(-0.02)}{(0.6)(0.3)(-0.05)} (1) \\
 & + \frac{(0.1)(-0.2)(0.03)}{(0.6)(0.3)(0.05)} (0.995) \\
 \approx & \frac{(-0.005)}{(-0.009)} + \frac{0.007461}{0.009} + \frac{0.001429}{0.0045} + \frac{(-0.0021285)}{(-0.0045)} \\
 & + \frac{0.0004}{(-0.009)} + \frac{(-0.000597)}{0.009} \\
 = & 0.5556 + 0.829 + 0.3176 \\
 & + 0.473 - 0.0444 - 0.0663 \\
 = & 2.0645.
 \end{aligned}$$

Since $2.074 - 2.0645 = 0.0095$,
it is true for two decimal place.

D. nm-Points-of-Fit

Let $f(x, y)$ be a function of two independent variables x and y with known values

$$\begin{aligned} f(x_0, y_0) &= f_{00}, f(x_0, y_1) = f_{01}, \\ f(x_0, y_2) &= f_{02}, \dots, f(x_0, y_m) = f_{0m}, \\ f(x_1, y_0) &= f_{10}, f(x_1, y_1) = f_{11}, \\ f(x_1, y_2) &= f_{12}, \dots, f(x_1, y_m) = f_{1m}, \\ \mathbf{N} \quad \quad \mathbf{N} \quad \quad \quad \mathbf{N} \\ f(x_n, y_0) &= f_{n0}, f(x_n, y_1) = f_{n1}, \\ f(x_n, y_2) &= f_{n2}, \dots, f(x_n, y_m) = f_{nm}, \end{aligned}$$

where $x_0, x_1, x_2, \dots, x_n$ are distinct;

$y_0, y_1, y_2, \dots, y_m$ are distinct.

Let (x, y) be a point such that

$$\min_{0 \leq i \leq 1} x_i \leq x \leq \max_{0 \leq i \leq 1} x_i \text{ and } \min_{0 \leq j \leq 1} y_j \leq y \leq \max_{0 \leq j \leq 1} y_j.$$

$$\begin{aligned} f(x_0, y) &\approx \frac{(y - y_1)(y - y_2)L(y - y_m)}{(y_0 - y_1)(y_0 - y_2)L(y_0 - y_m)} f_{00} \\ &+ \frac{(y - y_0)(y - y_2)L(y - y_m)}{(y_1 - y_0)(y_1 - y_2)L(y_1 - y_m)} f_{01} \\ &+ \dots \\ &+ \frac{(y - y_0)(y - y_1)L(y - y_{m-1})}{(y_m - y_0)(y_m - y_1)L(y_m - y_{m-1})} f_{0m} \\ f(x_1, y) &\approx \frac{(y - y_1)(y - y_2)L(y - y_m)}{(y_0 - y_1)(y_0 - y_2)L(y_0 - y_m)} f_{10} \\ &+ \frac{(y - y_0)(y - y_2)L(y - y_m)}{(y_1 - y_0)(y_1 - y_2)L(y_1 - y_m)} f_{11} \\ &+ \dots + \frac{(y - y_0)(y - y_1)L(y - y_{m-1})}{(y_m - y_0)(y_m - y_1)L(y_m - y_{m-1})} f_{1m} \\ f(x_n, y) &\approx \frac{(y - y_1)(y - y_2)L(y - y_m)}{(y_0 - y_1)(y_0 - y_2)L(y_0 - y_m)} f_{n0} \\ &+ \frac{(y - y_0)(y - y_2)L(y - y_m)}{(y_1 - y_0)(y_1 - y_2)L(y_1 - y_m)} f_{n1} \\ &+ \dots + \frac{(y - y_0)(y - y_1)L(y - y_{m-1})}{(y_m - y_0)(y_m - y_1)L(y_m - y_{m-1})} f_{nm} \\ f(x, y) &\approx \frac{(x - x_1)(x - x_2)L(x - x_n)}{(x_0 - x_1)(x_0 - x_2)L(x_0 - x_n)} f(x_0, y) \end{aligned}$$

$$\begin{aligned} &+ \frac{(x - x_0)(x - x_2)L(x - x_n)}{(x_1 - x_0)(x_1 - x_2)L(x_1 - x_n)} f(x_1, y) \\ &+ \dots + \frac{(x - x_0)(x - x_1)L(x - x_{n-1})}{(x_n - x_0)(x_n - x_1)L(x_n - x_{n-1})} f(x_n, y) \\ &\approx \left[\frac{(x - x_1)(x - x_2)L(x - x_n)(y - y_1)(y - y_2)L(y - y_m)}{(x_0 - x_1)(x_0 - x_2)L(x_0 - x_n)(y_0 - y_1)(y_0 - y_2)L(y_0 - y_m)} f_{00} \right. \\ &+ \frac{(x - x_1)(x - x_2)L(x - x_n)(y - y_0)(y - y_2)L(y - y_m)}{(x_0 - x_1)(x_0 - x_2)L(x_0 - x_n)(y_1 - y_0)(y_1 - y_2)L(y_1 - y_m)} f_{01} + \dots \\ &+ \left. \frac{(x - x_1)(x - x_2)L(x - x_n)(y - y_0)(y - y_1)L(y - y_{m-1})}{(x_0 - x_1)(x_0 - x_2)L(x_0 - x_n)(y_m - y_0)(y_m - y_1)L(y_m - y_{m-1})} f_{0m} \right] \\ &+ \left[\frac{(x - x_0)(x - x_2)L(x - x_n)(y - y_1)(y - y_2)L(y - y_m)}{(x_1 - x_0)(x_1 - x_2)L(x_1 - x_n)(y_0 - y_1)(y_0 - y_2)L(y_0 - y_m)} f_{10} \right. \\ &+ \frac{(x - x_0)(x - x_2)L(x - x_n)(y - y_0)(y - y_2)L(y - y_m)}{(x_1 - x_0)(x_1 - x_2)L(x_1 - x_n)(y_1 - y_0)(y_1 - y_2)L(y_1 - y_m)} f_{11} + \dots + \\ &\left. \frac{(x - x_0)(x - x_2)L(x - x_n)(y - y_0)(y - y_1)L(y - y_{m-1})}{(x_1 - x_0)(x_1 - x_2)L(x_1 - x_n)(y_m - y_0)(y_m - y_1)L(y_m - y_{m-1})} f_{1m} \right] \\ &+ \dots + \left[\frac{(x - x_0)(x - x_1)L(x - x_{n-1})(y - y_1)(y - y_2)L(y - y_m)}{(x_n - x_0)(x_n - x_1)L(x_n - x_{n-1})(y_0 - y_1)(y_0 - y_2)L(y_0 - y_m)} f_{n0} \right. \\ &+ \frac{(x - x_0)(x - x_1)L(x - x_{n-1})(y - y_0)(y - y_2)L(y - y_m)}{(x_n - x_0)(x_n - x_1)L(x_n - x_{n-1})(y_1 - y_0)(y_1 - y_2)L(y_1 - y_m)} f_{n1} \\ &+ \dots + \left. \frac{(x - x_0)(x - x_1)L(x - x_{n-1})(y - y_0)(y - y_1)L(y - y_{m-1})}{(x_n - x_0)(x_n - x_1)L(x_n - x_{n-1})(y_m - y_0)(y_m - y_1)L(y_m - y_{m-1})} f_{nm} \right] \end{aligned}$$

III. CONCLUSION

It has been presented about bivariate interpolation formulae starting with four- points -of -fit , two kinds of six-points-of-fit and mn-point-of-fit . Some practical problems has been solved by these interpolation formulae.

IV. REFERENCES

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Cite this article as :

Daw San San Nwe, Daw Hla Yin Moe, Daw Zin Nwe Khaing, "Bivariate Interpolation in Rectangular Form", International Journal of Scientific Research in Science, Engineering and Technology (IJSRSET), Online ISSN : 2394-4099, Print ISSN : 2395-1990, Volume 7 Issue 2, pp. 329-335, March-April 2020. Available at doi : <https://doi.org/10.32628/IJSRSET207270>
Journal URL : <http://ijsrset.com/IJSRSET207270>