

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ 0 & a_{22} & a_{23} & a_{24} \\ 0 & a_{32} & a_{33} & a_{34} \\ 0 & a_{42} & a_{43} & a_{44} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{bmatrix}$$

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ 0 & a_{22} & a_{23} & a_{24} \\ 0 & 0 & a_{33} & a_{34} \\ 0 & 0 & a_{43} & a_{44} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{bmatrix}$$

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ 0 & a_{22} & a_{23} & a_{24} \\ 0 & 0 & a_{33} & a_{34} \\ 0 & 0 & 0 & a_{44} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{bmatrix}$$

Stage 2: Back Substitutio

$$a_{11}x_1 + a_{12}x_2 + a_{13}x_3 + a_{14}x_4 = b_1$$

$$a_{22}x_2 +$$

$$a_{23}x_3 + a_{24}x_4 = b_2$$

$$a_{33}x_3 + a_{34}x_4 = b_3$$

$$a_{44}x_4 = b_4$$

$$x_4 = \frac{b_4}{a_{44}}$$

$$x_3 = \frac{(b_3 - a_{34}x_4)}{a_{33}}$$

$$x_2 = \frac{(b_2 - a_{23}x_3 - a_{24}x_4)}{a_{22}}$$

$$x_1 = \frac{(b_1 - a_{12}x_2 - a_{13}x_3 - a_{14}x_4)}{a_{11}}$$

B. Determination of the Inverse by the Gauss-Jordan Method

$$\begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \dots & \dots & \dots & \dots \\ a_{n1} & a_{n2} & \dots & a_{nn} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \dots \\ x_n \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ \dots \\ b_n \end{bmatrix}$$

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{bmatrix}$$

$$\begin{bmatrix} 1 & a_{12} & a_{13} & a_{14} \\ 0 & a_{22} & a_{23} & a_{24} \\ 0 & a_{32} & a_{33} & a_{34} \\ 0 & a_{42} & a_{43} & a_{44} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & a_{13} & a_{14} \\ 0 & 1 & a_{23} & a_{24} \\ 0 & 0 & a_{33} & a_{34} \\ 0 & 0 & a_{43} & a_{44} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{bmatrix}$$

General Formulas

Step 1. Transformation of the pivot row

$$a^*_{k,j} = \frac{a_{k,j}}{a_{k,k}}, b^*_k = \frac{b_k}{a_{k,k}} \quad k = 1, 2, \dots, n, j = k, \dots, n$$

Step 2: Transformation of non-pivot rows

$$a^*_{i,j} = a_{i,j} - a_{i,k} a^*_{k,j}, \quad b^*_i = b_i - a_{i,k} b^*_k$$

where, $k = 1, 2, \dots, n, i = 1, \dots, n,$
 $i \neq k, j = k, \dots, n$

1. If $a_{i,k} = 0$, skip row i .
2. Apply partial pivoting at each elimination.

III. RESULTS AND DISCUSSION

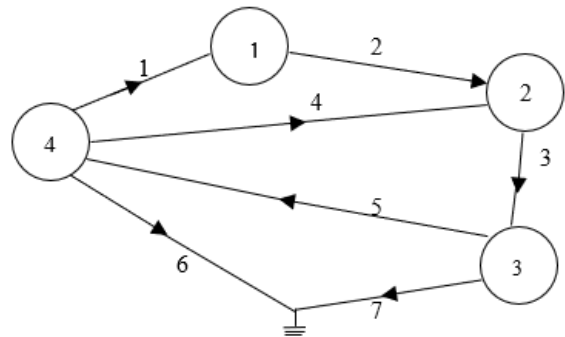


Figure 1. Electric Networks

Figure-1 presents an electrical network having seven branches and four nodes. One of the nodes is reference node. All the nodes are numbered except the reference node. We also numbered except the reference node. We also number and direct the branches.

We can now use 1.1 to form the nodal incidence matrices, we will need a computation table to relate the nodes.

Table 1.

Computation table for Electrical Network in Figure 1

No : of node	Bra h 1	Bra nch 2	Bra nch 3	Bra nch 4	Bra nch 5	Bra nch 6	Bra nch 7
No de	1	1	0	1	0	0	0

1							
No de 2	0	-1	1	-1	0	0	0
No de 3	0	0	1	0	1	0	1
No de 4	-1	0	0	1	-1	1	0

The nodal incidence matrix is now a 4 × 7 matrix given by

$$A = \begin{bmatrix} 1 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & -1 & 1 & -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 \\ -1 & 0 & 0 & 1 & 1 & 1 & 0 \end{bmatrix}$$

To construct the electrical network in figure 1 from the nodal incidence matrix above, we will follow the reverse operation of the step above,

From the nodal incidence matrix, we will construct table-1 to show us clearly now the nodes and branches are related.

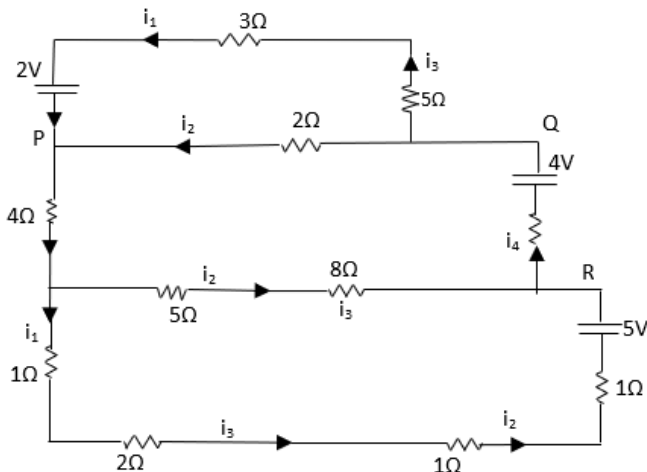


Figure 3. Network and relating the current

Node P: $3i_1 - 2i_2 + 5i_3 = 2$
 Node Q: $4i_1 + 5i_2 + 8i_3 + i_4 = 4$
 Node R: $4i_1 + 5i_2 + 8i_3 + i_4 = 4$
 Upper Loop: $i_1 + i_2 + 2i_3 + i_4 = 5$

Lower Loop: $2i_1 + 7i_2 + 6i_3 + 5i_4 = 7$

(A)Solve the following linear system by using Gauss-elimination.

$$\begin{aligned} 3x_1 - 2x_2 + 5x_3 &= 2 \\ 4x_1 + 5x_2 + 8x_3 + x_4 &= 4 \\ x_1 + x_2 + 2x_3 + x_4 &= 5 \\ 2x_1 + 7x_2 + 6x_3 + 5x_4 &= 7 \end{aligned}$$

$$\begin{bmatrix} 3 & -2 & 5 & 0 \\ 4 & 5 & 8 & 1 \\ 1 & 1 & 2 & 1 \\ 2 & 7 & 6 & 5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{bmatrix}$$

$$\begin{array}{l} 0 \\ 1 \\ 2 \\ 3 \end{array} \left[\begin{array}{cccc|c} 0 & 1 & 2 & 3 & b \\ 3 & -2 & 5 & 0 & 2 \\ 4 & 5 & 8 & 1 & 4 \\ 1 & 1 & 2 & 1 & 5 \\ 2 & 7 & 6 & 5 & 7 \end{array} \right]$$

$$\frac{a[0,0]}{a[1,0]} = \frac{3}{4} = 0.75 \times \text{row}[1]$$

$$\frac{a[0,0]}{a[2,0]} = \frac{3}{1} = 3 \times \text{row}[2]$$

$$\frac{a[0,0]}{a[3,0]} = \frac{3}{2} = 1.5 \times \text{row}[3]$$

$$\begin{array}{l} 0 \\ 1 \\ 2 \\ 3 \end{array} \left[\begin{array}{cccc|c} 0 & 1 & 2 & 3 & b \\ 3 & -2 & 5 & 0 & 2 \\ 3 & 3.75 & 6 & 0.75 & 3 \\ 3 & 3 & 6 & 3 & 15 \\ 3 & 10.5 & 9 & 7.5 & 10.5 \end{array} \right]$$

$$\frac{a[1,1]}{a[2,1]} = \frac{-5.75}{-5} = 1.15 \times \text{row}[2]$$

$$\frac{a[1,1]}{a[3,1]} = \frac{-5.75}{-12.5} = 0.46 \times \text{row}[3]$$

$$\begin{array}{l} 0 \\ 1 \\ 2 \\ 3 \end{array} \left[\begin{array}{cccc|c} 0 & 1 & 2 & 3 & b \\ 3 & -2 & 5 & 0 & 2 \\ 0 & -5.75 & -1 & -0.75 & -1 \\ 0 & -5 & -1 & -3 & -13 \\ 0 & -12.5 & -4 & -7.5 & -8.5 \end{array} \right]$$

$$\frac{a[1,1]}{a[2,1]} = \frac{-5.75}{-5} = 1.15 \times \text{row}[2]$$

$$\frac{a[1,1]}{a[3,1]} = \frac{-5.75}{-12.5} = 0.46 \times \text{row}[3]$$

$$\begin{array}{l} 0 \\ 1 \\ 2 \\ 3 \end{array} \left| \begin{array}{ccc|c} 0 & 1 & 2 & 3 & b \\ 3 & -2 & 5 & 0 & 2 \\ 0 & -5.75 & -1 & -0.75 & -1 \\ 0 & -5.75 & -1.15 & -3.45 & -14.95 \\ 0 & -5.75 & -1.84 & -3.45 & -3.91 \end{array} \right.$$

$$\begin{array}{l} 0 \\ 1 \\ 2 \\ 3 \end{array} \left| \begin{array}{ccc|c} 0 & 1 & 2 & 3 & b \\ 3 & -2 & 5 & 0 & 2 \\ 0 & -5.75 & -1 & -0.75 & -1 \\ 0 & 0 & 0.15 & 2.7 & 13.95 \\ 0 & 0 & 0.84 & 2.7 & 2.91 \end{array} \right.$$

$$\frac{a[2,2]}{a[3,2]} = \frac{0.15}{0.84} = 0.178571 \times \text{row}[3]$$

$$\begin{array}{l} 0 \\ 1 \\ 2 \\ 3 \end{array} \left| \begin{array}{ccc|c} 0 & 1 & 2 & 3 & b \\ 3 & -2 & 5 & 0 & 2 \\ 0 & -5.75 & -1 & -0.75 & -1 \\ 0 & 0 & 0.15 & 2.7 & 13.95 \\ 0 & 0 & 0.15 & 0.482143 & 0.519643 \end{array} \right.$$

$$\begin{array}{l} 0 \\ 1 \\ 2 \\ 3 \end{array} \left| \begin{array}{ccc|c} 0 & 1 & 2 & 3 & b \\ 3 & -2 & 5 & 0 & 2 \\ 0 & -5.75 & -1 & -0.75 & -1 \\ 0 & 0 & 0.15 & 2.7 & 13.95 \\ 0 & 0 & 0 & 2.217857 & 13.430357 \end{array} \right.$$

$$x[3] = \frac{b[3]}{a[3,3]} = \frac{13.430357}{2.217857} = 6.055556$$

$$x[2] = \frac{(b[2] - a[2,3] * x[3])}{a[2,2]} = \frac{[13.95 - 2.7 \times 6.055556]}{0.15} = -16$$

$$x[1] = \frac{(b[1] - a[1,3] * x[3] - a[1,2] * x[2])}{a[1,1]} = \frac{[-1 - (-0.75) \times 6.055556 - (-1) \times (-16)]}{(-5.75)} = 2.166667$$

$$x[0] = \frac{(b[0] - a[0,3] * x[3] - a[0,2] * x[2] - a[0,1] * x[1])}{a[0,0]} = \frac{[2 - 0 \times 6.055556 - 5 \times (-16) - (-2) \times 2.166667]}{3} = 28.777778$$

The solution is $\begin{bmatrix} x_0 \\ x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 28.777778 \\ 2.166667 \\ -16 \\ 6.055556 \end{bmatrix}$

(B) Solving the linear equation of Gauss-elimination by using Python Code

Code

```
[2,7,6,5]],float)
b=array([2,4,5,7],float)
n=len(b)
x=zeros(n,float)
#Elimination
for k in range(n-1):
    for i in range(k+1,n):
        if a[i,k]==0: continue
        factor=a[k,k]/a[i,k]
        for j in range(k,n):
            a[i,j]=a[k,j]-a[i,j]*factor
        b[i]=b[k]-b[i]*factor
print(a)
print(b)
#Back-substitution
x[n-1]=b[n-1]/a[n-1,n-1]
for i in range(n-2,-1,-1):
```

The solution of the system is [28.777777778, .1666668, -16, 6.0555556]

(C) Solve the following linear system by using Gauss-Jordan elimination.

$$a = \begin{bmatrix} 0 & 2 & 0 & 1 \\ 2 & 2 & 3 & 2 \\ 4 & -3 & 0 & 1 \\ 6 & 1 & -6 & -5 \end{bmatrix}, b = \begin{bmatrix} 0 \\ -2 \\ -7 \\ 6 \end{bmatrix}$$

$$\begin{array}{l} 0 \\ 1 \\ 2 \\ 3 \end{array} \left| \begin{array}{ccc|c} 0 & 1 & 2 & 3 & b \\ 0 & 2 & 0 & 1 & 0 \\ 2 & 2 & 3 & 2 & -2 \\ 4 & -3 & 0 & 1 & -7 \\ 6 & 1 & -6 & -5 & 6 \end{array} \right.$$

$$\begin{array}{l} 0 \\ 1 \\ 2 \\ 3 \end{array} \left| \begin{array}{ccc|c} 0 & 1 & 2 & 3 & b \\ 2 & 2 & 3 & 2 & -2 \\ 0 & 2 & 0 & 1 & 0 \\ 4 & -3 & 0 & 1 & -7 \\ 6 & 1 & -6 & -5 & 6 \end{array} \right.$$

Row[0] = Row[0]/2

$$\begin{array}{l} 0 \\ 1 \\ 2 \\ 3 \end{array} \left| \begin{array}{ccc|c} 0 & 1 & 2 & 3 & b \\ 1 & 1 & 1.5 & 1 & -1 \\ 0 & 2 & 0 & 1 & 0 \\ 4 & -3 & 0 & 1 & -7 \\ 6 & 1 & -6 & -5 & 6 \end{array} \right.$$

Row[2] = Row[2] - 4 * Row[0]

Row[3] = Row[3] - 6 * Row[0]

$$\begin{array}{l} 0 \\ 1 \\ 2 \\ 3 \end{array} \left| \begin{array}{cccc|c} 0 & 1 & 2 & 3 & b \\ 1 & 1 & 1.5 & 1 & -1 \\ 0 & 2 & 0 & 1 & 0 \\ 0 & -7 & -6 & -3 & -3 \\ 0 & -5 & -15 & -11 & 12 \end{array} \right.$$

Row[1] = Row[1]/2

$$\begin{array}{l} 0 \\ 1 \\ 2 \\ 3 \end{array} \left| \begin{array}{cccc|c} 0 & 1 & 2 & 3 & b \\ 1 & 1 & 1.5 & 1 & -1 \\ 0 & 1 & 0 & 0.5 & 0 \\ 0 & -7 & -6 & -3 & -3 \\ 0 & -5 & -15 & -11 & 12 \end{array} \right.$$

Row[0] = Row[0] - 1 * Row[1]

Row[2] = Row[2] - (-7) * Row[1]

Row[3] = Row[3] - (-5) * Row[1]

$$\begin{array}{l} 0 \\ 1 \\ 2 \\ 3 \end{array} \left| \begin{array}{cccc|c} 0 & 1 & 2 & 3 & b \\ 1 & 0 & 1.5 & 0.5 & -1 \\ 0 & 1 & 0 & 0.5 & 0 \\ 0 & 0 & 1 & -0.0833 & 0.5 \\ 0 & 0 & -15 & -8.5 & 12 \end{array} \right.$$

Row[0] = Row[0] - 1.5 * Row[2]

Row[3] = Row[3] - (-1.5) * Row[2]

$$\begin{array}{l} 0 \\ 1 \\ 2 \\ 3 \end{array} \left| \begin{array}{cccc|c} 0 & 1 & 2 & 3 & b \\ 1 & 0 & 0 & 0.625 & -1.75 \\ 0 & 1 & 0 & 0.5 & 0 \\ 0 & 0 & 1 & -0.0833 & 0.5 \\ 0 & 0 & 0 & -9.75 & 19.5 \end{array} \right.$$

Row[3] = Row[3]/(-9.75)

$$\begin{array}{l} 0 \\ 1 \\ 2 \\ 3 \end{array} \left| \begin{array}{cccc|c} 0 & 1 & 2 & 3 & b \\ 1 & 0 & 0 & 0.625 & -1.75 \\ 0 & 1 & 0 & 0.5 & 0 \\ 0 & 0 & 1 & -0.0833 & 0.5 \\ 0 & 0 & 0 & 1 & -2 \end{array} \right.$$

Row[0] = Row[0] - (0.625) * Row[3]

Row[1] = Row[1] - (0.5) * Row[3]

Row[2] = Row[2] - (-0.0833) * Row[3]

$$\begin{array}{l} 0 \\ 1 \\ 2 \\ 3 \end{array} \left| \begin{array}{cccc|c} 0 & 1 & 2 & 3 & b \\ 1 & 0 & 0 & 0 & -0.5 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0.3333 \\ 0 & 0 & 0 & 1 & -2 \end{array} \right.$$

The solution of the system $\begin{bmatrix} -0.5 \\ 1 \\ 0.3333 \\ -2 \end{bmatrix}$

(D) Solving the linear equation of Gauss-Jordan elimination by using Python Code

```
#main loop
for k in range (n):
    #Partial Pivoting
    if np.fabs (a[k, k])<1.0e-12: import numpy as np
def gassjrdn(a,b):
    a=np.array (a, float)
    b=np.array (b, float)
    n=len (b)
    For i in range (k+1,n):
        if np . fabs (a [i, k]) > np . fabs (a[k, k]) :
            for j in range (k, n) :
                a [k, j] , a [i, j ] = a [i, j] , a [k, j]
                b[k],b [i] =b [i] , b [k]
            break
    #Division of the pivot row
    pivot = a[k, k]
    for j in range (k, n):
        a [k, j] /= pivot
    b [k] /= pivot
    #Elimination loop
    for i in range (n) :
        if i == k or a [i, k] == 0: continue
        factor = a [i, k]
        a = [ [0,2,0,1] , [2,2,3,2] , [4,-3,0,1], [6,1,-6,-5] ]
        b = [0,-2,-7,6]
        x, A = g s s j r d n (a, b)
    print ("The solution:")
    print (X)
    print ("The transformed [a]: ")
    print(A)
```

The solution of the system is [-0.5, 1, 0.33333333, -2

IV. CONCLUSION

In this paper, firstly we have considered how to derive nodal incidence matrices from electrical networks, conversely how to sketch electrical networks from nodal incidence matrices. Secondly,

we calculate the linear equation by using python code with two methods. The paper aims to show that linear equation is calculated. The advantage of this paper is that it would be desirable to continuous the study of method engineering networks, networking in the computer studies and any other scientific field.

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