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Application of Option Pricing Framework for Analyzing and Predicting Stock Prices in Energy Markets

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ABSTRACT

In this study, the evaluation of the pricing framework for predicting West Texas Intermediate crude oil stock was implemented where detailed analysis with varying changepoint shows that an arbitrage-free forward price can be derived from the buy-and hold strategy in the energy market thereby enabling investors in the market willing to be salvage from the market uncertainties as well as Arrow-Debreu situations to execute a spot or forward contracts depending on the time and place the market becomes favorable.

Keywords : Energy, contract, volatility, lévy process, jumps.

2010 AMS Subject Classification : 91G10, 91G20, 91G80.

I. INTRODUCTION

The development and evaluation of pricing framework for an energy market has motivated several approaches for optimal estimation of results as [1] who extended the more direct ap- proach from one-factor model into a two-factor model for general commodity futures while capturing roughly term-structures (forward curves) features. Meanwhile the other line of re- search is to model futures markets directly, without considering spot prices uising Heath-Jarrow- Morton-type of model (HJM) which is given in the context of power in [2] while considering the problem of modeling the pricing dynamics of forward and futures contracts in the relation of spot, forward and swap-price dynamics.

However, [1] addressed the challenges around the pricing model in ther energy market using the two-factor model by modeling the futures price directly while fitting it to option data.

An energy forwards is the obligation to buy or sell a specific amount of energy derivative at a predetermined delivery price during a fixed delivery period. The forwards contract is set up such that initially no payment has to be made and while it is fixed, the price which makes the contract have a zero value will change over time. The price is called futures price and is quoted at the exchange as the component of the term-structure which is used to evaluate the direction of volatility of the energy forwards.

The approach of stochastic modeling of fixed-income markets can be roughly divided into two; either with a stochastic model for the spot interest rate or deriving bond prices based on no-arbitrage principles. However, because the HJM utilizes very many factors unlike the previous models explained by one factor, it serves as a better alternative to specify the complete forward rates curve dynamics directly. From general arbitrage theory the forward price can be derived as the risk-neutral expected present value of the spot price at the delivery time, assuming a liquid market for the spot and based on a stochastic model for the time evolution of the energy spot price, one can derive the forward and futures price dynamics by appealing to the arbitrage theory ([1]). The main objective of this study is to apply the pricing framework to predict stock prices in energy markets.

II. METHODS AND MATERIAL

2.1 Forward Curve Modelling and Representation.

Evaluating the energy forward curve requires a proper model framework to benchmark the jumps in the curves. As a result, we are interested in studying the dynamics of the forward curve $F(t, T)$, $0 \leq t \leq T$, of a contract delivering a commodity at time $T > 0$, and adopting the Heath-Jarrow-Merton (HJM) approach gives the dynamics of F as the solution of a stochastic differential equation

$$dF(t,T) = \alpha(t,T)dt + \sigma(t,T)dL(t) \qquad (1)$$

for some infinite-dimensional Lévy process $L(t)$ and appropriately defined parameters α and σ. Since the forward curve $F(t,T)_{T\geq t}$ changes its domain over time is more convenient to work with the Musiela parametrisation introduced by ;

$$f(t,\tau) = F(t,t+\tau), \tau \geq 0. \qquad (2)$$

We interpret $\tau = T - t$ as time to delivery, whereas T is the time of delivery and $T = \tau + t$. Then heuristically speaking, the forward curve follows the stochastic partial differential equation (SPDE)

$$f(t,T-t) = F(t,t+T-t) = F(t,T)$$

which is replaced in Equation (1) such that

$$df(t,\tau) = (\alpha(t,t+\tau) + \alpha_0 f(t,\tau))dt + \sigma(t,t+\tau)dL(t). \qquad (3)$$

However, in order to make sense of this SPDE, it is useful to work in an appropriate space of function which contain the entire forward curve $(f(t,\tau))_{\tau\geq 0}$ for any time $t \geq 0$. We shall mostly be concerned with the more convenient representation of F in terms of the Musiela parametrisation where we get

$$F(t,T) = f(t,T-t),$$

for a function $f(t,\tau), \tau \geq 0$, denoting the forward price at time t for a constant with time delivery τ. We shall interpret f as a stochastic process with values in a space of functions on $\mathbb{R}_+$. If we model the forward curve directly under the pricing measure $\mathbb{Q}$ which is the common strategy in the HJM approach, martingale conditions must be imposed in the dynamics. It is to be noted that the forward contracts are liquidly traded financial assets in most commodity markets, and therefore, it may sometimes be natural to model the dynamics under the market probability $\mathbb{P}$.

2.2 The Term Structure of Bond Markets

The most basic interest rate contract is a bond which pays the holder with certainty one unit of cash at a fixed future maturity date T with a price denoted by $P(t,T)$ at time $t \leq T$ where t is any present time before maturation. Therefore, the term structure of bond prices $\{P(t,T)|T \geq t\}$ is a deterministic non-increasing, positive curve with $P(t,t) = 1$. Whereas for fixed maturity T, $\{P(t,T)|t \in [0,T]\}$ is a stochastic process because the economy and the market beliefs about the future value of money, changes in time and is not certain in future. Therefore, a more informative measure of the current bond market at time t is defined as the term structure of interest rates or forward curve $\{f(t,T)|T \geq t\}$ given by

$$P(t,T) = exp\left(-\int_0^{T-t} r\,(t,s)ds\right) = exp\left(-\int_t^{T} f\,(t,s)ds\right), 0 \geq t \geq T \quad (4)$$

The function $f(t,\cdot)$ is a fortiori local integrable and $P(t,\cdot)$ is absolutely continuous. Hence, for the purpose of this study we call $f(t,T)$ the continuously compounded instantaneous forward rate for date T or an interest rate over the infinitesimal time interval $[T,T+dT]$ from time t. Therefore, Equation (4) shows that the forward curve contains all the original bond price information which can be completely recovered. And describing the bond prices in a complete market of a deterministic arbitrage free situation, we have that;

$$P(t,T) = P(t,S)P(S,T), \forall t \leq S \leq T \quad (5)$$

However, if $P(t,T) > P(t,S)P(S,T)$ for some $t \leq S \leq T$, then an arbitrage opportunity is described when at maturity T a bond bought $P(S,T)$ with maturity S at time t leads to a risk-less net gain at terminal time T. And as for arbitrage, it can be formulated such that a complete probability space $\left(\Omega, F, \{F_t\}_{t\in\mathbb{R}_+}, \mathbb{P}\right)$ satisfy the usual conditions, and under such appropriate assumptions, a particular no-arbitrage condition is equivalent to the existence of a probability measure $\mathbb{Q} \sim \mathbb{P}$ under which discounted bond price processes

$$P(t,T)/B(t), t \in [0,T]$$

follow local martingales. Here, we define the amount of cash accumulated $B(t)$ up to time t starting with one unit at time 0 and continually reinvesting at the short $R(s), s \in [0,t]$ by;

$$B(t) = exp\left(\int_0^t R\,(s)ds\right), t \in \mathbb{R}_+$$

where $R(s) = f(t,t)$. The measure $\mathbb{Q}$ is called an equivalent local martingale measure (ELMM) or risk neutral measure and by the absence of arbitrage we shall mean from now on the existence of such measure $\mathbb{Q}$. Assuming a frictionless market, i.e. there are no transaction costs, taxes, or short sale restrictions which we may have on an investment over the infinitesimal interval $[t,t+dt]$ if the contract is made at t, and the bonds are perfectly divisible then viewing the forwards rates $f(t,T)$ as an estimate of the future short rate $r(t,T-t)$ and assuming $m(t,U) = U - t$, that we had

$$L(t,T,U) = \left(\frac{P(t,T)}{P(t,U)} - 1\right)\frac{1}{m(T,U)} = -\frac{1}{P(t,U)}\left(\frac{P(t,U) - P(t,T)}{U-T}\right) \quad (6)$$

and when we assume U and T to be very close, then

$$\lim_{U\to T} L(t,T,U) = \lim_{U\to T}\frac{-1}{P(t,U)}\left(\frac{P(t,U) - P(t,T)}{U-T}\right) = \frac{-1}{P(t,U)}\frac{\partial P(t,T)}{\partial T} \quad (7)$$

because

$$\frac{\partial \log P(t,T)}{\partial T} = \frac{1}{P(t,T)}\frac{\partial P(t,T)}{\partial T}$$

We therefore define the instantaneous forward rate with maturity T

$$f(t,T) = -\frac{\delta \log P(t,T)}{\delta T} \quad (8)$$

and under the martingale measure $\mathbb{Q}$, the dynamics of the forward rates are of the Heath-Jarrow-Morton (HJM) form;

$$\boldsymbol{df(t,T) = \alpha(t,T)dt + \sigma(t,T)dL(t)} \quad (9)$$

where $df(t,T)$ is the instantaneous forward interest rate of zero-coupon bond with maturity T, and it is assumed to satisfy the stochastic differential equation (9]), L is an m-dimensional $\mathbb{Q}$ -Wiener process (Brownian motion) under the risk-neutral assumption, and the adapted α and σ are the drift and volatility functions respectively, which is investigated in subsequent sections of this study. We take into consideration that the use of the HJM model here is best for use in modeling forward interest rates which are then modeled to an existing term structure of interest rates which is used to determine appropriate prices for interest rate sensitive securities as well as to seek arbitrage opportunities in pricing the underlying contracts. Also, for each maturity date T , the evolution across calendar time t (where $t \leq T$) of the forward rate $f(t,T)$ is govern by (9).

2.3. Forward Curve and Factor Models

In practice, the forward curve cannot be observed directly on the market, hence it has to be estimated and common methods of estimating forward rates curves can be represented as a parametrized family on a forward curve manifold ϱ given by;

$$\boldsymbol{\varrho} = \{\boldsymbol{G}(\cdot,\boldsymbol{z}) \in \boldsymbol{C}[\boldsymbol{0},\infty) | \boldsymbol{z} \in \boldsymbol{Z}\} \tag{10}$$

where

$$\boldsymbol{G}: \boldsymbol{Z} \to \boldsymbol{C}[\boldsymbol{0},\infty) \tag{11}$$

is smooth curves with $Z \subset \mathbb{R}^m$ is a finite dimensional parameter set, for some $m \in \mathbb{N}$ and for each appropriate choice of parameter $z \in Z$. Hence, by slight change in notation, we write the optimal fit of the forward curve as

$$\tau \to G(\tau; z) \tag{12}$$

where the variable τ is interpreted as the time to maturity, as opposed to the time of maturity T, i.e. $\tau = T - t$. The main problem is to determine under which conditions the interest rate model ([eqtn1]) is consistent with the parametrized family of forward curves ([eqtn2]), in the following sense:

1. Assume that at an arbitrage chosen time $t = s$, we have fitted a forward curve G to market data. Technically, this means that we have specified an initial forward curve, i.e. for some $z_0 \in Z$ we have
$$f^*(s, s+\tau) = F(\tau, z_0), \forall \tau \geq 0. \tag{13}$$
2. Is it then the case that the subsequent forward curves produced by the interest rate model (9) always stay within the given forward curve family, i.e. does there at every fixed time $t \geq s$ exist some $z \in Z$ such that
$$f(t, t+\tau) = G(\tau; z), \forall \tau \geq 0? \tag{14}$$
3. Here, z may depend on t and on the elementary outcome $\omega \in \Omega$.

Now, suppose we introduce the Musiela parametrization $r_t(\tau) = f(t, t+\tau)$ where $\tau \geq 0$. Let $\{S(t) | t \in \mathbb{R}_+\}$ denote the semigroup of right shifts which is defined by

$$S(t)f(\tau) = f(t+\tau)$$

We denote the induced dynamics for the r-process by

$$dr(t,\tau) = \beta(t,\tau)dt + \sigma_0(t,\tau)dL(t), \tag{15}$$

and it is easy to see that there is a one-to-one correspondence between the formulations (9) and (15), namely

$$\begin{aligned} \beta(t,\tau) &= \frac{\partial}{\partial x} r(t,\tau) + \alpha(t, t+\tau) \text{ and} \\ \sigma_0(t,\tau) &= \sigma(t, t+\tau) \end{aligned}$$

Therefore, we can now transfer the HJM drift condition to the Musiela parameterization case.

Proposition 1. [5] Under the martingale measure $\mathbb{Q}$, the r-dynamics must be of the form

$$dr(t,\tau) = \left\{\frac{\partial}{\partial x} r(t,\tau) + \sigma_0(t,\tau)\int_0^{\tau} \sigma_0\,(t,z)' dz\right\} dt + \sigma_0(t,\tau) dL(t). \tag{16}$$

Thus the interest rate model is completely characterized by the initial curve and the volatility structure $\sigma_0(t,\tau)$ and hence with the presence of arbitrage situation with a martingale measure $\mathbb{Q}$, it clearly shows that the forward curves is being modelled in an incomplete market. However, as the focus of this current study, we make efforts to integrate the properties of completeness from Banach space in order to evaluate the forward rates in an arbitrage situation as well as time variation which is key while determining the volatility structure (jump diffusion) part $\{\sigma_0(t,\tau)dL(t): t \geq 0, \tau \geq 0\}$ in Equation (16) and evaluating the size and impact of the jumps as captured by the Lévy process ($L(t)$). In that case, according to [6], the forward rate process $\{r(t,\tau)dL(t): t \geq 0, \tau \geq 0\}$ must satisfy the following stochastic partial differential equation

$$dr(t,\tau) = \frac{\partial}{\partial x}\left(\left(r(t,\tau) + \frac{1}{2}|\sigma_0(t,\tau)|^2\right) dt + \sigma(t,\tau) \cdot dL(t)\right) \tag{17}$$

for all $t, \tau \geq 0$, where $L(t)$ is also define with the properties of a d-dimensional Brownian Motion, the volatility process $\{\sigma(t,\tau); t \geq 0\}$ is F_t-adapted with values in $\mathbb{R}^d$, while $||$ and $\cdot$ is the standard norm and inner product in $\mathbb{R}^d$, respectively. It is significant to say that (17) is sufficient for the non-arbitrage condition of a complete market which is a paramount characterization for this study.

2.4 Forward Curves with Lévy Jumps

In an incomplete market, energy commodity valuation is best described as stochastic process due to the volatile nature and its subsequent jumps in the forward curves because of the randomness in the pricing models. Hence, motivated by the approach of [7], the jumps in energy forwards is captured as an entity that is best described in a probability space of some adapted process hence; Property (2) implies that Markov property (i.e. conditional probability distribution of futures states depend only on the present state). Property (3) indicates that knowing the distribution of W_t for $t \leq \tau$ provides no predictive information about the process when $t > \tau$.

Definition 1 (Poisson Process). A Poisson process $N = (N_t)_{0 \leq t < \infty}$ satisfies the following three properties;

1. $N_0 = 0$
2. N has independent increments: $N_t - N_s$ is independent of F_s, $0 \leq s < t < \infty$
3. N has stationary increments: $P(N_t - N_s \leq \tau) \forall 0 \leq s < t < \infty$

In practice, stochastic differential equations formulated with only the Poisson process may be ineffective in describing the complex dynamics such as estimating jumps in forward curves. Therefore, the observable jumps in the forward curves of energy market follows a continuous sample path of stochastic processes, however, this study focuses on the discritization of the sample paths and hence we study the price volatility in a filtered probability space projected on some Hilbert space and this motivates the following definition;

Definition 2 (L´evy Process). Let $(\Omega, F, \{F_t\}_{t\geq 0}, \mathbb{P})$ be a filtered probability space, where $\mathbb{P}$ denotes the risk-neutral probability. An F_t-adapted process

Definition 2 (L´evy Process). : $\{L(t)\}_{t\geq 0} \subset \mathbb{R}$ with $L_0 = 0$ (almost surely) is called a Lévy process if L_t is continuous in probability and has stationary, independent increments with the following properties;

1. $\mathbb{P}\{L_0 = 0\} = 1$.
2. Stationary increments: the distribution $L_{t+s} - L_t$ over the interval $[t, t+s]$ does not depend on t but on length of interval s.
3. Independent increments: for every increasing sequence of times $t_0, \ldots, t_n$ the random variables: $L_{t_0}, L_{t_1} - L_{t_0}, \ldots, L_{t_{n+1}} - L_{t_n}$ are independent.
4. Stochastic continuity: $\forall \epsilon > 0$, $\lim_{h \to 0} \mathbb{P}(|L_{t+h} - L_t| \geq \epsilon)$ i.e. discontinuity occurs at random times.
5. The paths of L_t are $\mathbb{P}$ almost surely right continuous with left limits (Cadlag paths which is defined in the theorem below).

Definition 3. [8] Assume that $(\Omega, F, (F_t), \mathbb{P})$ is a filtered probability space and that L is a Wiener process in $\mathbb{R}^d$ adapted to (F_t). Then L is a Wiener process with respect to (F_t) or an (F_t)-Wiener process if, $\forall t, h \geq 0$, $L(t+h) - L(t)$ is independent of F_t.

Definition 4. [8] Let $q > 0$. A real-valued mean-zero Gaussian process $L = (L(t), t \geq 0)$ with continuous trajectories and covariance function

$$\mathbb{E}L(t)L(s) = (t \wedge s)q, t, s \geq 0 \tag{18}$$

is called a Wiener process with diffusion q. If the diffusion is equal to 1 i.e. if $q = 1$ and

$$\mathbb{E}L(t)\mathrm{L}(s) = (t \wedge s), t, s \geq 0 \tag{19}$$

then L is called standard Wiener process or standard Brownian motion.

Theorem 1. Let $\{L(t)\}$ be a Lévy process. Then L_t has a Cadlag version which is also a Lévy process.

Suppose we assuming that Theorem ([theorem1]) is consistent for all Lévy process, then it is safe to take the Lévy process for this study as cadlag so that the jump of the Lévy process L_t at $t \geq 0$ is defined by

$$\Delta L_t = L_{t^+} - L_{t^-} \tag{20}$$

Taking L_{t^+} and L_{t^-} as positive and negative (upward and downward) movement in forwards curves respectively in an incomplete energy market. Therefore, Theorem [theorem1] leads to the following assumption.

Theorem 2. [9] : Assume that $(L(t), t \geq 0)$ is a cadlag Lévy process in a Banach space B with jumps bounded by a fixed number $c > 0$; that is $|\Delta L(t)|_B \leq c$ for every $t \geq 0$. Then, for any $\beta > 0$ and $t \leq 0$

$$\mathbb{E}[e^{\beta|L(t)|_B}] < \infty \tag{21}$$

This processes further strengthens L_t , $\forall t \geq 0$ therefore, suppose that B_0 be the family of Borel sets $U \subset \mathbb{R}$ whose closure $\overline{U}$ does not contain 0. For $U \in B_0$, let $N(t, U)$ represent the number of jumps in the forwards curves described in a Lévy market of size $\Delta L_x \in U$ which occur before or at time t so that

$$N(t, U) = N(t, U, \omega) = \textstyle\sum_{x:0<x\leq 1} \chi_U (\Delta L_x) \tag{22}$$

$N(t, U)$ is a Poisson random (jumps) measure for a forward curve in energy market in time $t_1, t_2 \geq 0$. It measures the sizes of the continuous sample path movement between two consecutive prices in the energy market. The differential form of this measure is written in the form of $N(dt, dz)$, and for times $0 \leq t_1 \leq t_2 < \infty$, we therefore denote the differential $N(dt, dz)$ as $N(dt, dz) = N(t_2, U) - N(t_1, U), 0 \leq t_1 \leq t_2 < \infty$.

In (22) χ is generated measure while N is the number of jumps of electricity forwards prices in a Lévy market.

Remark 1. Taking a finite $N(t,U)$ for all $U \in B_0$, and define

$$T_1(\omega) = inf\{t > 0, L_t \in U\}$$

for all $T_1(\omega) > 0$ (almost surely). Then by right continuity of paths we have

$$\lim_{t \to 0^+} L(t) = L(0) = 0, \text{almost surely}$$

Therefore, for all $\epsilon > 0$ there exist $t(\epsilon) > 0$ such that $|L(t)| < \epsilon$ for all $t < t(\epsilon)$. This implies that the Lévy process $L(t) \notin U$ for all $t < t(\epsilon)$, if $\epsilon < dist(0,U)$. By inductive definition

$$T_{n+1}(\omega) = inf\{t > T_n(\omega); \Delta L_t \in U\}.$$

Then by the above argument, $T_{n+1} > T_n$ (almost surely). Hence $T_\text{n} \to \infty$ as $n \to \infty$ (almost surely). Assume then that $T_n \to T \to \infty$. But then $\lim_{x \to T^-} L(S)$ cannot exist. Thereby contradicting the existence of left limits of the paths. It is well-known that Brownian motion $\{B(t)\}_{t \geq 0}$ has stationary and independent increments. Thus $B(t)$ is also a Lévy process.

Theorem 3.

1. The set function $U \to N(t,U,\omega)$ defines a σ-finite measure on B_0 for each fixed t,ω.
2. The set function
$$v(U) = E[N(1,U)]$$
3. where $E = E_P$ denotes expectation with respect to P, also defines a σ-finite measure on B_0, called the Lévy measure of $\{L_t\}$.
4. Fix $U \in B_0$. Then the process $\pi_U(t) = \pi_U(t,\omega) = N(t,U,\omega)$ is a Poisson process of intensity $\lambda = v(U)$.

2.5 Estimation of Jump Sizes

The size of jumps of the forward curves in the Poisson random measure is defined by some Lévy processes which upon decomposition helps to measure the impact size of each continuous sample path movement of the curve. Therefore, we start by introducing the Lévy process decomposition of the measure.

2.5.1 Lévy-Ito Decomposition

Let $L = (L(t), t \geq 0)$ be an U-valued Lévy process with characteristics (α, σ^2, v). The jump at time t is $\Delta L(t) = L(t) - L(t^-)$. Hence we obtain a Poisson random measure N on $\mathbb{R}^+ \times (U - 0)$, which has an intensity measure λv, by the definition;

$$N(t,A) = \{0 \leq s \leq t, \Delta L(t) \in A\},$$

for each $A \in \mathrm{B}(U)$. The associated compensator is denoted by $\widetilde{N}$, so

$$\widetilde{N}(dt,dz) = N(dt,dz) - dtv(dz).$$

We say that $A \in \mathrm{B}(U)$ is bounded below if $0 \notin \tilde{A}$ and $t \geq 0$ so that the compound Poisson process;

$$Y_k(t) = \sum_{0 \leq s \leq t} \Delta\, L(s) 1_{\{\Delta L(s) \in A\}}$$
$$= \int_A z\, N(t,dz) \text{is finite a.s}$$

If A is bounded below and $A \subseteq B_\sigma(0)$ for some $\sigma > 0$, we may define

$$Z_k(t) = Y_k(t) - \int_A z\, v(dz)$$
$$= \int_A z\, N(t,dz) - \int_A z\, v(dz)$$
$$Z_k(t) = \int_A z\, \widetilde{N}(t,dz).$$

and according to , we write

$$Z(t) = \int_{B_1} z\, \widetilde{N}(t,dz),$$

where B_1 is a Borel set containing $(A_n, n \in \mathbb{N})$ which itself is a sequence of Borel sets and $A_\text{n}^c = B_1 - A_n$ is bounded below. We then obtain the Lévy-Ito Decomposition as presented in Theorem (4);

Theorem 4. Let $\{L_t\}$ be a U-valued Lévy process with characteristics (α, σ^2, v) , there exist a Brownian motion B with covariance σ^2 and an independent Poisson random measure N and $\mathbb{R}^+ \times (U - \{0\},)$, with intensity measure λv so that for each $t \geq 0$, the L_t decomposition is given by

$$L_t = \alpha t + \sigma B(t) + \int_{|z|<1} z\,\tilde{N}(t, dz) + \int_{|z|\geq 1} z\,N(t, dz) \tag{23}$$

for some constants $\alpha, \beta \in \mathbb{R}$, $R \in [0, \infty]$.

Here the measure $\tilde{N}(dt, dz) = N(dt, dz) - v(dz)dt$ is called the compensated Poisson random measure of $L(.)$ and $B(t)$ is an independent Brownian motion, $v(dz)$ is Lévy measure of the Lévy process respectively. Also, it is convenient to say that $\int_{|z|>1} z\,N(t, dz)$ is the sum of all jumps (finite many) of size bigger than one and $\int_{|z|<1} z\,\tilde{N}(t, dz)$ process is the sum of small jumps (of size smaller than 1). For any given constants $R > 0$ small jumps and big jumps can be defined as $|z| < R$ and $|z| \geq R$ respectively, such that the corresponding Lévy-Ito decomposition is given as

$$L_t = \alpha t + \sigma B(t) + \int_{|z|<R} z\,\tilde{N}(t, dz) + \int_{|z|\geq R} z\,N(t, dz) \tag{24}$$

Also, suppose $A \in \mathbb{B}_0$ the process

$$M_t = \tilde{N}(t, A) \text{is a martingale} \tag{25}$$

If $\alpha = 0$ and $R = \infty$, we call L_t a Lévy martingale. Note that for this study, we can always assign $R = 1$. In this study, we will be considering jumps of sizes greater than 1. Therefore, we have a measure

$$\tilde{N}(dt, dz) = \begin{cases} N(dt, dz) - v(dz)dt, if|z| < 1 \\ N(dt, dz), if|z| \geq 1 \end{cases} \tag{26}$$

For each $A \in B_0$ the process $M_t = \tilde{N}(t, A)$ is a martingale. Also, if $\alpha = 0$ and $R = \infty$, then L_t is a Lévy martingale.

Theorem 5. Choosing $R = 1$ if

$$E[L_t] < \infty, \forall t \geq 0,$$

then

$$\int_{|z|\geq 1} |z|\,v(dz) \leq \infty$$

and we may choose $R = \infty$ and hence write

$$L_t = \alpha_1 t + \beta B(t) + \int_{\mathbb{R}} z\,\tilde{N}(t, dz).$$

where

$$\alpha_1 = \alpha + \int_{|z|\geq 1} |z|\,v(dz).$$

which reduces Equation ([eqn3.5]) because the jump size is greater than 1 at $\mathbb{R} = \infty$.

2.6 Lévy-Khintchine Representation

A fundamental theorem concerning Lévy processes is the so called Lévy-Khintchine representation:

Theorem 6. Let $\{L_t\}$ be a Lévy process with Lévy measure v. Then

$$\int_{\mathbb{R}} m\,in(1, z^2) v(dz)$$

and its characteristic function is of the form

$$\phi L_t = E[e^{tu} L_t] = \int_{\mathbb{R}} e^{iuL_t}\,v(dx) = e^{t\psi(u)}, u \in \mathbb{R} \tag{27}$$

If there exist a triplet (a, σ^2, v), $a \in \mathbb{R}$, $\sigma \geq 0$, v a measure concentrated on $\mathbb{R}\{0\}$ that satisfies $\int_{\mathbb{R}} (1 \wedge x^2)\,v(dx) \leq \infty$ such that

$$\psi(u) = -\frac{1}{2}\sigma^2 u^2 + i\alpha u + \int_{|z|<R} \{e^{iuz} - 1 - iuz\} v(dz) + \int_{|z|\geq R} (e^{iuz} - 1) v dz \tag{28}$$

where ψ is called the characteristics exponent. Conversely, given constants α, σ^2 and a measure v on $\mathbb{R}$ such that

$$\int_{\mathbb{R}} m\,in(1, z^2) v(dz) < \infty.$$

There exist a Lévy process $L(t)$ (unique in law) such that ([eq9]) and ([eq10]) hold and it is possible that

$$\int_{|z|\geq 1} |z|\, v(dz) = \infty.$$

We also know that a Lévy process is a semimartingale. This theorem is the core of the theory of Lévy processes, and deserves much attention. It states that every Lévy process has a characteristic function of the form of equation (27). Thus, they can be parameterized using the triplet (a, σ^2, v), where a is the drift, σ^2 is the variance of a Brownian motion and $v(dx)$ is the so called Lévy measure or jump measure.

Definition 5. Let D_{ucp} denote the space of cadlag adapted processes, equipped with the topology of uniform convergence on compacts probability (ucp): $H_n \to H_{ucp}$ if for all $t > 0$ and $sup_{0\leq s\leq t}|H_n(s) - H(s)| \to 0$ in probability. Also, Let L_{ucp} denotes the space of adapted cadlaq process (left continuous with right limits) equipped with the ucp topology. If $H(t)$ is a step function of the from

$$H(t) = H_0 \chi_{(0)}(t) + \sum_t H_i\, \chi_{T_i, T_{i+1}}(t).$$

where $H_t \in F_t$ and $0 = T_0 \leq T_1 \leq \ldots \leq T_{n+1} \leq \infty$ are F_t-stopping times and χ is cadlag, we define

.

$$J_\chi H(t) = \int_0^t H_s\, dL_s = H_0 L_0 + \sum_i H_i\, (L_{T_{i+1\Lambda t}} - L_{T_{i\Lambda t}}); t \geq 0.$$

Theorem 7. Let L be a semimartingale. Then the mapping J_x can be extended to a continuous linear map

$$J_x : L_{ucp} \to D_{ucp}.$$

This constructions allows us to define stochastic integrals of the form

$$\int_0^t H\,(s) d\eta_s$$

for all $H \in Lucp$. In view of the decomposition ([eq6]) this integral can be split into integrals with respect to ds, $dB(s)$, $\widetilde{N}(ds, dz)$ and $N(ds, dz)$. This makes it natural to consider the more general stochastic integral of the form

$$L(t) = L(0) + \int_0^t \alpha\,(s, \omega) ds + \int_0^s \sigma\,(s, \omega) dB(s) + \int_0^t \int_{\mathbb{R}} \gamma\,(s, z, \omega) \widetilde{N}(ds, dz) \tag{29}$$

where the integrads are $\mathbb{F}$-predictable are satisfying the appropriate growth conditions

$$\int_0^t \left\{ |\alpha(s)| + \sigma^2(s) + \int_{\mathbb{R}} \gamma^2\,(s, z) v(dz) \right\} ds < \infty, \forall t > 0$$

for the integrals to exist and for simplicity we put

$$\widetilde{N}(dt, dz) = \begin{cases} N(dt, dz) - v(dz)dt, if |z| < R \\ N(dt, dz), if |z| \geq R \end{cases} \tag{30}$$

with R as in Theorem 3, then we use the following short hand differential notation for process $L(t)$ satisfying ([eq11])

$$dL(t) = \alpha(t)dt + \sigma dB(t) + \int_{\mathbb{R}} \gamma\,(t,z)\tilde{N}(dt,dz) \tag{31}$$

where ([eq12]) is called the Ito-Lévy process and $B(t)$ is a Brownian motion with $\alpha(t)$, $\sigma(t)$ and $\gamma(t)$ satisfying the necessary growth conditions for which the given Ito-Lévy process has unique strong solution $L(t)$.

Since the semimartingale $M(t)$ is called a local martingale up to time T (with respect to P) then there exists an increasing sequence of F_t-stopping times τ_n such that $\lim_{n\to\infty}\tau_n = T$.a.s and

$M(t \wedge \tau_n)$,is a martingale with respect to P for all n

Such that

1. If
$$E\left[\int_0^T \int_{\mathbb{R}} \gamma^2\,(t,z)v(dz)dt\right] < \infty \tag{32}$$
2. then the process
$$M(t) = \int_0^T \int_{\mathbb{R}} \gamma\,(s,z)\tilde{N}(ds,dz), 0 \le t \le T$$
3. is a martingale.
4. If
$$\int_0^T \int_{\mathbb{R}} \gamma^2\,(t,z)v(dz)dt < \infty,\text{a.s.} \tag{33}$$
5. then $M(t)$ is a local martingale, $0 \le t \le T$.

The above conditions therefore leads to the important Ito-Lévy process if for $L(t)$ in Equation ([eq12]) and $f: \mathbb{R}^2 \to \mathbb{R}$ is a C^2 function, is the process $Y(t) = f(t, L(t))$ again an Ito-Lévy process and let $L^{(c)}(t)$ be the continuous part of $L(t)$, i.e., $L^{(c)}(t)$ is obtained by removing the jumps from $L(t)$. Then an increment in $Y(t)$ stems from an increment in $X^{(c)}(t)$ plus the jumps in Equation ([eq3.8]). Therefore, based on the classical Ito formula we guess that;

$$dY(t) = \frac{\partial f}{\partial t}(t, L(t))dt + \frac{\partial f}{\partial x}(t, L(t))dX^{(c)}(t) + \frac{1}{2}\frac{\partial^2 f}{\partial x^2}(t, L(t)) \cdot \sigma^2(t)dt + \int_{\mathbb{R}} \{f(t, L(t^+) + \gamma(t,z)) - f(t, L(t^-))\}\, N(dt,dz).$$

which can support the guess since

$$dL^{(c)}(t) = \left(\alpha(t) - \int_{|z|<R} \gamma\,(t,z)v(dz)\right)dt + \sigma(t)dB(t),$$

which gives th following results; Suppose that $L(t) \in \mathbb{R}$ is an Ito-Lévy process of the form

$$dL(t) = \alpha(t,\omega)dt + \sigma(t,\omega)dB(t) + \int_{\mathbb{R}} \gamma\,(t,z,\omega)\tilde{N}(dt,dz) \tag{34}$$

where

$$\tilde{N}(dt,dz) = \begin{cases} N(dt,dz) - v(dz)dt, if|z| < R \\ N(dt,dz), if|z| \ge R \end{cases} \tag{35}$$

for some $R \in [0,\infty]$. Therefore, suppose $f \in C^2(\mathbb{R}^2)$ and define $Y(t) = f(t, L(t))$. Then $Y(t)$ is again an Ito-Lévy process and

$$\begin{aligned} dY(t) \quad &= \frac{\partial f}{\partial t}(t, L(t))dt + \frac{\partial f}{\partial x}(t, L(t))[\alpha(t,\omega)dt + \sigma(t,\omega)dB(t)] + \frac{1}{2}\sigma^2(t,\omega)\frac{\partial^2 f}{\partial x^2}(t, L(t))dt \\ &+ \int_{\mathbb{R}} \{f(t, L(t^+) + \gamma(t,z,\omega)) - f(t, L(t^-))\} - \frac{\partial f}{\partial t}(t, L(t^-))\gamma(t,z,\omega)v(dz)\mathrm{d}t \\ &+ \int_{\mathbb{R}} \{f(t, L(t^+) + \gamma(t,z)) - f(t, L(t^-))\} \widetilde{N}(dt, dz). \end{aligned} \tag{36}$$

However, solving the stochastic differential equation (SDE) of a Geometric Lévy;

$$dL(t) = L(t^-)\left[\alpha dt + \sigma dB(t) + \int_{\mathbb{R}} \gamma\,(t,z)\widetilde{N}(dt,dz)\right] \tag{37}$$

where α, σ are constants and $\gamma(t,z) \geq -1$. Therefore, in other to estimate $L(t)$ we rewrite Equation (37) as follows;

$$\frac{dL(t)}{L(t^+)} = \alpha dt + \sigma dB(t) + \int_{\mathbb{R}} \gamma\,(t,z)\widetilde{N}(dt,dz)$$

Now define

$$Y(t) = \ln L(t).$$

Then estimating by Ito's formula, we have;

$$\begin{aligned} dY(t) = \frac{dL(t)}{L(t)} &= [\alpha dt + \sigma dB(t)] - \frac{1}{2}\sigma^2 L^{-2}(t)L^2(t)dt \\ + \int_{|z|<R} \{\ln(L(t^-) + \gamma(t,z)L(t^-)) &- \ln(L(t^-)) - L^{-1}(t^-)\gamma(t,z)L(t^-)\}\, v(dz)dt \\ &+ \int_{\mathbb{R}} \{\ln(L(t^-) + \gamma(t,z)L(t^-)) - \ln(L(t^-))\}\, \widetilde{N}(dz)dt \end{aligned} \tag{38}$$

$$\begin{aligned} dY(t) = \left(\alpha - \frac{1}{2}\sigma^2\right)dt + \sigma dB(t) + \int_0^t \int_{|z|<R} \{\ln(1 + \gamma(s,z)) - \gamma(s,z)\}\, v(dz)ds \\ + \int_{\mathbb{R}} \ln\,(1 + \gamma(s,z))\widetilde{N}(ds,dz) \end{aligned} \tag{39}$$

Hence

$$\begin{aligned} Y(t) = Y(0) + \left(\alpha - \tfrac{1}{2}\sigma^2\right)t + \sigma dB(t) \\ + \int_0^t \int_{|z|<R} \{\ln(1 + \gamma(s,z)) - \gamma(s,z)\}\, v(dz)ds \\ + \int_0^t \int_{\mathbb{R}} \ln\,(1 + \gamma(s,z))\widetilde{N}(ds,dz) \end{aligned} \tag{40}$$

However, we recall that

$$Y(t) = \ln(L(t))$$

therefore, this gives the results

$$L(t) = L(0)e^W \tag{41}$$

where

$$W = \left(\alpha - \frac{1}{2}\sigma^2\right)t + \sigma dB(t) + \int_0^t \int_{|z|<R} \{\ln(1+\gamma(s,z)) - \gamma(s,z)\}\, v(dz)ds + \int_0^t \int_{\mathbb{R}} \ln(1+\gamma(s,z))\tilde{N}(ds,dz)$$

Therefore, Equation ([eqn125]) is called the geometric Lévy process which is analogous with the diffusion case where $N = 0$ and it is the expected model for estimating the stock prices with the presence of jumps in the forward curve of energy market.

III. Main Results

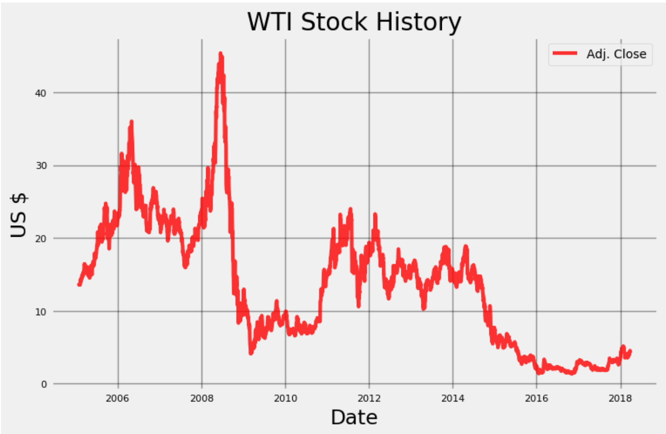

FIGURE 1. Default value plot of the Adjusted Closing price of West Texas Intermediate (WTI) crude oil futures prices from between 2005 to 2018

3.1 Stock Price Analysis - West Texas Intermediate

Default value plot of the Adjusted Closing price of West Texas Intermediate (WTI) crude oil futures prices from between 2005 to 2018

Default value plot of the Adjusted Closing price of West Texas Intermediate (WTI) crude oil futures prices from between 2005 to 2018

In Figure (1), the default value plotted is the Adjusted Closing price of West Texas Intermediate (WTI) crude oil futures prices which accounts for splits in the stock and compares the Daily Change in price of WTI with the Adjusted Volume i.e. the number of shares traded for the entire period.

With an interactively selected new start date: 2000-05-09 for the WTI futures prices it is observed that when the Daily Change in price is compared with number of shares traded over between the entire period from 2005 to 2018, the following results were realized that the Maximum Daily Change = 3.31 on 2011-04-26 and Minimum Daily Change = -3.81 on 2008-07-22 with a Current Daily Change = -0.31 on 2018-03-27. Also, the Maximum Adj. Volume = 9910953.00 on 2016-12-20 and Minimum Adj. Volume = 34700.00 on 2005-05-02 also Current Adj. Volume = 1352638.00 on 2018-03-27.

This clearly shows that the stock price although not at its best, is currently still not doing well at the moment as current daily change is at the lower negative compared to its minimum daily change. Similarly, the current adjusted volume is less than the maximum adjusted volume strictly although more than the minimum adjusted volume and this is obvious because the demand for WTI crude oil declined sharply between 2009 and 2010 with an early rise in 2011 and a constant digression from 2016 till 2018. The 2 years of WTI stock price depression is clearly informs by the decrease in demand of stock volumes and hence price devaluation as seen in Figure (1).

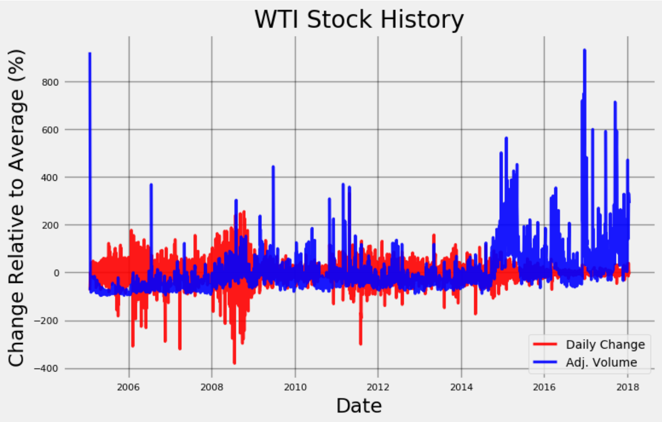

FIGURE 2. Percentage daily change relative to average of West Texas Intermediate (WTI) crude oil futures prices from between 2005 to 2018

From Figure (2), the y-axis is in percentage change relative to the average value for the statistics and the chart clearly shows the jumps in the forward curve clearly constricted to a bare minimum benchmark as seen in the Daily Change prices. The boundary point or limiting change point by the clearly showcases the impact of the properties of Hilbert space of function in limiting the spikes in WTI crude oil daily price change. The scale is necessary because the daily volume is originally in shares, with a range in the hundreds of millions, while daily price change typically is a few dollars. Meanwhile, by converting to percentage change we can look at both datasets on a similar scale. The plot in Figure (2) shows there is no correlation between the number of shares traded and the daily change in price and therefore the forward prices distribution over the curve is asymmetric to the volume of WTI stocks traded. This is contrary to expectation that more shares would be traded on days with larger price changes as demand would increase when investors take advantage of the fluctuation in prices. However, the only real trend seems to be that the volume traded decrease with high prices and increases as price goes downwards.

On this note, suppose an investor invest in 100 shares of WTI at the company's Initial Public Offer (IPO), the buy and hold value from investing in 100 shares between 2005 and 2018 will be a lost of over 1000USD as seen in Figure (3) below;

Buy and Hold profit for West Texas Intermediate (WTI) crude oil futures prices from between 2005 to 2018

Buy and Hold profit for West Texas Intermediate (WTI) crude oil futures prices from between 2005 to 2018

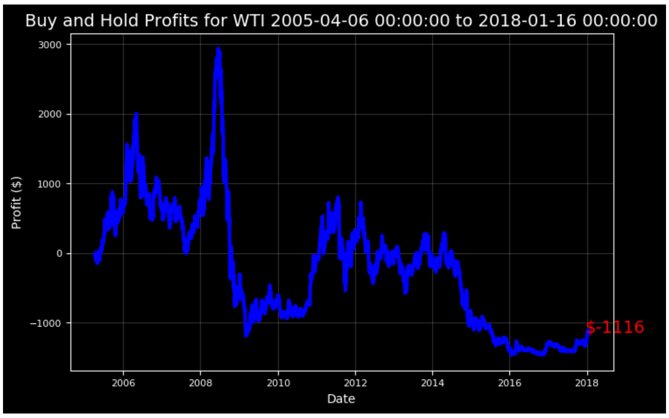

FIGURE 3. Buy and Hold profit for West Texas Intermediate (WTI) crude oil futures prices from between 2005 to 2018

However, for a different trading period of say between 2005 and 2012, the investor who invests 100 shares of WTI at the company's IPO will have a buy and hold profit of about USD350 despite the price fluctuation/depreciation from 2009 to 2011 as seen in Figure (4) below;

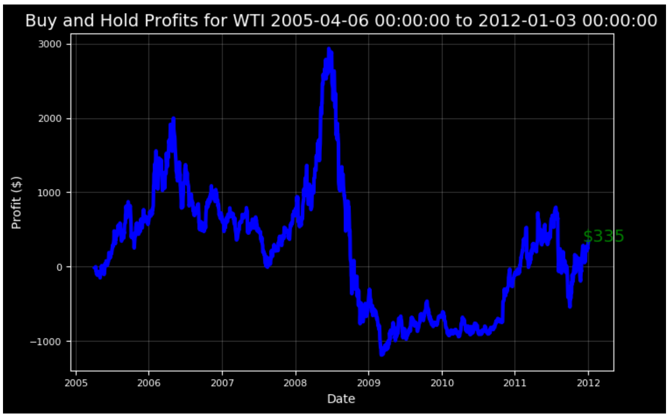

FIGURE 4. Buy and Hold profit for West Texas Intermediate (WTI) crude oil futures prices from between 2005 to 2012

Then, the next step is to introduce the concept of an additive model which is a tool used for analyzing and predicting time series data and hence represents a time series as a combination of patterns on different time scales and an overall trend.

Therefore, it is of importance to note that an additive model smooths out the noise present in our stock data due to the jump volatility as observed in the forward curve in Figure (1) and also as seen in Figure (5) the modeled line does not exactly line up with the observations. Similarly, with the large confidence interval observed in data as well as the variation between the observed and modeled data as described in Figure (5) it is important to estimate the uncertainties caused by the Levy jumps in the model data because it forms an essential part of the modeling procedure due to the fact that the fluctuation in WTI stock prices affects the future prediction and investors buy and hold decisions.

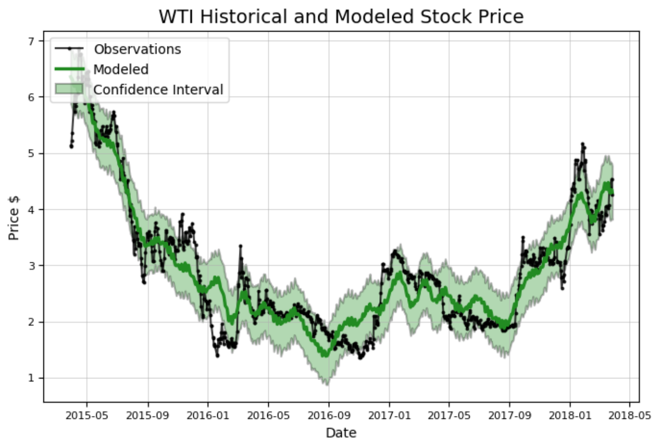

FIGURE 5. Additive model smoothening jumps for West Texas Intermediate (WTI) crude oil futures prices from between 2005 to 2018

However, for a granulated forward curve analysis to understand the behavior of WTI stock price data over time and with the presence of jump-diffusion and no-arbitrage situations, its is important that we analyse the data frequency for trends as well as for different periods like monthly and yearly as described in Figure (6)

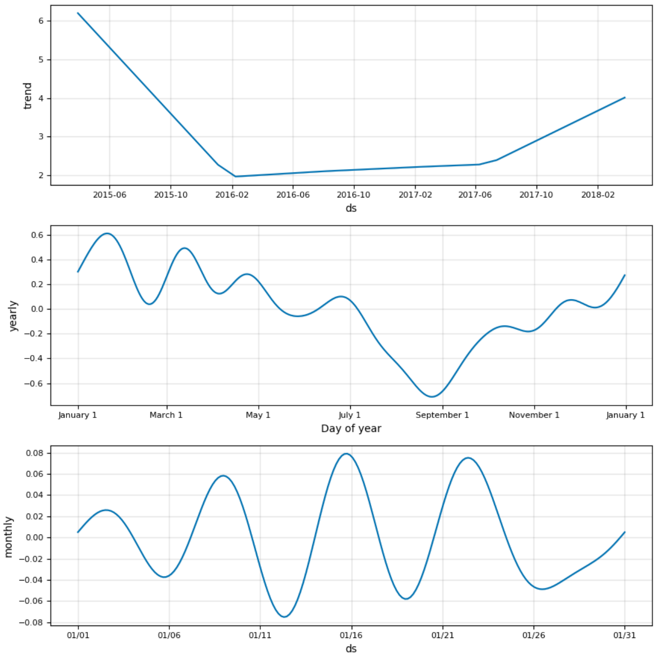

FIGURE 6. An overall trend ofWest Texas Intermediate (WTI) crude oil futures prices from between 2005 to 2018

The overall trend is a definitive increase over the past three years. There also appears to be noticeable yearly pattern with prices bottoming out in September and reaching a peak in October and eventually January. As the time-scale decreases, the data gets noiser and the model data begin to show the presence of drift and the jump sizes increases. However, over the course of a typical month, there appears to exist more signal than jumps in the model data. And these signals are important also to show the direction which the model pricing framework leads to for an investor in WTI crude oil stock.

3.2 Change points in the Model Data

Change points occur when a time-series goes from increasing to decreasing or vice versa and they are point of decision for the model data that are located where the change in the rate of the time series is greatest. These times are very significant due to the fact that the knowledge of when a stock will reach a peak or is about to take off due to spikes and jump volatility could significantly impact investors buy and hold profit margins and benefit the economy. Also, identifying the causes of change points mostly in an Arrow Debreu market can lead to the predicting of future swings in the value of a stock.

Change points sorted by slope rate of change (2nd derivative)			
	Date	Adj. Close	delta
217	2016-02-08	1.65	1.542875
193	2016-01-04	2.37	0.862472
578	2017-07-14	1.98	0.465588
554	2017-06-09	2.02	0.441055
337	2016-07-29	1.99	-0.040519

Change points sorted by slope rate of change (2nd derivative) Stock price with Change points of West Texas Intermediate (WTI) crude oil futures prices from between 2005 to 2018

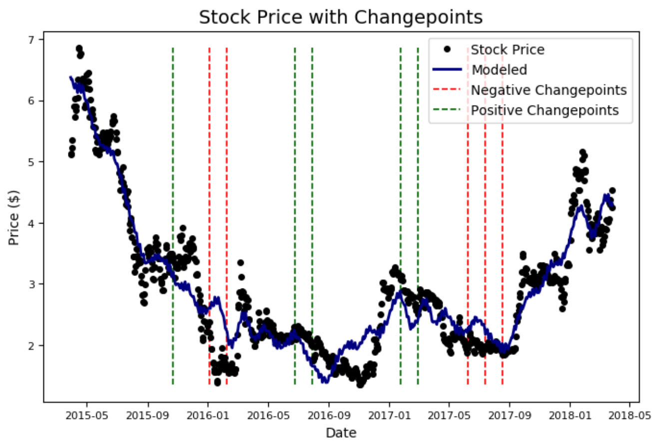

FIGURE 7. Stock price with Change points of West Texas Intermediate (WTI) crude oil futures prices from between 2005 to 2018

The change points tend to line up with weak peaks and strong valleys in the stock prices and this clearly shows the direction at which the changepoints happen in the curve. From Figure (7) there exist a concentration of negative changepoints from December 2015 to January 2016 as well as from June 2017 to August 2017 and stock values at that range tends to reveal a trend of loses for an investor in WTI crude oil stock.

3.3 Stock Price Prediction - West Texas Intermediate

In this section we try to predict the stock market for the data frequency of WTI using model pricing framework taking into consideration the presence of jumps produced by volatility and fluctuation (Daily Changes) in an incomplete market. The daily ups and downs of the stock market gives a hint to the patterns the model framework can learn in order to beat all the loses caused by jumps in the market, which although sounds enticing is unrealistic in the live daily stock market. The data frequency for WTI in this study range from between mid 2005 to early 2018. However, using our model we predict 90, 365 and 730 days into the future and observed the trends in the confidence interval, observed and modeled data.

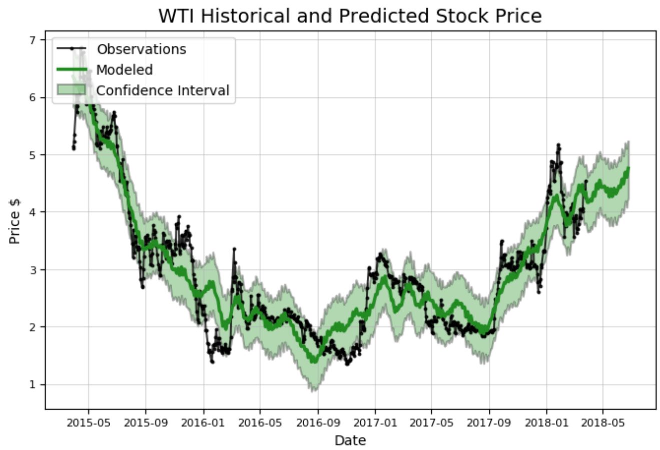

FIGURE 8. 90 days prediction of West Texas Intermediate (WTI) crude oil futures prices from between 2005 to 2018

Here in Figure (8), notice that the prediction, the green line, contains a confidence interval and a predicted WTI crude oil price of USD3.79. This represents the model's uncertainty in the forecast. In this case, the confidence interval width is set at 80 percent, meaning we expect that this range will contain the actual value 80 percent of the time. The confidence interval grows wide further out in time because the estimate has more uncertainty as it gets further away from the data. Similarly, in Figures (9) and (10) and predicted WTI crude oil prices USD5.60

for a future 365days (1 year) and USD7.12 for a future 730 (2 years) respectively.

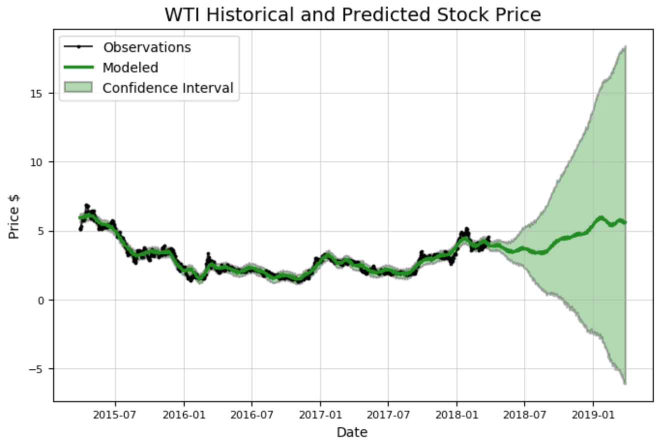

FIGURE 9. 1 year prediction ofWest Texas Intermediate (WTI) crude oil futures prices from between 2015 to 2019

As much as the predicted prices from 90 days to 730 days in the future appears to grow from USD3.79 to USD7.12 which looks enticing to an investor buying and holding shares in WTI stock, it is significant to note the growth in confidence intervals further away in time from the estimated data which shows the uncertainty level as the price is projected further in the future.

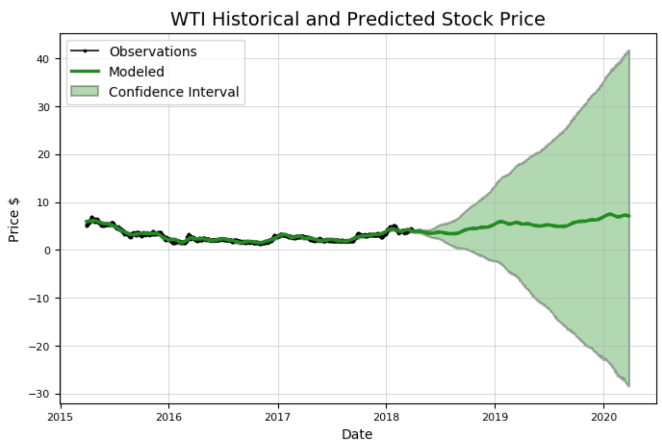

FIGURE 10. 2 year prediction of West Texas Intermediate (WTI) crude oil futures prices from between 2015 to 2020

Therefore, anytime predictions in stock prices are made, it is extremely significant to include a confidence interval and this is due to the fact that the model considers the presence of possible jumps which in our study is bench marked with the properties of Hilbert space of functions for proper price evaluation.

3.4 Evaluating Predictions of Stock - WTI

To evaluate the accuracy of stock predictions, it is significant to split the data frequency in two (a test set and training set). In modeling the training actual stock price data before using the test data for prediction, we use the past one year historical data (2017 in our case). However, when training the data we block the model from the test data, hence we use three years of data previous to the testing time frame (2014-2016). In machine learning as in our case using supervised learning to train the model to learn the patterns and relationships in the data from the training set and then is able to correctly reproduce them for the test data which is quantified using the predictions for the test set and the actual values and estimating the metrics including error on the testing and training set, the percentage of the time the stock price correctly predicted the direction of a price change, and the percentage of the time the actual price fell within the predicted 80 percent confidence interval.

Prediction Range: 2017-03-27 00:00:00 to 2018-03-27 00:00:00.

Predicted price on 2018-03-24 00:00:00 = 6.14.
Actual price on 2018-03-23 00:00:00 = 4.38.
Average Absolute Error on Training Data = 0.48.
Average Absolute Error on Testing Data = 2.04.

When the model predicted an increase, the price increased 45.00 percent of the time.

When the model predicted a decrease, the price decreased 47.71 percent of the time.

The actual value was within the 80 percent confidence interval 35.20 percent of the time.

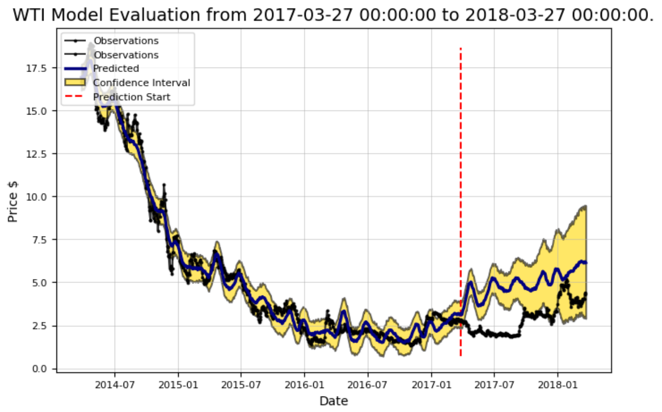

FIGURE 11. Prediction of West Texas Intermediate (WTI) crude oil futures prices from between 2017 to 2018 using training and test data.

3.5 Changepoint Prior Selection

As an improvement to the changepoint studied in previous section, the changepoint prior scale represents the amount of emphasis given to the changepoints in the model. This is used to control overfitting vs. underfitting (also known as the bias vs. variance tradeoff). In this case a higher prior creates a model with more weight on the changepoints and a more flexible fit. This may lead to overfitting because the model will closely stick to the training data and not able to generalize to new set data. However, lowering the prior decrease the model flexibility which can cause the opposite problem i.e. underfitting. This occurs when the model does not follow the training data closely enough and fails to learn the underlying patterns.

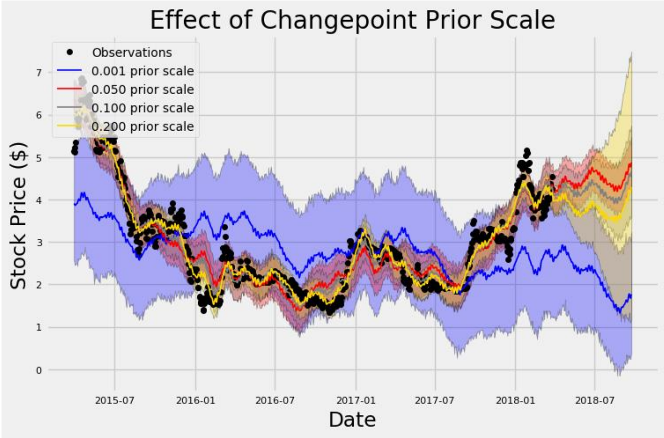

FIGURE 12. Effect of Changepoint Prior Scale for West Texas Intermediate (WTI) crude oil futures prices from between 2015 to 2018.

In Figure (12) we are training on three years data and then showing predictions for six months. We do not quantify the predictions here because we are just trying to udnerstand the role of the changepoint pior. The graph shows that the lowest prior, the blue line, does not follow the training data, the black observations, very closely. However, by creating its own route it picks a path through the general vicinity of the data. In contrast, the highest prior, the yellow line, sticks to the training observations as closely as possible. Meanwhile, the default value for the changepoint prior is 0.05 which falls above both extremes.

Similarly, notice the shaded intervals which defines the difference in uncertainty for the priors. The lowest prior has the largest uncertainty on the training data and the least uncertainty o the test data. In contrast the highest prior has the smallest uncertainty on the training data but the greatest uncertainty on the test data. Therefore, the higher the prior, the more confident it is on the training data because it closely follows each observation. When it comes to the test data however, an outfit model is lost without any data points to anchor it. As the WTI stocks have quite a bit of volatility, it is important to use a more flexible model than the default so the model can capture as many patterns as possible. In so doing, we evaluate four priors with four metrics:

training error, training range (confidence interval), testing error, and testing range (confidence interval). As can be seen in Figure (12), the higher the prior, the lower the training error and the lower the uncertainty on the training data. Also, the higher prior the lower the testing error, which supports the argument that closely fitting to the data is a good idea with the WTI crude oil stocks.

Meanwhile, in exchange for greater accuracy on the test set, we get a greater range of uncertainty on the test data with the increase prior.

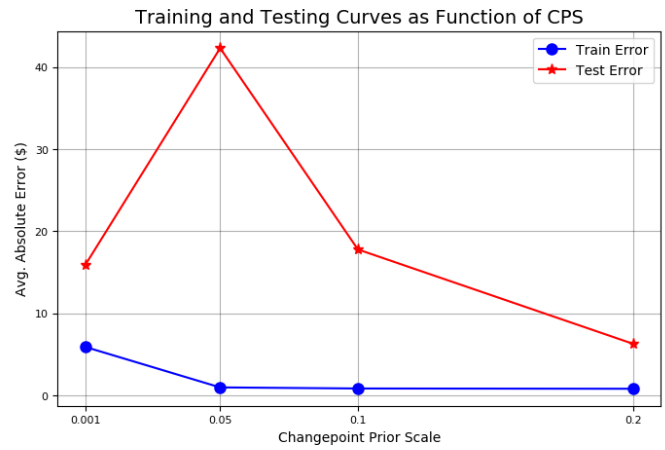

FIGURE 13. Training and Testing Accuracy Curves for Different Changepoint Prior Scales

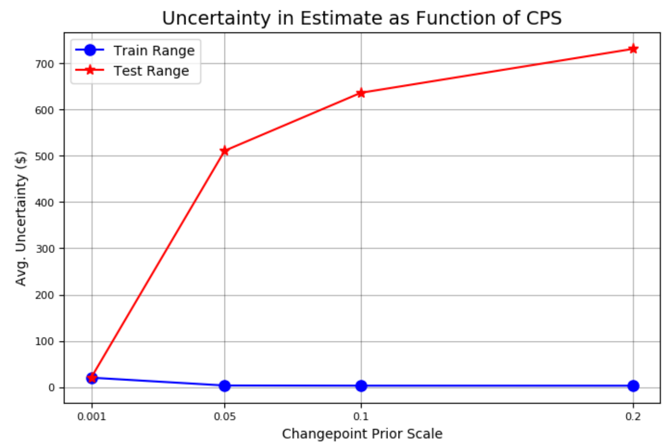

FIGURE 14. Uncertainty for Different Changepoint Prior Scales

In this case, Figures (13) and (14) shows clearly that the highest prior produced the lowest testing error and highest uncertainty respectively. Therefore, increasing the prior might give a better performance as can be seen in Figures (15) and (16)

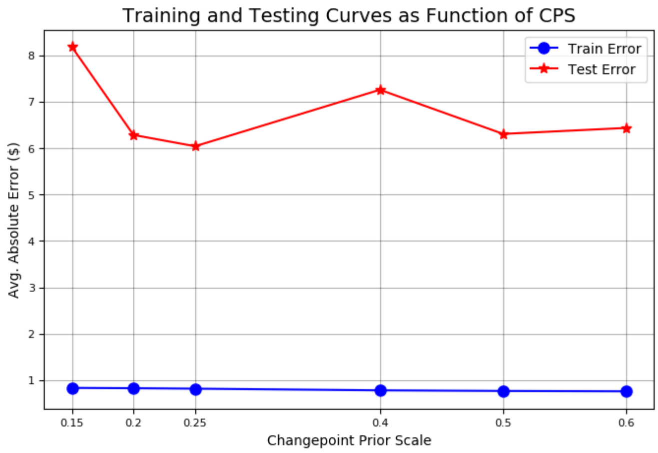

FIGURE 15. Training and Testing Accuracy Curves for increased Changepoint Prior Scales

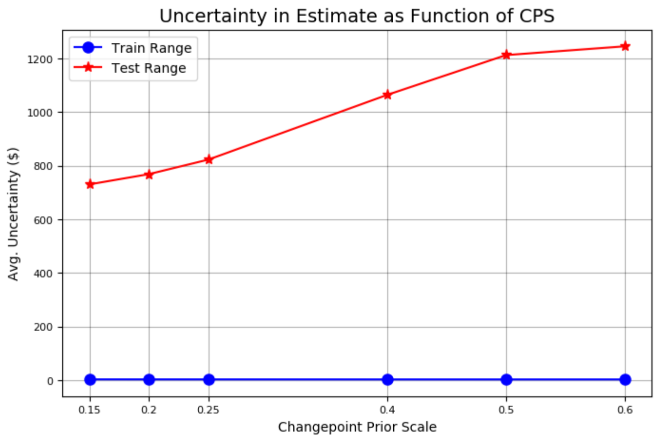

FIGURE 16. Uncertainty for increased Changepoint Prior Scales

Therefore, with the variation in prior and introduction of different changepoint prior scales on the training and testing data set, the model is optimized to predict better stock prices for WTI crude oil stock.

Prediction Range: 2017-03-27 00:00:00 to 2018-03-27 00:00:00.

Predicted price on 2018-03-24 00:00:00 = 3.23.
Actual price on 2018-03-23 00:00:00 = 4.38.
Average Absolute Error on Training Data = 0.37.
Average Absolute Error on Testing Data = 0.71.

When the model predicted an increase, the price increased 44.88percent of the time.
When the model predicted a decrease, the price decreased 47.54percent of the time.
The actual value was within the 80percent confidence interval 94.00percent of the time.
Prediction of West Texas Intermediate (WTI) crude oil futures prices from between 2017 to 2018 with optimized model.

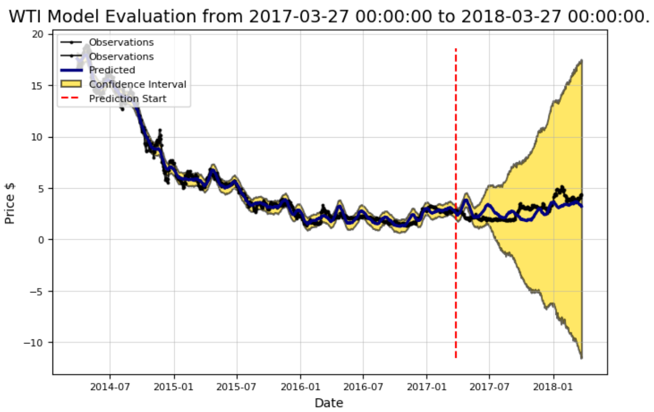

FIGURE 17. Prediction of West Texas Intermediate (WTI) crude oil futures prices from between 2017 to 2018 with optimized model.

Hence, with the optimized model on the WTI dataset, the confidence interval increased from about 35 percent to 94percent and the training data as well as the testing data seems to converge and meet as expected which shows the significance of model optimization.

Conflict of Interests

The authors declare that there is no conflict of interests.

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