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Limitations of a ZN-PI Controller for Liquid Level Control of Horizontal Cylindrical Tank

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ABSTRACT

Most chemical processes are inherently nonlinear. However, because of their simplicity, linear control algorithms have been used for the control of nonlinear processes. In this work, horizontally placed cylindrical tanks whose parameters vary with respect to process variable are considered for study. [Anandanatarajan, R.,Chidambaram,M., and Jayasingh, T., Limitations of PI controller for a first- order nonlinear process with dead time. ISA transactions 45, 185-199(2006)]. The time constant and gain of the chosen process vary as a function of level. The limitations of the conventional PI controller tuned using Ziegler Nichols (ZN) settings for the chosen process are carried out. The servo and regulatory responses for various magnitudes of set point changes and load changes at various operating points with the controller tuned only at a chosen nominal operating point are obtained. Simulation is conducted within MATLAB environment to verify the performances of the system in terms of Settling Time, Overshoot (OS) and Simulation is conducted within MATLAB environment to verify the performances of the system in terms of Settling Time and Overshoot (OS)
Keywords : Horizontal Cylindrical Tank, ZN-PI Controller, Liquid Level Control, Nonlinear Process

I. INTRODUCTION

Industries such as petro-chemical industries, paper making industries, waste management and others are the vital industries where liquid level and flow control are essential. Liquids will be processed by chemical or mixing treatment in the tanks, but always the level fluid in the tanks must be controlled, failing to do so may lead to serious shutdown process. So it is necessary to maintain the level of tank at particular set point.

Most of the industries deal with nonlinear process tanks such as conical, spherical, hemispherical horizontal cylindrical tanks. But the majority of the findings in control theory deal only with the linear system designs. So the control of nonlinear shape of process tank presents a challenging task mainly due to its non-linearity and constantly changing cross section. Hence, horizontal cylindrical tank level process is taken up for present study.

II. HORIZONTAL CYLINDRICAL TANK LEVEL PROCESS AND ITS MATHEMATICAL MODEL

The horizontal tank such as oil, chemical liquid in its surge drum level control system has shown Figure.1. The purpose of the surge vessel is for smooth variations in the flow rate from one process to another process. The level can vary substantially from the set point, as long as the vessel does not overflow or go dry. Surge vessels are used to help reduce the effect of flow rate variations between interconnected process units.

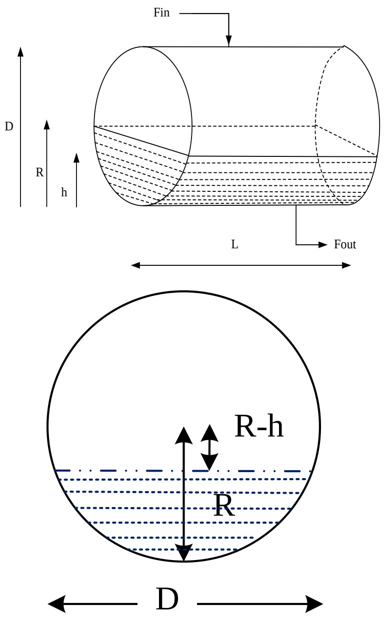


Fig.1 : Structure of horizontal cylindrical Tank level process

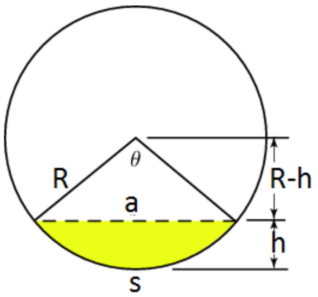
The mathematical model of the horizontal cylindrical tank liquid level system considered for the study is expressed as,

Let R, be radius of cross section.

h, be level of liquid inside the tank.

D, be diameter of cross section.

L, be length of the tank.



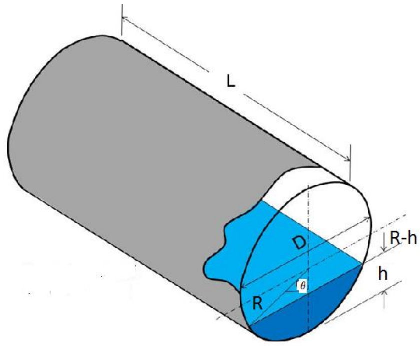


Fig. 2: cross section of horizontal cylindrical tank

Base area between 0 to R : (If the level of the tank is below centre of the circle)

Let R be the radius of the circle, a be the chord length, s be the arc length, h be the height of the arced portion, and r be the height of the triangular portion. Then the radius is

$$R = r + h$$

The area A of the (shaded) segment in fig.2(a) is then simply given by the area of the circular sector (the entire wedge-shaped portion) minus the area of the triangular portion,

A area = area of circular sector – area of triangular portion.

$$= \frac{1}{2} R^2 2\cos^{-1}\left(\frac{R-h}{R}\right) - \frac{1}{2} R^2 \sin 2\theta$$

$$= R^2 \cos^{-1}\left(\frac{R-h}{R}\right) - \frac{1}{2} R^2 2\sin\theta\cos\theta$$

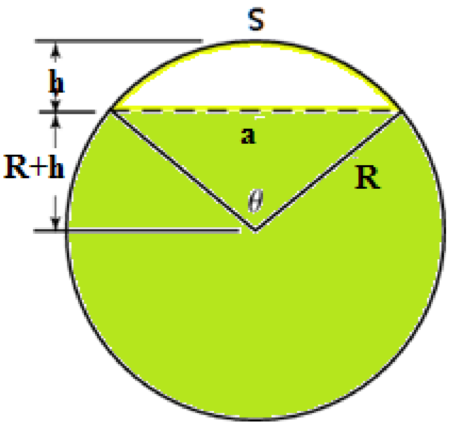
$$= R^2 \cos^{-1}\left(\frac{R-h}{R}\right) - R^2 \frac{\sqrt{R^2-(R-h)^2}}{R}\left(\frac{R-h}{R}\right)$$

$$= R^2 \cos^{-1}\left(\frac{R-h}{R}\right) - (R-h)\sqrt{2Rh-h^2}$$

the volume V of the cylindrical segment is given by multiplying the area of a circular segment of height h by the length of the tank L,

$$V = L\left(R^2 \cos^{-1}\left(\frac{R-h}{R}\right) - (R-h)\sqrt{2Rh-h^2}\right) \qquad (2)$$

Base area between R to 2R: (If the level of the tank is above centre of the circle)



$$A = \frac{\pi R^2}{2} - (h-R)\sqrt{2Rh-h^2}\cos^{-1}\left(\frac{h-R}{R}\right)$$

$$= \pi r^2 - \left[\pi - \cos^{-1}\left(\frac{R-h}{R}\right)\right]$$

$$= \pi r^2 - (R-h)\sqrt{2Rh-h^2} - \pi r^2 + R^2\cos^{-1}\left(\frac{R-h}{R}\right)$$

$$A = R^2\cos^{-1}\left(\frac{R-h}{R}\right) - (R-h)\sqrt{2Rh-h^2} \qquad (3)$$

the volume V of the cylindrical segment is given by multiplying the area of a circular segment of height h by the length of the tank L,

$$V = LR^2\cos^{-1}\left(\frac{R-h}{R}\right) - (R-h)\sqrt{2Rh-h^2}$$

(4)

Both the equations (2) and (4) for the level from 0 to R and level R to 2R respectively are identical

Note that the above equation gives

When $h = 0$

$$V = 0$$

When h=R

$$V = \frac{\pi R^2 L}{2}$$

When h=2R

$V = \pi R^2 L$ as expected.

Therefore the equation holds good for all the different level of the tank. Differentiating with respect to h will get,

$$\frac{dV}{dh} = L\left[R^2 \frac{-1}{\sqrt{1-\left(\frac{R-h}{R}\right)^2}}\frac{-1}{R} - \left\{(R-h)\frac{1}{2}\left(2Rh-h^2\right)^{\frac{-1}{2}}(2R-h) + \sqrt{2Rh-h^2}(-1)\right\}\right]$$

$$= L\left[\frac{R^2}{\sqrt{\frac{R^2-(R-h)^2}{R}}} - \left\{(R-h)\frac{1}{2}\left(2Rh-h^2\right)^{\frac{-1}{2}}(2R-h) + \sqrt{2Rh-h^2}\right\}\right]$$

$$= L\left[\frac{R^2}{\sqrt{2Rh-h^2}} - \left\{\frac{(R-h)^2}{\sqrt{2Rh-h^2}} - \sqrt{2Rh-h^2}\right\}\right]$$

$$= L\left[\frac{R^2}{\sqrt{2Rh-h^2}} - \left\{\frac{R^2+h^2-2Rh-\left(2Rh-h^2\right)}{\sqrt{2Rh-h^2}}\right\}\right]$$

$$= L\left[\frac{R^2}{\sqrt{2Rh-h^2}} - \left\{\frac{R^2+2h^2-4Rh}{\sqrt{2Rh-h^2}}\right\}\right]$$

$$= \left[\frac{L}{\sqrt{2Rh-h^2}} - \left\{\frac{R^2+2h^2-4Rh}{\sqrt{2Rh-h^2}}\right\}\right]$$

$$= 2L * \frac{(2Rh-h^2)}{\sqrt{2Rh-h^2}}$$

$$= 2L * \sqrt{2Rh-h^2}$$

$$dV = 2L * \sqrt{2Rh-h^2}\,dh$$

(5)

Consider the horizontal cylindrical tank level process, as per the conservation of mass, we have

$$\frac{dV}{dt} = F_{in} - F_{out}$$

(6)

Where dV/dt is change of volume of liquid in tank with respect to time

F_{in} - is Volume flow rate at inlet

F_{out} - is Volume flow rate at outlet

in conservation of mass,

$$2L\sqrt{Rh - h^2}\,\frac{dh}{dt} = F_{in} - F_{out} \quad (7)$$

$$\frac{dh}{dt} = \frac{F_{in} - F_{out}}{2L\sqrt{2Rh - h^2}} \quad (8)$$

Where $F_{out} = c\sqrt{h}$, c is valve coefficient. A delay time (T_d) is introduced in the inflow F_{in} to incorporate a dead time in the process and u is a linear function. The equation becomes

$$\frac{dh}{dt} = \frac{u(t - T_d) - c\sqrt{h}}{2L\sqrt{2Rh - h^2}} \quad (9)$$

This is the mathematical model of the horizontal cylindrical tank level process.

III. TUNING OF PI CONTROLLER

The goal of PI controller tuning is to determine parameters that meet closed loop system performance specifications, and to ensure the robust performance of the control loop over a wide range of operating conditions. Practically, it is often difficult to simultaneously achieve all of these desirable qualities. For example, if the PI controller is adjusted to provide better transient response to set point change, it usually results in a sluggish response when under disturbance conditions. On the other hand, if the control system is made robust to disturbance by choosing conservative values for the PI controller, it may result in a slow closed loop response to a set point change. A number of tuning techniques that take into consideration the nature of the dynamics present within a process control loop have been proposed (Ziegler and Nichols, 1942; Cohen and Coon, 1953; Åström and Hägglund, 1984; and Atherton, 1993).[1-3] All these methods are based upon the dynamical behavior of the system under either open-loop or closed-loop conditions

The Simulink model of horizontal cylindrical tank level process is shown in figure 3.. The height, end radius and length of the tank are 30 cm, 15 cm and 30cm respectively. A dead time (T_d) of 30 seconds is introduced in the process through software. The time constant and the gain of the process increase as the level increases.. To obtain the transfer function model, reaction curves for various magnitudes of input at 50% nominal operating point are obtained by MATLAB Simulink software as shown in the figure 4. Different step changes in input (say) 10%, 20 %, 30% and 40% given to the process to obtain reaction curves. The corresponding process gain and time constant are tabulated in the Table 1.

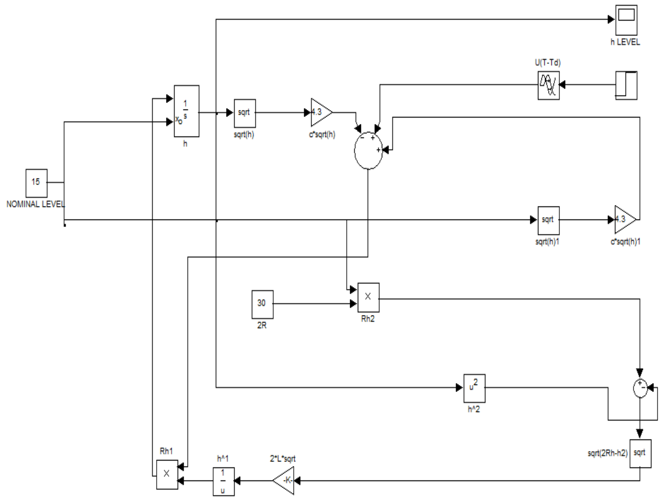


Fig. 3 Simulink model of horizontal cylindrical tank level process

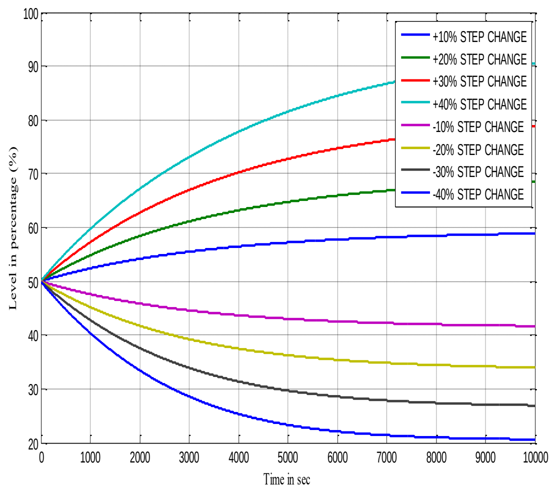


Fig 4. Reaction curve for different set values

Table 1. Model parameters obtained from simulated reaction curves for horizontal cylindrical tank

Step change	Process gain	Time constant	Time delay
40%	2.125	3745	39
30%	2.04	3724	24.9
20%	1.963	3615	12.9
10%	1.88	3457	9.3
-10%	1.72	3015	17
-20%	1.64	2767	42.5
-30%	1.55	2547	59
-40%	1.477	2313	81

For the simulation study, the reaction curve for +10% change at 50% nominal operating point is considered to tune the PI controller. The tuning parameters obtained are K_c=1.4407 and K_i=0.0023

IV. PERFORMANCE OF PI CONTROLLER FOR THE HORIZONTAL CYLINDRICAL TANK LEVEL PROCESS

The servo response on the Horizontal cylindrical tank level process for increase in set points, and decrease in set point at different time interval but all of them tuned at the nominal operating point of 50% shown in Figure 5 & 6. Figure 7& 8 shows the regulatory response for 10% increasing load change and 10% decreasing load change from its nominal operating point. The performance of ZN PI controller is oscillatory response and produce high ISE and IAE values given in the table.2.

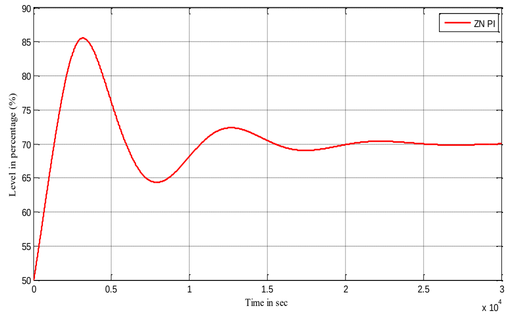


Figure.5 Servo Response of hemispherical tank level process for 20% increase in set point from nominal operating point of 50% using ZNPI controller

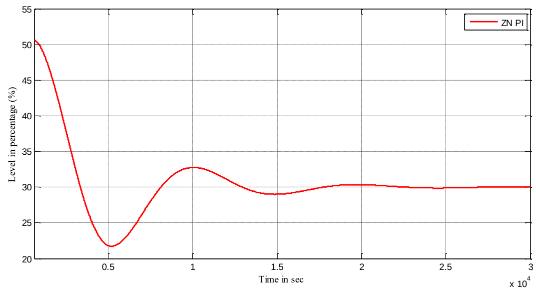


Figure.6 Servo Response of hemispherical tank level process for 20% decrease in set point from nominal operating point of 50% using ZNPI controller

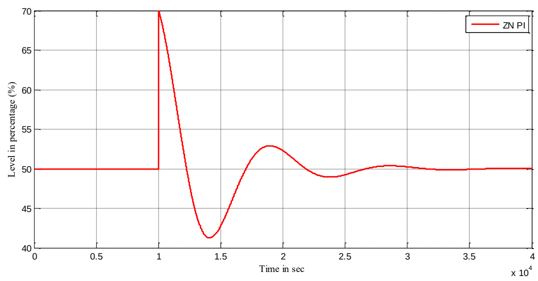


Figure.7 Regulatory response of hemispherical tank level process for increase in 20% load at t=10000 sec at 50% nominal operating point using ZNPI controller

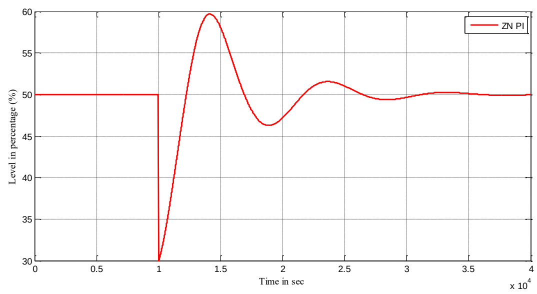


Figure.8 Regulatory response of hemispherical tank level process for decrease in 20% load at t=10000 sec at 50% nominal operating point using ZNPI controller

Table .2 Performance index for Horizontal cylindrical Tank at nominal operating point of 50%

Controller		Servo Response		Regulatory Response	
		ISE	IAE	ISE	IAE
ZN PI	20%Increase in SP or load	7.117×10^4	2.557×10^4	4.855×10^4	1.897×10^4
	20%Decrease in SP or load	8.406×10^4	2.46×10^4	5.327×10^4	2.12×10^4

Controller	Servo Response				Regulatory Response			
	20% increase in SP		20% decrease in SP		20% increase in load		20% decrease in load	
	over shoot (%)	Settling time (s)	under shoot (%)	Settling time (s)	under shoot (%)	Settling time (s)	over shoot (%)	Settling time (s)
ZNPI	76.5	25140	41.35	21560	43.6	22280	48.45	26600

V. ANALYSIS OF ROBUSTNESS OF ZN PI CONTROLLER FOR HORIZONTAL CYLINDRICAL TANK LEVEL PROCESS

The change in nominal operating point from 50% to 60% for increase in set points, and decrease in set point at different time interval but all of them tuned at the nominal operating point of 50% horizontal cylindrical tank level process are shown in figure 9to 10. Figure 11 & 12 show the regulatory response for increase in 10% and decrease in 10% load from its 60% of nominal operating point. The PI controller gives large undershoot for negative load and takes large time to reaches steady state value and lower overshoot for positive load changes for set point changes controller exhibits overshoot for increasing set point. The performance index of the controller is given in the table 3. Similarly Figure 13 to 16 shows the servo and regulatory response of Horizontal cylindrical tank level process for changing nominal operating point from 50% to 40%. Table 4 represents the performance index of the PI controller for change in nominal operating point from 50% to 40%.

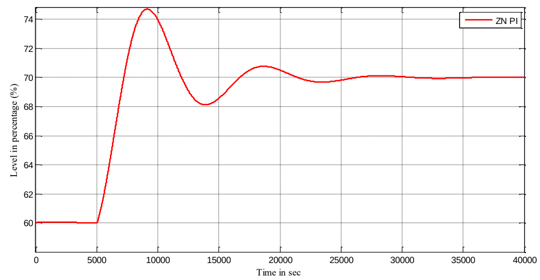


Figure.9 Servo Response of Horizontal cylindrical tank level process for 10% increase in set point from nominal operating point of 60% using ZNPI controller but tuned at 50% of nominal operating point

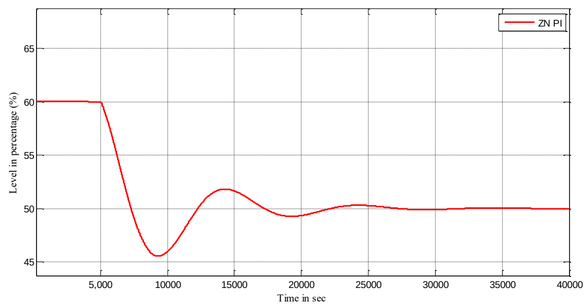


Figure.10 Servo Response of Horizontal cylindrical tank level process for 10% decrease in set point from nominal operating point of 60% using ZNPI controller but tuned at 50% of nominal operating point

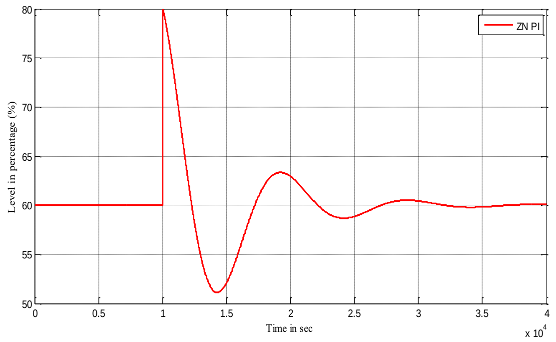


Figure.11 Regulatory response of hemispherical tank level process for increase in 20% load at t=10000 sec at 60% nominal operating point using ZNPI controller but tuned at 50% of nominal operating point

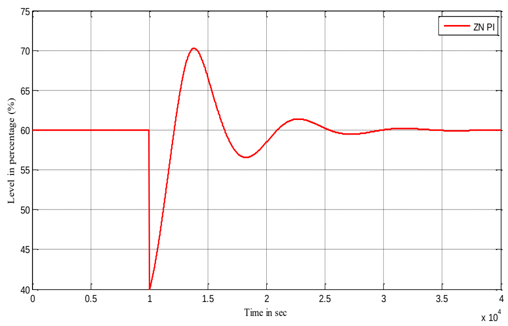


Figure.12 Regulatory response of hemispherical tank level process for decrease in 20% load at t=10000 sec at 60% nominal operating point using ZNPI controller but tuned at 50% of nominal operating point

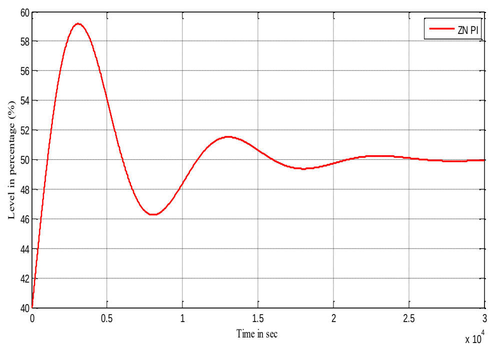


Figure.13 Servo Response of Horizontal cylindrical tank level process for 10% increase in set point from nominal operating point of 40% using ZNPI controller but tuned at 50% of nominal operating point

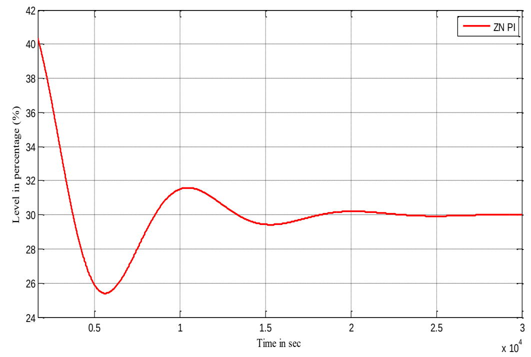


Figure.14 Servo Response of Horizontal cylindrical tank level process for 10% decrease in set point from nominal operating point of 40% using ZNPI controller but tuned at 50% of nominal operating point

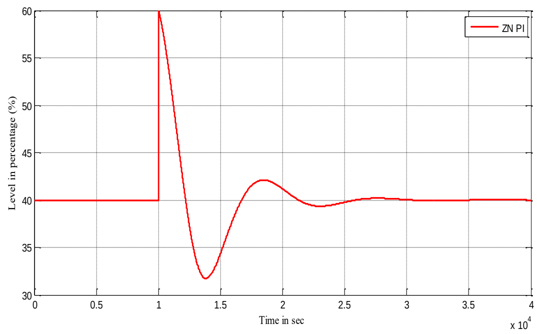


Figure.15 Regulatory response of hemispherical tank level process for increase in 20% load at t=10000 sec at 40% nominal operating point using ZNPI controller but tuned at 50% of nominal operating point

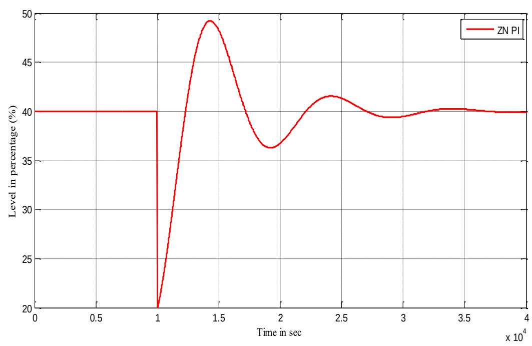


Figure. 16 Regulatory response of hemispherical tank level process for decrease in 20% load at t=10000 sec at 40% nominal operating point using ZNPI controller but tuned at 50% of nominal operating point.

Table 3 Performance index for Horizontal cylindrical Tank at nominal operating point of 60%

Controller		Servo Response		Regulatory Response	
		ISE	IAE	ISE	IAE
ZN PI	10% 20% Increase in SP or load	4.639x10^{4}	2.057x10^{4}	5.065x10^{4}	2.045x10^{4}
	10% or 20%Decrease in SP or load	5.576x10^{4}	2.203x10^{4}	5.146x10^{4}	1.966x10^{4}

Controller	Servo Response				Regulatory Response			
	10% increase in SP		10% decrease in SP		20% increase in load		20% decrease in load	
	over shoot (%)	Settling time (s)	under shoot (%)	Settling time (s)	under shoot (%)	Settling time (s)	over shoot (%)	Settling time (s)
ZNPI	135	24610	60	24580	44.45	22680	51.35	23630

Table 4. Performance index for Horizontal cylindrical Tank at nominal operating point of 40%

Controller		Servo Response		Regulatory Response	
		ISE	IAE	ISE	IAE
ZN PI	10% or 20% Increase in SP or load	2.516x10^{4}	1.591x10^{4}	4.3889x10^{4}	1.63x10^{4}
	10% or 20% Decrease in SP or load	3.181x10^{4}	1.537x10^{4}	5.265x10^{4}	2.146x10^{4}

Controller	Servo Response				Regulatory Response			
	10% increase in SP		10% decrease in SP		20% increase in load		20% decrease in load	
	over shoot (%)	Settling time (s)	under shoot (%)	Settling time (s)	under shoot (%)	Settling time (s)	over shoot (%)	Settling time (s)
ZNPI	91.7	25640	46	23190	41.3	20490	45.9	26150

VI. CONCLUSION

The reaction curves for different magnitudes of inflow rate u of the horizontal tank level process at various operating points are obtained and the FOPDT model parameters are computed. The time constant of the process is found to vary as a function of the level. Both the time constant and gain will increase towards top of the tank. This shows that control of the level of the tank around bottom level is very difficult than controlling it at the top level.

The servo responses of horizontal cylindrical tank level process are obtained for shifted higher operating points and shifted lower operating point. The larger overshoot for increasing set point at the higher operating region. The regulatory responses for changes in input load at shifted lower operating points exhibit higher oscillations and slowly settle to steady value. The performance of ZN PI controller exhibits a higher oscillation and higher IAE ,ISE value when the operating point shifting to any particular value other than the controller which was tuned . For the both hemispherical and horizontal cylindrical level process results shows the limitations of PI controllers for the chosen process. It is difficult to control the system near bottom level of the tank. The reasons for difficulty in controlling the process are due to the dead time of the process and nonlinearity. So, to effectively control the process a nonlinear controller is required.

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