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L(R) Cyclic Semigroups Satisfying The Identity : $abc = ac$

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ABSTRACT

Semigroups being one of the algebraic structures are sets with associative binary operation defined on them. The theory of semigroups satisfy additional properties like commutative, Left(Right) cyclic i.e., L(R) cyclic , Left(Right) identity, Left(Right) cancellative and many others. In this paper we determine different structures of semigroups like normal, seminormal, quasinormal, semiregular and others by using the identity $abc = ac$ with the concept of L(R) cyclic properties of semigroups.

Keywords : Semigroup, Normal, Seminormal, Regular, Semiregular, Quasinormal

I. INTRODUCTION

Semigroup is an important algebraic structure with a binary operation satisfying closure and associative properties [1,2]. The main attempt of theory of semigroups is to generalize the concept of the groups. Also the study of the theory consists of algebraic abstraction of the properties of composition of transformation on a set [3].

From the past few decades the theory of semigroups had become a self-established branch of modern algebra linked strongly with different fields in Mathematics such as Group theory and Ring theory, Functional analysis and Differential geometry [4,5].

Applications of semigroups are of high interest that can be seen in Automata theory, Formal languages, Sociology, Biology and Biochemistry [6,7].

In the present work we use L (R) cyclic properties of semigroups with an identity $abc = ac$ and study different structures of semigroups involved [8].

Definition: A semigroup is a nonempty set S together with a binary operation '.' from $S \ X \ S \rightarrow S$. Thus we state the condition of (S,.) to be a semigroup as : $(a.b).c = a.(b.c) \ or \ (ab)c = a(bc) \ for \ all \ a,b,c \ in \ S$

Definition: If a semigroup (S,.) satisfies the identity $x.(y.z) = y.(z.x) = z.(x.y)$ for all $x,y \ and \ z$ in S then S is said to be L- cyclic.

Definition: If a semigroup (S,.) satisfies the identity $(x.y).z = (y.z).x = (z.x).y$ for all $x,y \ and \ z$ in S then S is said to be R- cyclic.

II. METHODS AND MATERIAL

Theorem 1: Let (S, .) be a L(R) cyclic semigroup satisfying the identity $abc = ac$, $where \ a, b, c \in S$

then (S, .) is left (right) seminormal if and only if it is left(right) semiregular.

Proof: Let (S, .) be a L (R) cyclic semigroup with identity $\boldsymbol{abc = ac, where\ a, b, c \in S}$

Case-1 : Let S be left seminormal.

$abca = acbca$
$= (ac)bca$ [Using associativity]
$= (abc)bca$ [Using the identity]
$= abc(bc)a$ [Using associativity]
$= abc(bac)a$[Using the identity]
$= ab(cba)ca$ [Using L(R)-cyclic]
$= ab(acb)ca$ [Using L(R)-cyclic]
$= aba(cb)ca$[Using associativity]
$= aba(cab)ca$[Using the identity]
$\Rightarrow abca = abacabca$
⇒ S is left semiregular.

Case-2:

Now let S be leftt semiregular then
$abca = abacabca$
$= a(bac)abca$ [Using associativity]
$= a(bc)abca$[Using the identity]
$= a(bca)bca$ [Using associativity]
$= a(ba)bca$ [Using the identity]
$= ab(abc)a$ [Using associativity]
$= ab(cab)a$ [Using L(R)-cyclic]
$= (abc)aba$ [Using associativity]
$= (ac)aba$ [Using the identity]
$= aca(ba)$ [Using associativity]
$= aca(bca)$ [Using the identity]
$= a(cab)ca$ [Using associativity]
$= a(cb)ca$ [Using the identity]
$\Rightarrow abca = acbca$
⇒ S is left seminormal.

Theorem 2: Let (S, .) be a L(R) cyclic semigroup satisfying the identity $\boldsymbol{abc = ac, where\ a, b, c \in S}$ then (S, .) is left (right) semiregular, if and only if it is regular.

Proof: Let (S, .) be a L(R) cyclic semigroup with identity $\boldsymbol{abc = ac, where\ a, b, c \in S}$

Case-1 : Let S be regular, then
$abca = abaca$
$= (ab)(ac)a$ [Using associativity]
$= (acb)(abc)a$ [Using the identity]
$= a(cba)bca$ [Using associativity]
$= a(bac)bca$ [Using L(R)-cyclic]
$= aba(cb)ca$ [Using associativity]
$= aba(cab)ca$ [Using the identity]
$\Rightarrow abca = abacabca$
⇒ S is left semiregular.

Case-2 : Let S be left semiregular, then
$abca = abacabca$
$= a(bac)(abc)a$ [Using associativity]
$= a(bc)(ac)a$ [Using the identity]
$= a(bca)(ca)$ [Using associativity]
$= a(cab)ca$ [Using L(R) -cyclic]
$= (ac)abca$ [Using associativity]
$= (abc)abca$ [Using the identity]
$= a(bca)bca$ [Using associativity]
$= a(ba)bca$ [Using the identity]
$= ab(abc)a$ [Using associativity]
$= ab(ac)a$ [Using the identity]
$\Rightarrow abca = abaca$
⇒ S is regular.

Theorem 3: Let (S, .) be a L(R) cyclic semigroup satisfying the identity $\boldsymbol{abc = ac, where\ a, b, c \in S}$ then (S, .) is left (right) seminormal, if and only if it is normal.

Proof: Let (S, .) be a L(R) cyclic semigroup with identity $\boldsymbol{abc = ac, where\ a, b, c \in S}$

Case-1 : If S is normal, then
$abca = acba$
$= (ac)ba$ [Using associativity]
$= (abc)ba$ [Using the identity]
$= a(bcb)a$ [Using associativity]
$= a(cbb)a$ [Using L(R) -cyclic]
$= (acb)(ba)$ [Using associativity]
$= (acb)(bca)$ [Using the identity]
$= a(cbb)ca$ [Using associativity]
$= a(cb)ca$ [Using associativity]
$\Rightarrow abca = acbca$
⇒ S is left (right) seminormal.

Case-2 : Let S be left (right) seminormal, then
$abca = acbca$
$= ac(bca)$ [Using associativity]
$= ac(abc)$ [Using L(R) -cyclic]
$= a(cab)c$ [Using associativity]
$= a(cb)c$ [Using the identity]
$= ac(bc)$ [Using associativity]
$= ac(bac)$ [Using the identity]
$= ac(cba)$ [Using L(R) -cyclic]
$= a(ccb)a$ [Using associativity]
$= a(cb)a$ [Using the identity]
$\Rightarrow abca = acba$
⇒ S is normal.

Theorem 4: Let (S, .) be a L(R) cyclic semigroup satisfying the identity $\boldsymbol{abc = ac}$, $\boldsymbol{where\ a, b, c \in S}$ then (S, .) is left (right) semiregular if and only if it is right(left) semiregular.
Proof: Let (S, .) be a L(R) cyclic semigroup with identity $\boldsymbol{abc = ac}$, $\boldsymbol{where\ a, b, c \in S}$

Case-1 : If S is left semiregular, then
$abca = abacabca$
$= aba(cab)ca$ [Using associativity]
$= aba(bca)ca$ [Using L(R)-cyclic]
$= a(ba)bcaca$ [Using associativity]
$= a(bca)bcaca$ [Using the identity]
$= abca(bca)ca$ [Using associativity]
$= abca(ba)ca$ [Using the identity]
$= ab(cab)(aca)$ [Using associativity]
$\Rightarrow abca = abcabaca$
⇒ S is right semiregular.

Case-2:
Now let S be right semiregular, then
$abca = abcabaca$
$= abca(bac)a$ [Using associativity]
$= abca(cba)a$ [Using L(R)-cyclic]
$= abc(acb)aa$ [Using associativity]
$= abc(ab)aa$ [Using the identity]
$= a(bc)a(ba)a$ [Using associativity]
$= a(bac)a(bca)a$ [Using the identity]
$= abacab(caa)$ [Using associativity]
$= abacab(ca)$ [Using the identity]
$\Rightarrow abca = abacabca$
⇒ S is left semiregular.

Theorem 5: Let (S, .) be a L(R) cyclic semigroup satisfying the identity $\boldsymbol{abc = ac}$, $\boldsymbol{where\ a, b, c \in S}$ then (S, .) is left (right) seminormal if and only if it is right(left) seminormal.
Proof: Let (S, .) be a L(R) cyclic semigroup with identity $\boldsymbol{abc = ac}$, $\boldsymbol{where\ a, b, c \in S}$

Case-1 : If S is (left) seminormal, then
$abca = acbca$
$= ac(bca)$ [Using associativity]
$= ac(abc)$ [Using L(R)-cyclic]
$= (ac)abc$ [Using associativity
$= (abc)abc$ [Using the identity]
$= abca(bc)$ [Using associativity]
$= abca(bac)$ [Using the identity]
$= abca(cba)$ [Using L(R)-cyclic]
$= abc(acb)a$ [Using associativity]
$= abc(ab)a$ [Using the identity]
$= ab(cab)a$ [Using associativity]
$= ab(cb)a$ [Using the identity]
$\Rightarrow abca = abcba$
⇒ S is right seminormal

Case-2:
Now let S be right seminormal, then
$abca = abcba$
$= abc(ba)$ [Using associativity]
$= abc(bca)$ [Using the identity]
$= a(bcb)ca$ [Using associativity]
$= a(cbb)ca$ [Using L(R)-cyclic]

$= a(cb)ca$ [Using the identity]
$\Rightarrow abca = acbca$
⇒ S is left seminormal.

Theorem 6: Let (S, .) be a L(R) cyclic semigroup satisfying the identity $\boldsymbol{abc = ac}$, $\boldsymbol{where\ a, b, c \in S}$ then (S, .) is normal if and only if it is left (right) quasi normal.
Proof: Let (S, .) be a L(R) cyclic semigroup with identity $\boldsymbol{abc = ac}$, $\boldsymbol{where\ a, b, c \in S}$

Case-1 : If S is (left) quasi normal, then
$abc = acbc$
$\Rightarrow\ abca = ac(bc)$ [Using associativity]
$= ac(bac)$ [Using the identity]
$= ac(cba)$ [Using L(R)-cyclic]
$= a(ccb)a$ [Using associativity]
$= a(cb)a$ [Using the identity]
$\Rightarrow abca = acba$
⇒ S is normal.

Case-2:
Now let S be normal, then
$abca = acba$
$= ac(ba)$ [Using associativity]
$= ac(bca)$ [Using the identity]
$= ac(abc)$ [Using L(R)-cyclic]
$= a(cab)c$ [Using associativity]
$= a(cb)c$ [Using the identity]
$\Rightarrow abc = acbc$
⇒ S is left quasi normal.

Theorem 7: Let (S, .) be a L(R) cyclic semigroup satisfying the identity $\boldsymbol{abc = ac, where\ a, b, c \in S}$ then (S, .) is regular if and only if it is left (right) quasi normal.

Proof: Let (S, .) be a L(R) cyclic semigroup with identity $\boldsymbol{abc = ac, where\ a, b, c \in S}$

Case-1 : If S is left quasi normal, then
$abc = acbc$
$\Rightarrow\ abca = (ac)(bc)$ [Using associativity]
$= (abc)(bac)$ [Using the identity]
$= ab(cba)c$ [Using associativity]
$= ab(acb)c$ [Using L(R)-cyclic]
$= ab(ab)c$ [Using the identity]
$= aba(bc)$ [Using associativity]
$= aba(bac)$ [Using the identity]
$= aba(cba)$ [Using L(R)-cyclic]
$= aba(ca)$ [Using the identity]
$\Rightarrow abca = abaca$
⇒ S is regular.

Case-2:
Now let S be regular semigroup , then
$abca = abaca$
$= a(bac)a$ [Using associativity]
$= a(cba)a$ [Using L(R)-cyclic]
$= ac(baa)$ [Using associativity]
$= ac(ba)$ [Using the identity]
$= ac(bca)$ [Using the identity]
$= ac(abc)$ [Using L(R)-cyclic]
$= a(cab)c$ [Using associativity]
$= a(cb)c$ [Using the identity]
$\Rightarrow\ abc = acbc$
⇒ S is left quasi normal.

III. CONCLUSION

The present paper mainly focuses on different structures of semigroups which are studied through L (R) cyclic properties with an identity $abc = ac$. The work can also be extended with different properties and stuctures of semigroups.

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