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Method for Evaluating Fractional Derivatives of Fractional Functions

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ABSTRACT

This paper studies the fractional differential problem of fractional functions, regarding the modified Riemann-Liouvellie (R-L) fractional derivatives. A new multiplication and the fractional power series method are used to obtain any order fractional derivatives of some elementary fractional functions.

Keywords: Fractional Functions, Modified R-L Fractional Derivatives, New Multiplication, Fractional Power Series

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I. INTRODUCTION

Fractional derivatives of non-integer orders [1-2] have wide applications in physics and mechanics [3-8]. The tools of fractional derivatives and integrals allow us to investigate the behaviour of objects and systems that are characterized by power-law non-locality, power-law long-term memory or fractal properties. Historically, it emerged almost at the same time of the genesis of classical calculus and owes its origin to an inquiry raised by L'Hospital, in a letter sent to Leibniz, of whether the meaning of a derivative to an integer order could be extended to a non-integer order. For further details on the history of fractional calculus, see [9-11]. But the rule of fractional derivative is not unique, the definition of fractional derivative is given by many authors. The commonly used definition is the Riemann-Liouvellie (R-L) fractional derivative [12-13]. Other useful definition includes Caputo definition of fractional derivative, the Grunwald-Letinikov (G-L) fractional derivative [12], and Jumarie's modified R-L fractional derivative is used to avoid nonzero fractional derivative of a constant functions [14].

In this article, we can find any order fractional derivatives of some elementary fractional functions such as fractional exponential function, trigonometric functions and hyperbolic functions, regarding the Jumarie type of modified R-L fractional derivatives. The method we used is to introduce a new multiplication and the fractional power series expansions of these fractional functions. The fractional differentiation term by term theorem plays an important role in this study, and our results are the generalizations of the results obtained by the traditional calculus.

II. PRELIMINARIES

In this section, we introduce some fractional functions and their fractional power series expansions.

Notation 2.1: If α is a real number, then

$$[\alpha] = \begin{cases} 0 & \text{if } \alpha < 0, \\ \text{the greatest integer less than or equal to } \alpha & \text{if } \alpha \geq 0. \end{cases}$$

Definition 2.2: Let α be a real number, m be a positive integer, and $f(x) \in C^{[\alpha]}([a,b])$. The modified Riemann-Liouville fractional derivatives of Jumarie type ([14]) is defined by ${}_aD_x^{\alpha}[f(x)]$

$$= \begin{cases} \frac{1}{\Gamma(-\alpha)}\int_a^x (x-\tau)^{-\alpha-1} f(\tau)d\tau, & \text{if } \alpha < 0 \\ \frac{1}{\Gamma(1-\alpha)}\frac{d}{dx}\int_a^x (x-\tau)^{-\alpha}[f(\tau)-f(a)]\,d\tau & \text{if } 0 \leq \alpha < 1 \\ \frac{d^m}{dx^m}\left({}_aD_x^{\alpha-m}\right)[f(x)], & \text{if } m \leq \alpha < m+1 \end{cases} \quad (1)$$

where $\Gamma(y) = \int_0^{\infty} t^{y-1}e^{-t}dt$ is the gamma function defined on $y > 0$, and $\left({}_aD_x^{\alpha}\right)^n = \left({}_aD_x^{\alpha}\right)\left({}_aD_x^{\alpha}\right)\cdots\left({}_aD_x^{\alpha}\right)$ is the n-th order fractional derivative of ${}_aD_x^{\alpha}$. We note that $\left({}_aD_x^{\alpha}\right)^n \neq {}_aD_x^{n\alpha}$ in general, and we have the following property [15].

Proposition 2.3: *Let* α, β, c *be real numbers and* $\beta \geq \alpha > 0$, *then*

$${}_0D_x^{\alpha}\left[x^{\beta}\right] = \frac{\Gamma(\beta+1)}{\Gamma(\beta-\alpha+1)}x^{\beta-\alpha}, \quad (2)$$

and

$${}_0D_x^{\alpha}[c] = 0. \quad (3)$$

The followings are the power series expansions of trigonometric functions and hyperbolic functions [16] used in this paper.

Proposition 2.4: *Let* B_k, E_k *be* k*-th Bernoulli number and Euler number respectively for all non-negative integers* k. *Then*

$$exp(x) = \sum_{k=0}^{\infty}\frac{1}{k!}x^k, \ -\infty < x < \infty \quad (4)$$

$$sinx = \sum_{k=0}^{\infty}\frac{(-1)^k}{(2k+1)!}x^{2k+1}, \ -\infty < x < \infty \quad (5)$$

$$cosx = \sum_{k=0}^{\infty}\frac{(-1)^k}{(2k)!}x^{2k}, \ -\infty < x < \infty \quad (6)$$

$$tanx = \sum_{k=1}^{\infty}\frac{(-1)^{k-1}2^{2k}(2^{2k}-1)B_{2k}}{2k(2k-1)!}x^{2k-1}, \ -\frac{\pi}{2} < x < \frac{\pi}{2} \quad (7)$$

$$cotx = \frac{1}{x} + \sum_{k=1}^{\infty}\frac{(-1)^k 2^{2k}B_{2k}}{2k(2k-1)!}x^{2k-1}, \ 0 < x < \pi \quad (8)$$

$$secx = \sum_{k=0}^{\infty}\frac{(-1)^k E_{2k}}{(2k)!}x^{2k}, \ -\frac{\pi}{2} < x < \frac{\pi}{2} \quad (9)$$

$$cscx = \frac{1}{x} + \sum_{k=1}^{\infty}\frac{(-1)^{k+1}2(2^{2k-1}-1)B_{2k}}{2k(2k-1)!}x^{2k-1}, \ 0 < x < \pi \quad (10)$$

$$sinhx = \sum_{k=0}^{\infty}\frac{1}{(2k+1)!}x^{2k+1}, \ -\infty < x < \infty \quad (11)$$

$$coshx = \sum_{k=0}^{\infty}\frac{1}{(2k)!}x^{2k}, \ -\infty < x < \infty \quad (12)$$

$$tanhx = \sum_{k=1}^{\infty}\frac{2^{2k}(2^{2k}-1)B_{2k}}{2k(2k-1)!}x^{2k-1}, \ -\frac{\pi}{2} < x < \frac{\pi}{2} \quad (13)$$

$$cothx = \frac{1}{x} + \sum_{k=1}^{\infty}\frac{2^{2k}B_{2k}}{(2k)!}x^{2k-1}, \ 0 < x < \pi \quad (14)$$

$$sechx = \sum_{k=0}^{\infty}\frac{E_{2k}}{(2k)!}x^{2k}, \ -\frac{\pi}{2} < x < \frac{\pi}{2} \quad (15)$$

$$cschx = \frac{1}{x} - \sum_{k=1}^{\infty}\frac{2(2^{2k-1}-1)B_{2k}}{(2k)!}x^{2k-1}, \ 0 < x < \pi \quad (16)$$

In the following, we define a new multiplication of fractional functions.

Definition 2.5: Let λ, μ, z be complex numbers, $0 < \alpha \leq 1$, j, l, k be non-negative integers, and a_k, b_k be real numbers, $p_k(z) = \frac{1}{\Gamma(k\alpha+1)}z^k$ for all k. The $\otimes$ multiplication is defined by

$$p_j(\lambda x^{\alpha}) \otimes p_l(\mu y^{\alpha}) = \frac{1}{\Gamma(j\alpha+1)}(\lambda x^{\alpha})^j \otimes \frac{1}{\Gamma(l\alpha+1)}(\mu y^{\alpha})^l = \frac{1}{\Gamma((j+l)\alpha+1)}\binom{j+l}{j}(\lambda x^{\alpha})^j(\mu y^{\alpha})^l, \quad (17)$$

where $\binom{j+l}{j} = \frac{(j+l)!}{j!l!}$.

If $f_{\alpha}(\lambda x^{\alpha})$ and $g_{\alpha}(\mu y^{\alpha})$ are two fractional functions,

$$f_{\alpha}(\lambda x^{\alpha}) = \sum_{k=0}^{\infty} a_k\, p_k(\lambda x^{\alpha}) = \sum_{k=0}^{\infty}\frac{a_k}{\Gamma(k\alpha+1)}(\lambda x^{\alpha})^k, \quad (18)$$

$$g_{\alpha}(\mu y^{\alpha}) = \sum_{k=0}^{\infty} b_k\, p_k(\mu y^{\alpha}) = \sum_{k=0}^{\infty}\frac{b_k}{\Gamma(k\alpha+1)}(\mu y^{\alpha})^k, \quad (19)$$

then we define

$$f_{\alpha}(\lambda x^{\alpha}) \otimes g_{\alpha}(\mu y^{\alpha}) = \sum_{k=0}^{\infty} a_k\, p_k(\lambda x^{\alpha}) \otimes \sum_{k=0}^{\infty} b_k\, p_k(\mu y^{\alpha}) = \sum_{k=0}^{\infty}\left(\sum_{m=0}^{k} a_{k-m}b_m p_{k-m}(\lambda x^{\alpha}) \otimes p_m(\mu y^{\alpha})\right). \quad (20)$$

Proposition 2.6: $f_{\alpha}(\lambda x^{\alpha}) \otimes g_{\alpha}(\mu y^{\alpha})$

$$= \sum_{k=0}^{\infty}\frac{1}{\Gamma(k\alpha+1)}\sum_{m=0}^{k}\binom{k}{m} a_{k-m}b_m\,(\lambda x^{\alpha})^{k-m}(\mu y^{\alpha})^m. \quad (21)$$

Definition 2.7: Let $\left(f_{\alpha}(\lambda x^{\alpha})\right)^{\otimes n} = f_{\alpha}(\lambda x^{\alpha}) \otimes \cdots \otimes f_{\alpha}(\lambda x^{\alpha})$ be the n times $\otimes$ product of the fractional function $f_{\alpha}(\lambda x^{\alpha})$. If $f_{\alpha}(\lambda x^{\alpha}) \otimes g_{\alpha}(\lambda x^{\alpha}) = 1$, then

$g_\alpha(\lambda x^\alpha)$ is called the $\otimes$ reciprocal of $f_\alpha(\lambda x^\alpha)$, and is denoted by $\left(f_\alpha(\lambda x^\alpha)\right)^{\otimes -1}$.

Remark 2.8: The $\otimes$ multiplication satisfies the commutative law and the associate law, and is the generalization of ordinary multiplication, since the $\otimes$ multiplication becomes the ordinary multiplication if $\alpha = 1$.

Definition 2.9: Let $0 < \alpha \le 1$, $f(x) = \sum_{k=0}^{\infty} \frac{a_k}{k!} x^{k\alpha}$, then

$$F_\alpha(x^\alpha) = \sum_{k=0}^{\infty} \frac{a_k}{k!} \left(\frac{1}{\Gamma(\alpha+1)} x^\alpha\right)^{\otimes k} = \sum_{k=0}^{\infty} \frac{a_k}{\Gamma(k\alpha+1)} x^{k\alpha} \quad (22)$$

is called the α-order fractional function with respect to $f(x)$. And $\sum_{k=0}^{\infty} \frac{a_k}{\Gamma(k\alpha+1)} x^{k\alpha}$ is called the fractional power series expansion of $F_\alpha(x^\alpha)$.

Proposition 2.10: *Let* $0 < \alpha \le 1$, $f(x) = \sum_{k=0}^{\infty} \frac{a_k}{k!} x^{k\alpha}$ *and* $g(x) = \sum_{k=0}^{\infty} \frac{b_k}{k!} x^{k\alpha}$. *If* $f(x) \cdot g(x) = 1$, *then* $F_\alpha(x^\alpha) \otimes G_\alpha(x^\alpha) = 1$.

Proof Since

$$f(x) \cdot g(x) = \sum_{k=0}^{\infty} \frac{1}{k!} \left(\sum_{p=0}^{k} a_{k-p} b_p\right) x^{k\alpha} = 1,$$

it follows that

$$a_0 b_0 = 1 \text{ and } \sum_{p=0}^{k} a_{k-p} b_p = 0 \text{ for all } k \ge 1.$$

By Proposition 2.6, we have

$$F_\alpha(x^\alpha) \otimes G_\alpha(x^\alpha)$$
$$= \sum_{k=0}^{\infty} \frac{1}{\Gamma(k\alpha+1)} \sum_{p=0}^{k} \binom{k}{p} a_{k-p} b_p \, (x^\alpha)^{k-p} (y^\alpha)^p$$
$$= 1. \quad \text{q.e.d.}$$

Remark 2.11: $E_\alpha(x^\alpha)$ is the α-order fractional exponential function with respect to exponential function $exp(x)$. And the α-order fractional function with respect to related trigonometric functions and hyperbolic functions are as follows:

$sin_\alpha(x^\alpha)$, $cos_\alpha(x^\alpha)$, $tan_\alpha(x^\alpha)$, $cot_\alpha(x^\alpha)$, $sec_\alpha(x^\alpha)$, $csc_\alpha(x^\alpha)$; $sinh_\alpha(x^\alpha)$, $cosh_\alpha(x^\alpha)$, $tanh_\alpha(x^\alpha)$, $coth_\alpha(x^\alpha)$, $sech_\alpha(x^\alpha)$, $csch_\alpha(x^\alpha)$.

Proposition 2.12 (fractional Euler's formula) ([17]): *Let* $0 < \alpha \le 1$, , $i = \sqrt{-1}$, *then*

$$E_\alpha(ix^\alpha) = cos_\alpha(x^\alpha) + i sin_\alpha(x^\alpha) . \quad (23)$$

Proposition 2.13: *Let* $0 < \alpha \le 1$, *then*

$$sin_\alpha(x^\alpha + y^\alpha)$$
$$= sin_\alpha(x^\alpha) \otimes cos_\alpha(y^\alpha) + cos_\alpha(x^\alpha) \otimes sin_\alpha(y^\alpha), \quad (24)$$

and

$$cos_\alpha(x^\alpha + y^\alpha)$$
$$= cos_\alpha(x^\alpha) \otimes cos_\alpha(y^\alpha) - sin_\alpha(x^\alpha) \otimes sin_\alpha(y^\alpha). \quad (25)$$

Notation 2.14: Let $0 < \alpha \le 1$. The smallest positive real number T_α such that $E_\alpha(iT_\alpha) = 1$ is called the period of $E_\alpha(ix^\alpha)$.

Remark 2.15: By Proposition 2.12, we have $sin_\alpha(T_\alpha)$=0 and $cos_\alpha(T_\alpha) = 1$. Taking advantage of Proposition 2.13 yields $sin_\alpha(x^\alpha + T_\alpha) = sin_\alpha(x^\alpha)$ and $cos_\alpha(x^\alpha + T_\alpha) = cos_\alpha(x^\alpha)$, i.e., the periods of both $sin_\alpha(x^\alpha)$ and $cos_\alpha(x^\alpha)$ are T_α. On the other hand, all the periods of $tan_\alpha(x^\alpha)$, $cot_\alpha(x^\alpha)$, $sec_\alpha(x^\alpha)$, $csc_\alpha(x^\alpha)$ are $\frac{1}{2}T_\alpha$.

III. MAIN RESULTS

In the following, we can obtain the fractional power series of fractional trigonometric functions and hyperbolic functions.

Theorem 3.1: *Let* $0 < \alpha \le 1$, *then*

$$E_\alpha(x^\alpha) = \sum_{k=0}^{\infty} \frac{1}{\Gamma(k\alpha+1)} x^{k\alpha} , \; -\infty < x < \infty \quad (26)$$

$$sin_\alpha(x^\alpha) = \sum_{k=0}^{\infty} \frac{(-1)^k}{\Gamma((2k+1)\alpha+1)} x^{(2k+1)\alpha} , -\infty < x < \infty \quad (27)$$

$$cos_\alpha(x^\alpha) = \sum_{k=0}^{\infty} \frac{(-1)^k}{\Gamma(2k\alpha+1)} x^{2k\alpha}, -\infty < x < \infty \quad (28)$$

$$tan_\alpha(x^\alpha) = \sum_{k=1}^{\infty} \frac{(-1)^{k-1} 2^{2k} (2^{2k}-1) B_{2k}}{2k \cdot \Gamma((2k-1)\alpha+1)} x^{(2k-1)\alpha} , \; -\frac{1}{4}T_\alpha < x < \frac{1}{4}T_\alpha \quad (29)$$

$$cot_\alpha(x^\alpha) = \left(\frac{1}{\Gamma(\alpha+1)} x^\alpha\right)^{\otimes -1} + \sum_{k=1}^{\infty} \frac{(-1)^k 2^{2k} B_{2k}}{2k \cdot \Gamma((2k-1)\alpha+1)} x^{(2k-1)\alpha}, \;\; 0 < x < \frac{1}{2}T_\alpha \quad (30)$$

$$sec_\alpha(x^\alpha) = \sum_{k=0}^{\infty} \frac{(-1)^k E_{2k}}{\Gamma(2k\alpha+1)} x^{2k\alpha}, -\frac{1}{4}T_\alpha < x < \frac{1}{4}T_\alpha \quad (31)$$

$$csc_\alpha(x^\alpha) = \left(\frac{1}{\Gamma(\alpha+1)} x^\alpha\right)^{\otimes -1} + \sum_{k=1}^{\infty} \frac{(-1)^{k+1} 2 (2^{2k-1}-1) B_{2k}}{2k \cdot \Gamma((2k-1)\alpha+1)} x^{(2k-1)\alpha} , 0 < x < \frac{1}{2}T_\alpha \quad (32)$$

$$sinh_\alpha(x^\alpha) = \sum_{k=0}^{\infty} \frac{1}{\Gamma((2k+1)\alpha+1)} x^{(2k+1)\alpha} , -\infty < x < \infty$$

(33)

$$cosh_{\alpha}(x^{\alpha})=\sum_{k=0}^{\infty}\frac{1}{\Gamma(2k\alpha+1)}x^{2k\alpha}\ ,-\infty<x<\infty \quad (34)$$

$$tanh_{\alpha}(x^{\alpha})=\sum_{k=1}^{\infty}\frac{2^{2k}(2^{2k}-1)B_{2k}}{2k\cdot\Gamma((2k-1)\alpha+1)}x^{(2k-1)\alpha},\ -\frac{1}{4}T_{\alpha}<x<\frac{1}{4}T_{\alpha} \quad (35)$$

$$coth_{\alpha}(x^{\alpha})=\left(\frac{1}{\Gamma(\alpha+1)}x^{\alpha}\right)^{\otimes -1}+\sum_{k=1}^{\infty}\frac{2^{2k}B_{2k}}{2k\cdot\Gamma((2k-1)\alpha+1)}x^{(2k-1)\alpha},\ 0<x<\frac{1}{2}T_{\alpha} \quad (36)$$

$$sech_{\alpha}(x^{\alpha})=\sum_{k=0}^{\infty}\frac{E_{2k}}{\Gamma(2k\alpha+1)}x^{2k\alpha},\ -\frac{1}{4}T_{\alpha}<x<\frac{1}{4}T_{\alpha} \quad (37)$$

$$csch_{\alpha}(x^{\alpha})=\left(\frac{1}{\Gamma(\alpha+1)}x^{\alpha}\right)^{\otimes -1}-\sum_{k=1}^{\infty}\frac{2(2^{2k-1}-1)B_{2k}}{2k\cdot\Gamma((2k-1)\alpha+1)}x^{(2k-1)\alpha}\ ,0<x<\frac{1}{2}T_{\alpha} \quad (38)$$

Proof Using Proposition 2.4 and Proposition 2.10 yields the desired results hold. q.e.d.

The following is the major result in this article. We can evaluate any order fractional derivatives of these fractional functions discussed above by using fractional differentiation term by term theorem.

Theorem 3.2: *Let* $0<\alpha\leq 1$, n *be any positive integer, then*

$$\left({}_{0}D_{x}^{\alpha}\right)^{n}[E_{\alpha}(x^{\alpha})]=E_{\alpha}(x^{\alpha})=\sum_{k=0}^{\infty}\frac{1}{\Gamma(k\alpha+1)}x^{k\alpha}\ ,\ -\infty<x<\infty \quad (39)$$

$$\left({}_{0}D_{x}^{\alpha}\right)^{n}[sin_{\alpha}(x^{\alpha})]=\sum_{k=\left[\frac{n}{2}\right]}^{\infty}\frac{(-1)^{k}}{\Gamma((2k+1-n)\alpha+1)}x^{(2k+1-n)\alpha}\ ,\ -\infty<x<\infty \quad (40)$$

$$\left({}_{0}D_{x}^{\alpha}\right)^{n}[cos_{\alpha}(x^{\alpha})]=\sum_{k=\left[\frac{n+1}{2}\right]}^{\infty}\frac{(-1)^{k}}{\Gamma((2k-n)\alpha+1)}x^{(2k-n)\alpha},\ -\infty<x<\infty \quad (41)$$

$$\left({}_{0}D_{x}^{\alpha}\right)^{n}[tan_{\alpha}(x^{\alpha})]=\sum_{k=\left[\frac{n+2}{2}\right]}^{\infty}\frac{(-1)^{k-1}2^{2k}(2^{2k}-1)B_{2k}}{2k\cdot\Gamma((2k-1-n)\alpha+1)}x^{(2k-1-n)\alpha}\ ,\ -\frac{1}{4}T_{\alpha}<x<\frac{1}{4}T_{\alpha} \quad (42)$$

$$\left({}_{0}D_{x}^{\alpha}\right)^{n}[cot_{\alpha}(x^{\alpha})]=(-1)^{n}n!\left(\frac{1}{\Gamma(\alpha+1)}x^{\alpha}\right)^{\otimes -1-n}+\sum_{k=\left[\frac{n+2}{2}\right]}^{\infty}\frac{(-1)^{k}2^{2k}B_{2k}}{2k\cdot\Gamma((2k-1-n)\alpha+1)}x^{(2k-1-n)\alpha}\ ,0<x<\frac{1}{2}T_{\alpha} \quad (43)$$

$$\left({}_{0}D_{x}^{\alpha}\right)^{n}[sec_{\alpha}(x^{\alpha})]=\sum_{k=\left[\frac{n+1}{2}\right]}^{\infty}\frac{(-1)^{k}E_{2k}}{\Gamma((2k-n)\alpha+1)}x^{(2k-n)\alpha},\ -\frac{1}{4}T_{\alpha}<x<\frac{1}{4}T_{\alpha} \quad (44)$$

$$\left({}_{0}D_{x}^{\alpha}\right)^{n}[csc_{\alpha}(x^{\alpha})]=(-1)^{n}n!\left(\frac{1}{\Gamma(\alpha+1)}x^{\alpha}\right)^{\otimes -1-n}+\sum_{k=\left[\frac{n+2}{2}\right]}^{\infty}\frac{(-1)^{k+1}2(2^{2k-1}-1)B_{2k}}{2k\cdot\Gamma((2k-1-n)\alpha+1)}x^{(2k-1-n)\alpha}\ ,0<x<\frac{1}{2}T_{\alpha} \quad (45)$$

$$\left({}_{0}D_{x}^{\alpha}\right)^{n}[sinh_{\alpha}(x^{\alpha})]=\sum_{k=\left[\frac{n}{2}\right]}^{\infty}\frac{1}{\Gamma((2k+1-n)\alpha+1)}x^{(2k+1-n)\alpha}\ ,\ -\infty<x<\infty \quad (46)$$

$$\left({}_{0}D_{x}^{\alpha}\right)^{n}[cosh_{\alpha}(x^{\alpha})]=\sum_{k=\left[\frac{n+1}{2}\right]}^{\infty}\frac{1}{\Gamma((2k-n)\alpha+1)}x^{(2k-n)\alpha},\ -\infty<x<\infty \quad (47)$$

$$\left({}_{0}D_{x}^{\alpha}\right)^{n}[tanh_{\alpha}(x^{\alpha})]=\sum_{k=\left[\frac{n+2}{2}\right]}^{\infty}\frac{2^{2k}(2^{2k}-1)B_{2k}}{2k\cdot\Gamma((2k-1-n)\alpha+1)}x^{(2k-1-n)\alpha},\ -\frac{1}{4}T_{\alpha}<x<\frac{1}{4}T_{\alpha} \quad (48)$$

$$\left({}_{0}D_{x}^{\alpha}\right)^{n}[coth_{\alpha}(x^{\alpha})]=(-1)^{n}n!\left(\frac{1}{\Gamma(\alpha+1)}x^{\alpha}\right)^{\otimes -1-n}+\sum_{k=\left[\frac{n+2}{2}\right]}^{\infty}\frac{2^{2k}B_{2k}}{2k\cdot\Gamma((2k-1-n)\alpha+1)}x^{(2k-1-n)\alpha},\ 0<x<\frac{1}{2}T_{\alpha} \quad (49)$$

$$\left({}_{0}D_{x}^{\alpha}\right)^{n}[sech_{\alpha}(x^{\alpha})]=\sum_{k=\left[\frac{n+1}{2}\right]}^{\infty}\frac{E_{2k}}{\Gamma((2k-n)\alpha+1)}x^{(2k-n)\alpha}\ ,\ -\frac{1}{4}T_{\alpha}<x<\frac{1}{4}T_{\alpha} \quad (50)$$

$$\left({}_{0}D_{x}^{\alpha}\right)^{n}[csch_{\alpha}(x^{\alpha})]=(-1)^{n}n!\left(\frac{1}{\Gamma(\alpha+1)}x^{\alpha}\right)^{\otimes -1-n}-\sum_{k=\left[\frac{n+2}{2}\right]}^{\infty}\frac{2(2^{2k-1}-1)B_{2k}}{2k\cdot\Gamma((2k-1-n)\alpha+1)}x^{(2k-1-n)\alpha}\ ,0<\ x<\frac{1}{2}T_{\alpha} \quad (51)$$

IV. CONCLUSION

The fractional functions studied in this paper are generalizations of traditional elementary functions. And the fractional differential problem of these fractional functions is also the generalization of classical differential problem. The methods we used in this article are fractional power series expansions and fractional differentiation term by term theorem. In the future, we will use the modified R-L fractional derivatives and the new multiplication $\otimes$ to study the engineering mathematics problems.

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