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≈ g(1,2)*-Closed and ≈ g(1,2)*-Open Maps

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ABSTRACT

General topology plays vital role in many fields of applied sciences as well as in all branches of mathematics. In reality it is used in data mining, computational topology for geometric design and molecular design, computer-aided design, computer-aided geometric design, digital topology, information systems, particle physics and quantum physics etc. By researching generalizations of closed sets, some new separation axioms have been founded and they turn out to be useful in the study of digital topology. Therefore, all bi-topological sets and functions defined will have many possibilities of applications in digital topology and computer graphics.

Keywords : Closed Sets, Open Maps, bi-topological sets.

I. INTRODUCTION

Malghan [40] introduced the concept of generalized closed maps in topological spaces. Devi [18] introduced and studied sg-closed maps and gs-closed maps. Recently, Sheik John [76] defined -closed maps and studied some of their properties. We introduce ≈g(1,2)*-closed maps, ≈g(1,2)*-open maps, ≈ *g(1,2)*-closed maps and ≈*g(1,2)*-open maps in bitopological spaces and obtainertain characterizations of these classes of maps. In last section, we introduce ≈ *g(1,2)*-homeomorphisms and prove that the set of all≈*g(1,2)*- homeomorphisms forms a group under the operation composition of functions.

II. PRELIMINARIES

Definition : A map f : (X, τ_1, τ_2) → (Y, σ_1, σ_2) is called

- (1,2)*-g-closed [68] if f(V) is (1,2)*-g-closed in Y, for every $\tau_{1,2}$-closed set V of X.
- (1,2)*-sg-closed [65] if f(V) is (1,2)*-sg-closed in Y, for every $\tau_{1,2}$-closed set V of X.
- (1,2)*-gs-closed [65] if f(V) is (1,2)*-gs-closed in Y, for every $\tau_{1,2}$-closed set V of X.
- (1,2)*-ψ-closed [51] if f(V) is (1,2)*-ψ-closed in Y, for every $\tau_{1,2}$-closed set V of X.

III. ≈ g(1,2)*-CLOSED MAPS

Definition : A map f : (X, τ_1, τ_2) → (Y, σ_1, σ_2) is said to be ≈ g(1,2)*-closed if the image of every $\tau_{1,2}$-closed set in X is ≈g(1,2)*-closed in Y.

Example 3.1

Let X = Y = {a, b, c}, τ_1 = { φ, X, {a}} and τ_2 = {φ, X, {b}}. Then the sets in{φ, X, {a}, {b}, {a, b}} are called $\tau_{1,2}$-open and the sets in {φ, X, {c}, {a, c}, {b, c}} are called $\tau_{1,2}$-closed. Let σ_1 = {φ, Y} and σ_2 = {φ, Y, {a, b}}. Then the sets in {φ, Y,{a, b}} are called $\sigma_{1,2}$-open in Y and the sets in {φ, Y, {c}} are called $\sigma_{1,2}$-closed inY. Let f : (X, τ_1, τ_2) → (Y, σ_1, σ_2) be the identity map. Then f is an ≈ g(1,2)*-closed map.

Proposition 3.1

A map f : (X, τ_1, τ_2) → (Y, σ_1, σ_2) is ≈g(1,2)*-closed if and only if ≈g(1,2)*-cl(f(A)) ⊆ f($\tau_{1,2}$-cl(A)) for every subset A of X.

Proof

Suppose that f is ≈ g(1,2)*-closed and A ⊆ X. Then $\tau_{1,2}$-cl(A) is $\tau_{1,2}$-closed in X and so f($\tau_{1,2}$-cl(A)) is ≈g(1,2)*-closed in Y. We have f(A) ⊆ f($\tau_{1,2}$-cl(A)) and by Propositions 1.6.9 and 1.6.10, ≈g(1,2)*-cl(f(A)) ⊆ ≈ g(1,2)*-cl(f($\tau_{1,2}$-cl(A))) = f($\tau_{1,2}$- cl(A)).Conversely, let A be any $\tau_{1,2}$-closed set in X. Then A = $\tau_{1,2}$-cl(A) and so f(A) = f($\tau_{1,2}$-cl(A)) ⊇ ≈g(1,2)*-cl(f(A)), by hypothesis. We have f(A) ⊆ ≈g(1,2)*-cl(f(A)). Therefore f(A) = ≈g(1,2)*-cl(f(A)). That is f(A) is ≈g(1,2)*-closed by proposition 1.6.9 and hence f is ≈ g(1,2)*-closed.

Proposition 3.2

Let f : (X, τ_1, τ_2) → (Y, σ_1, σ_2) be a map such that ≈g(1,2)*-cl(f(A)) ⊆ f($\tau_{1,2}$- cl(A)) for every subset A ⊆ X. Then the image f(A) of a $\tau_{1,2}$-closed set A in X is ≈ g(1,2)*-closed in Y.

Proof

A be a $\tau_{1,2}$-closed set in X. Then by hypothesis ≈ g(1,2)*-cl(f(A)) ⊆ f($\tau_{1,2}$- cl(A)) = f(A) and so ≈ g(1,2)*-cl(f(A))=f(A). Therefore f(A) is ≈ g(1,2)*-closed in Y.

Theorem 3.1

A map f : (X, τ_1, τ_2) → (Y, σ_1, σ_2) is ≈ g(1,2)*-closed if and only if for each subset S of Y and each $\tau_{1,2}$-open set U containing f^{-1}(S) there is an ≈ g(1,2)*-open set V of Y such that S ⊆ V and f^{-1}(V) ⊆ U.

Proof

Suppose f is ≈ g(1,2)*-closed. Let S ⊆ Y and U be an $\tau_{1,2}$-open set of X such thatf^{1}(S)⊆U.Then V = $(f(U^c))^c$ is an ≈ g(1,2)*-open set containing S such that f^{-1}(V) ⊆ U.

For the converse, let F be a $\tau_{1,2}$-closed set of X. Then $f^{-1}((f(F))^c)$ ⊆ F^c and F^c is $\tau_{1,2}$-open. By assumption, there exists an ≈ g(1,2)*-open set V in Y such that $(f(F))^c$ ⊆ V and f^{-1}(V) ⊆ F^c and so F ⊆ $(f^{-1}(V))^c$. Hence V^c ⊆ f(F) ⊆ $f((f^{-1}(V))^c)$ ⊆ V^c which implies f(F) = V^c. Since V^c is ≈ g(1,2)*-closed, f(F) is ≈ g(1,2)*-closed and therefore f is ≈g(1,2)*-closed.

Proposition 3.3

If f : (X, τ_1, τ_2) → (Y, σ_1, σ_2) is (1,2)*-sg-irresolute ≈g(1,2)*-closed and A is an ≈g(1,2)*-closed subset of X, then f(A) is ≈g(1,2)*-closed in Y.

Proof

Let U be an (1,2)*-sg-open set in Y such that f(A) ⊆ U. Since f is (1,2)*-sg- irresolute, f^{-1}(U) is an (1,2)*-sg-open set containing A. Hence $\tau_{1,2}$-cl(A) ⊆ f^{-1}(U) as A is ≈ g(1,2)*-closed in X. Since f is ≈ g(1,2)*-closed, f($\tau_{1,2}$-cl(A)) is an ≈ g(1,2)*- closed set contained in the (1,2)*-sg-open set U, which implies that $\tau_{1,2}$-cl(f($\tau_{1,2}$-cl(A))) ⊆ U and hence $\tau_{1,2}$-cl(f(A)) ⊆ U. Therefore, f(A) is an ≈g(1,2)*-closed set in Y.

The following example shows that the composition of two ≈ g(1,2)*-closed maps need not be a ≈ g(1,2)*-closed.

Example3.2

Let X, Y and f be as in Example 5.3.2. Let Z = {a, b, c} and η_1 = {φ, Z, {c}} and η_2 = {φ, Z, {a, b}}. Then the sets in {φ, Z, {c}, {a, b}} are called $\eta_{1,2}$-open and the sets in {φ, Z, {c}, {a, b}} are called $\eta_{1,2}$-closed. Let g : (Y, σ_1, σ_2) → (Z, η_1, η_2) be the identity map. Then both f and g are ≈ g(1,2)*-closed maps but their composition g o f : (X, τ_1, τ_2) → (Z, η_1, η_2) is not an ≈g(1,2)*-closed map, since for the $\tau_{1,2}$-closed set {b, c} in X, (g o f) ({b, c}) = {b, c}, which is not an ≈ g(1,2)*- closed set in Z.

Corollary 3.1

Let $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ be ≈ g(1,2)*-closed and $g : (Y, \sigma_1, \sigma_2) \to (Z, \eta_1, \eta_2)$ be ≈g(1,2)*-closed and (1,2)*-sg-irresolute, then their composition g o f : $(X, \tau_1, \tau_2) \to (Z, \eta_1, \eta_2)$ is ≈ g(1,2)*-closed.

Proof

Let A be a $\tau_{1,2}$-closed set of X. Then by hypothesis f(A) is an ≈ g(1,2)*-closed set in Y. Since g is both ≈g(1,2)*-closed and (1,2)*-sg-irresolute by Proposition 5.3.6, g(f(A)) = (g o f) (A) is ≈g(1,2)*-closed in Z and therefore g o f is ≈g(1,2)*- closed.

Proposition 3.4

Let $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ and $g : (Y, \sigma_1, \sigma_2) \to (Z, \eta_1, \eta_2)$ be ≈g(1,2)*- closed maps where Y is a T ≈ $g_{(1,2)^*}$-space. Then their composition g o f : $(X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is ≈g(1,2)*-closed.

Proof

Let A be a $\tau_{1,2}$-closed set of X. Then by assumption f(A) is ≈g(1,2)*-closed inY. Since Y is a T ≈ $g_{(1,2)^*}$-space, f(A) is $\sigma_{1,2}$-closed in Yand again by assumptiong(f(A))is ≈g(1,2)*closedinZ.That is (g o f) (A) is ≈ g(1,2)*-closed in Z and so g o f is ≈g(1,2)*-closed.

Proposition 3.5

If $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is ≈ g(1,2)*-closed, $g : (Y, \sigma_1, \sigma_2) \to (Z, \eta_1,\eta_2)$ is ≈ g(1,2)*-closed (resp. (1,2)*-g-closed, (1,2)*-ψ-closed, (1,2)*-sg-closed and (1,2)*-gs-closed) and Y is a T ≈$g_{(1,2)^*}$-space, then their composition g o f : $(X, \tau_1, \tau_2)\to (Z, \eta_1, \eta_2)$ is ≈ g(1,2)*-closed (resp. (1,2)*-g-closed, (1,2)*-ψ-closed, (1,2)*-sg-closed and (1,2)*-gs-closed).

Proposition 3.6

Let $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ be a (1,2)*-closed map and $g : (Y, \sigma_1, \sigma_2) \to (Z, \eta_1, \eta_2)$ be an ≈g(1,2)*-closed map, then their composition g o f : $(X, \tau_1, \tau_2) \to (Z, \eta_1, \eta_2)$ is ≈ g(1,2)*-closed.

Remark 3.1

If $f : (X, \tau 1, \tau 2) \to (Y, \sigma 1, \sigma 2)$ is an ≈ g(1,2)*-closed and $g : (Y, \sigma 1,\to (Z,\eta 1, \eta 2)$ is (1,2)*-closed, then their composition need not be an ≈ g(1,2)*-closed map as seen from the following example.

Example 3.3

Let X, Y and f be as in Example 5.3.2. Let Z = {a, b, c} and η_1 = {φ, Z, {a}} and η= {φ,Z, {a, b}}.Then the sets in {φ, Z, {a}, {a, b}} are called$\eta_{1,2}$open and the sets in {φ, Z, {c}, {b, c}} are called $\eta_{1,2}$-closed. Let $g : (Y, \sigma_1, \sigma_2) \to (Z, \eta_1, \eta_2)$ be the identity map. Then f is an ≈g(1,2)*-closed map and g is a (1,2)*-closed map. But their composition g o f : $(X, \tau_1, \tau_2) \to (Z, \eta_1, \eta_2)$ is not an ≈g(1,2)*-closed map, since for the $\tau_{1,2}$-closed set {a, c} in X, (g o f) ({a, c}) = {a, c}, which is not an≈ g(1,2)*-closed set in Z.

Theorem 3.2

Let $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ and $g : (Y, \sigma_1, \sigma_2) \to (Z, \eta_1, \eta_2)$ be two maps such that their composition gof:$(X,\tau_1,\tau_2)\to(Z,\eta_1,\eta_2)$ is an ≈g(1,2)*-closed map. Then the following statements are true.

- If f is (1,2)*-continuous and surjective, then g is ≈g(1,2)*-closed.
- If g is ≈g(1,2)*irresolute and injective, then f is ≈g(1,2)*-closed.
- If f is (1,2)*-ĝ-continuous, surjective and (X, τ) is a (1,2)*-T_ω-space, then g is≈ g(1,2)*-closed.
- If g is strongly ≈g(1,2)*-continuous and injective, then f is (1,2)*-closed.

Proof

- Let A be a $\sigma_{1,2}$-closed set of Y. Since f is (1,2)*-continuous, $f^{-1}(A)$ is $\tau_{1,2}$- closed in X and since g o f is ≈ g(1,2)*-closed, (g o f)($f^{-1}(A)$) is ≈ g(1,2)*-closed in Z. That is g(A) is ≈ g(1,2)*-closed in Z, since f is surjective. Therefore g is an ≈ g(1,2)*-closed map.
- Let B be a $\tau_{1,2}$-closed set of X. Since g o f is ≈ g(1,2)*-closed, (g o f) (B) is≈ g(1,2)*-closed in Z. Since g is ≈ g(1,2)*-irresolute, g^{-1}((g o f)(B)) is≈ g(1,2)*-closed set in Y. That is f(B) is ≈g(1,2)*-

closed in Y, since g is injective. Thus f is an ≈ g(1,2)*-closed map.

- Let C be a $\sigma_{1,2}$-closed set of Y. Since f is (1,2)*-ĝ-continuous, $f^{-1}(C)$ is (1,2)*- ĝ-closed in X. Since X is a (1,2)*-T_ω-space, $f^{-1}(C)$ is $\tau_{1,2}$-closed in X and so as in (i), g is an ≈g(1,2)*-closed map.
- Let D be a $\tau_{1,2}$-closed set of X. Since g o f is ≈ g(1,2)*-closed, (g o f)(D) is≈ g(1,2)*-closed in Z. Since g is strongly ≈g(1,2)*-continuous, g^{-1}((g o f)(D)) is $\sigma_{1,2}$-closed in Y. That is f(D) is $\sigma_{1,2}$-closed set in Y, since g is injective. Therefore f is a (1,2)*-closed map.In the next theorem we show that (1,2)*-normality is preserved under (1,2)*-continuous ≈ g(1,2)*-closed maps.

Theorem 3.3

If f : (X, τ_1, τ_2) → (Y, σ_1, σ_2) is a (1,2)*continuous, ≈g(1,2)*closed map from a (1,2)*-normal space X onto a space Y, then Y is (1,2)*-normal.

Proof

Let A and B be two disjoint $\sigma_{1,2}$-closed subsets of Y. Since f is (1,2)*-continuous, $f^{-1}(A)$ and $f^{-1}(B)$ are disjoint $\tau_{1,2}$-closed sets of X. Since X is (1,2)*- normal, there exist disjoint $\tau_{1,2}$-open sets U and V of X such that $f^{-1}(A) \subseteq U$ and $f^{-1}(B) \subseteq V$.Since f is ≈g(1,2)*-closed, by Theorem 5.3.5, there exist disjoint ≈ g(1,2)*-open sets G and H in Y such that A ⊆ G, B ⊆ H, $f^{-1}(G) \subseteq U$ and $f^{-1}(H) \subseteq V$. Since U and V are disjoint, $\sigma_{1,2}$-int(G) and $\sigma_{1,2}$-int(H) are disjoint $\sigma_{1,2}$-open sets in Y. Since A is $\sigma_{1,2}$- closed, A is (1,2)*-sg-closed and therefore we have by Theorem 3.3.3, A ⊆ $\sigma_{1,2}$- int(G). Similarly B ⊆ $\sigma_{1,2}$-int(H) and hence Y is (1,2)*-normal.Analogous to an ≈ g(1,2)*-,we have defined an ≈ g(1,2)*-open map as follows:

Definition

A map f : (X, τ_1, τ_2) → (Y, σ_1, σ_2) is said to be an ≈ g(1,2)*-open map if the image f(A) is ≈ g(1,2)*-open in Y for each $\tau_{1,2}$-open set A in X.

Proposition 3.7

For any bijection f : (X, τ_1, τ_2) → (Y, σ_1, σ_2), the following statements are equivalent:

(i) f^{-1} : (Y, σ_1, σ_2)→ (X, τ_1, τ_2) is ≈ g(1,2)*-continuous.

(ii) f is ≈ g(1,2)*-open map.

(iii) f is ≈ g(1,2)*-closed map.

Proof

- i⇒ (ii). Let U be an $\tau_{1,2}$-open set of X. By assumption, $(f^{-1})^{-1}(U)$ = f(U) is ≈g(1,2)*- open in Y and so f is ≈g(1,2)*-open.
- i⇒ (iii). Let F be a $\tau_{1,2}$-closed set of X. Then F^c is $\tau_{1,2}$-open set in X. By assumption, $f(F^c)$ is ≈ g(1,2)*-open in Y. That is $f(F^c) = (f(F))^c$ is ≈ g(1,2)*-open in Y and therefore f(F) is ≈ g(1,2)*-closed in Y. Hence f is ≈ g(1,2)*-closed.
- i⇒ (i). Let F be a $\tau_{1,2}$-closed set of X. By assumption, f(F) is ≈g(1,2)*-closed in Y. But f(F) = $(f^{-1})^{-1}(F)$ and therefore f^{-1} is ≈ g(1,2)*-continuous.

Theorem 3.4

Assume that the collection of all ≈ g(1,2)*-open sets of Y is closed under arbitrary union. Let f : (X, τ_1, τ_2) → (Y, σ_1, σ_2) be a map. Then the following statements are equivalent:

- f is an ≈ g(1,2)*-open map.
- For a subset A of X, f($\tau_{1,2}$-int(A)) ⊆ ≈g(1,2)*-int(f(A)).
- For each x ∈ X and for each $\tau_{1,2}$-neighborhood U of x in X, there exists an≈ g(1,2)*- neighborhood W of f(x) in Y such that W ⊆ f(U).

Proof

- i⇒ (ii). Suppose f is ≈ g(1,2)*-open. Let A ⊆ X. Then $\tau_{1,2}$-int(A) is $\tau_{1,2}$-open in X and so f($\tau_{1,2}$-int(A)) is ≈ g(1,2)*-open in Y. We have f($\tau_{1,2}$-int(A)) ⊆ f(A). Therefore by Proposition 1.5.3, f($\tau_{1,2}$-int(A)) ⊆ ≈g(1,2)*-int(f(A)).
- i⇒ (iii). Suppose (ii) holds. Let x ∈ X andU be an arbitrary $\tau_{1,2}$-neighborhood of x in X. Then there

exists an $\tau_{1,2}$-open set G such that x ∈ G ⊆ U. By assumption, f(G) = f($\tau_{1,2}$-int(G)) ⊆ ≈ g(1,2)*-int(f(G)). This implies f(G) = ≈ g(1,2)*-int(f(G)). By Proposition 1.5.3, we have f(G) is ≈ g(1,2)*-open in Y. Further, f(x) ∈ f(G) ⊆ f(U) and so (iii) holds, by taking W = f(G).

- i⇒ (i). Suppose (iii) holds. Let U be any $\tau_{1,2}$-open set in X, x ∈ U and f(x) = y. Then y ∈ f(U) and for each y ∈ f(U), by assumption there exists an ≈g(1,2)*- neighborhood W_y of y in Y such that W_y ⊆ f(U). Since W_y is an ≈g(1,2)*- neighborhood of y, there exists an ≈ g(1,2)*-open set Vy in Y such that y ∈ V_y ⊆ W_y. Therefore, f(U) = ∪ {V_y : y ∈ f(U)} is an ≈g(1,2)*-open set in Y. Thus f is an≈ g(1,2)*-open map.

Theorem 3.5

A map f : (X, τ_1, τ_2) → (Y, σ_1, σ_2) is ≈ g(1,2)*-open if and only if for any subset S of Y and for any $\tau_{1,2}$-closed set F containing f^{-1}(S), there exists an ≈ g(1,2)*-closed set K of Y containing S such that f^{-1}(K) ⊆ F.

Corollary 3.2

A map f : (X, τ_1, τ_2) → (Y, σ_1, σ_2) is ≈ g(1,2)*-open if and only if f^{-1}(≈g(1,2)*-cl(B)) ⊆ $\tau_{1,2}$-cl(f^{-1}(B)) for each subset B of Y.

Proof

Suppose that f is ≈g(1,2)*-open. Then for any B ⊆ Y, f^{-1}(B) ⊆ $\tau_{1,2}$-cl(f^{-1}(B)). By Theorem 5.3.19, there exists an ≈ g(1,2)*-closed set K of Y such that B ⊆ K and f^{-1}(K) ⊆ $\tau_{1,2}$-cl(f^{-1}(B)). Therefore, f^{-1}(≈ g(1,2)*-cl(B)) ⊆ (f^{-1}(K)) ⊆ $\tau_{1,2}$-cl(f^{-1}(B)), since K is an ≈ g(1,2)*-closed set in Y.

Conversely, let S be any subset of Y and F be any $\tau_{1,2}$-closed set containing f^{-1}(S). Put K = ≈ g(1,2)*-cl(S). Then K is an ≈ g(1,2)*-closed set and S ⊆ K. By assumption, f^{-1}(K) = f^{-1}(≈g(1,2)*-cl(S)) ⊆ $\tau_{1,2}$-cl(f^{-1}(S)) ⊆ F and therefore by Theorem 5.3.19, f is ≈g(1,2)*-open.

Finally in this section, we define another new class of maps called ≈ *g(1,2)*- closed maps which are stronger than ≈ g(1,2)*-closed maps.

Definition

A map f : (X, τ_1, τ_2) → (Y, σ_1, σ_2) is said to be ≈ *g(1,2)*-closed if the image f(A) is ≈ g(1,2)*-closed in Y for every ≈ g(1,2)*-closed set A in X.For example the map f in Example 5.3.2 is an ≈*g(1,2)*-closed map.

Example 2.4

Let X = Y = {a, b, c} τ_1 = { φ, X, {a, b}} and τ_2 = {φ, X}. Then the sets in

{φ, X, {a, b}} are called $\tau_{1,2}$-open and the sets in {φ, X, {c}} are called $\tau_{1,2}$-closed. Let σ_1 = {φ, Y, {a}} and σ_2 = {φ, Y, {a, b}}. Then the sets in {φ, Y, {a}, {a, b}} are called $\sigma_{1,2}$-open and the sets in {φ, Y, {c}, {b, c}} are called $\sigma_{1,2}$-closed. Let f : (X, τ_1, τ_2) → (Y, σ_1, σ_2) be the identity map. Then f is an ≈ g(1,2)*-closed but not≈ *g(1,2)*-closed map. Since {a, c} is ≈g(1,2)*-closed set in X, but its image under f is {a, c} which is not ≈g(1,2)*-closed set in Y.

Proposition 3.7

A map f : (X, τ_1, τ_2) → (Y, σ_1, σ_2) is ≈*g(1,2)*-closed if and only if≈ g(1,2)*-cl(f(A)) ⊆ f(≈ g(1,2)*-cl(A)) for every subset A of X.

Analogous to ≈*g(1,2)*-closed map we can also define ≈ *g(1,2)*-open map.

Proposition 3.8

For any bijection f : (X, τ_1, τ_2) → (Y, σ_1, σ_2), the following statements are equivalent:

- ✓ f^{-1} : (Y, σ_1, σ_2)→ (X, τ_1, τ_2) is ≈ g(1,2)*-irresolute.
- ✓ f is ≈ *g(1,2)*-open map.
- ✓ f is ≈ *g(1,2)*-closed map.

Proposition 3.9

If f : (X, τ_1, τ_2) → (Y, σ_1, σ_2) is (1,2)*-sg-irresolute and ≈ g(1,2)*-closed, then it is an ≈ *g(1,2)*-closed map.

IV. ≈ *g(1,2)*-HOMEOMORPHISMS

The notion of (1,2)*-homeomorphisms plays a very important role in bitopological spaces. By definition, an (1,2)*-homeomorphism between two bitopological spaces (X, τ_1, τ_2) and (Y, σ_1, σ_2) is a bijective map $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ when f and f^{-1} are (1,2)*-continuous. We introduce the following definition:

Definition

A bijection $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ is said to be

- ≈ g(1,2)*-homeomorphism if f is both≈g(1,2)*continuous and ≈g(1,2)*- open.
- ≈ *g(1,2)*-homeomorphism if both f and f^{-1} are ≈g(1,2)*-irresolute.

We denote the family of all ≈ *g(1,2)*-homeomorphisms of a bitopological space (X, τ_1, τ_2) onto itself by ≈ (1,2)*-g*-h(X).

Theorem 4.1

Let $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ be a bijective ≈ g(1,2)*-continuous map. Then the following are equivalent:

- f is an ≈g(1,2)*-open map.
- f is an ≈ g(1,2)*-homeomorphism.
- f is an ≈g(1,2)*-closed map.

Proposition 4.1

If $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ and $g : (Y, \sigma_1, \sigma_2) \rightarrow (Z, \eta_1, \eta_2)$ are ≈*g(1,2)*- homeomorphisms, then their composition $g \circ f : (X, \tau_1, \tau_2) \rightarrow (Z, \eta_1, \eta_2)$ is also ≈ *g(1,2)*-homeomorphism.

Proof

Let U be ≈ g(1,2)*-open set in (Z, η_1, η_2). Now, $(g \circ f)^{-1}(U) = f^{-1}(g^{-1}(U)) = f^{-1}(V)$, where $V = g^{-1}(U)$. By hypothesis, V is ≈ g(1,2)*-open in Y and so again by hypothesis, $f^{-1}(V)$ is ≈ g(1,2)*-open in X. Therefore, g o f is ≈ g(1,2)*-irresolute.Also for an ≈ g(1,2)*-open set G in X, we have (g o f)(G) =g(f(G)) = g(W), where W= f(G). By hypothesisf(G)is≈g(1,2)*open in Yand so again by hypothesis, g(f(G)) is ≈ g(1,2)*-open in Z. That is (g o f) (G) is ≈ g(1,2)*-open in Z and therefore $(g \circ f)^{-1}$ is ≈g(1,2)*-irresolute. Hence g o f is a ≈*g(1,2)*- homeomorphism.

Theorem 4.2

The set ≈(1,2)*-g*-h(X) is a group under the composition of maps.

Proof

Define a binary operation * : ≈(1,2)*-g*-h(X) × ≈(1,2)*-g*-h(X) → ≈(1,2)*- g*-h(X) by f * g = g o f for all f, g ∈ ≈ (1,2)*-g*-h(X) and o is the usual operation of composition of maps. Then by Proposition 5.4.3, g o f ∈ ≈ (1,2)*-g*-h(X). We know that the composition of maps is associative and the identity map $I : (X, \tau_1, \tau_2) \rightarrow (X, \tau_1, \tau_2)$ belonging to ≈ (1,2)*-g*-h(X) serves as the identity element. If f ∈ ≈ (1,2)*-g*- h(X), then f^{-1} ∈ ≈(1,2)*-g*-h(X) such that $f \circ f^{-1} = f^{-1} \circ f = I$ and so inverse exists foreach element of ≈ (1,2)*-g*-h(X). Therefore, (≈ (1,2)*-g*- h(X), o) is a group under the operation of composition of maps.

Theorem 4.3 Let $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ be an ≈ *g(1,2)*-homeomorphism. Then f induces an (1,2)*-isomorphism from the group ≈ (1,2)*-g*-h(X) on to the group≈ (1,2)*-g*-h(Y).

Proof

Using the map f, we define a map θ_f : ≈ (1,2)*-g*-h(X) → ≈ (1,2)*-g*-h(Y) by $\theta_f(h) = f \circ h \circ f^{-1}$ for every h ∈ ≈(1,2)*-g*-h(X). Then θ_f is a bijection. Further, for all h_1, h_2 ∈ ≈ (1,2)*-g*-h(X), $\theta_f (h_1 \circ h_2) = f \circ (h_1 \circ h_2) \circ f^{-1} = (f \circ h_1 \circ f^{-1}) \circ (f \circ h_2 \circ f^{-1}) = \theta_f(h_1) \circ \theta_f(h_2)$. Therefore, θ_f is a (1,2)*-homomorphism and so it is an (1,2)*-isomorphism induced by f.

V. CONCLUSION

Various interesting problems arise when one considers continuity and generalized continuity. General topology plays vital role in many fields of applied sciences as well as in all branches of mathematics. In reality it is used in data mining, computational topology for geometric design and molecular design, computer-aided design, computer-aided geometric design, digital topology, information systems, particle physics and quantum physics. This paper stated the concept of generalized closed maps in topological spaces. This paper also introduced and studied sg-closed maps and gs-closed maps.

VI. REFERENCES

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