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Mathematical Study of Temperature, Pressure and Density of Generated Cylindrical Shock Waves

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Abstract - The phenomenon of shock waves is commonly associated with aerospace engineering, astronautics and commonly with supersonic flight. Shockwave appear in nature whenever the different elements in a fluid approach one another with a velocity higher than the local speed of sound dissipation of energy, rapid changes in velocity temp. pressure and flow turning are some of the features associated with shockwaves. In the present work, a further study is carried out to assess the influence of several parameters including the converging angle θ_0, the incident planar shock Mach number M_0, and the shock tube height h on the wall profile and the converging shock wave. Cylindrical converging shock waves and thermodynamic properties for different controllable parameters are analysed. The changes of the shock Mach number, pressure, temperature, and density are obtained quantitatively.

Keywords:- Cylindrical, Shock, Converging, self-similar.

INTRODUCTION

Shock focusing is one of the most effective means available to produce high pressure and high temperature at the center of convergence. Much attention has been paid to the shock focusing phenomena due to the importance in study of the Richtmyer-Meshkov (RM) instability and extensive physical applications such as shock-wave lithotripsy, inertial confinement fusion, turbulent mixing in scramjet, collapse in supernova, and others. Specifically, it is of great interest to generate initially smooth converging shock waves and investigate their interaction with different flow configurations. In the pioneer study, Guderley analyzed theoretically the process of converging shockwaves and obtained self-similar solutions of cylindrical and spherical shock waves. In recent years many techniques have been used to tackle propagation of shock waves using similarity method and CCW (chestor-chisnell-whitham) method, the propagation of cylindrical shock waves through a rotating gas has received a considerable attention in the recent post. The similarity method is based on a series expansion in powers of inverse square of mach numbers. The conclusion drawn from these investigations should be reliable for only strong shocks. The propagation of cylindrical weak shocks through a rotating gas is studied by CCW method. Numerical simulations were also launched by Payne in study of the cylindrical converging shocks. From then on, plenty of researches have been performed. The improved version of the shock tube could produce converging shock waves with minimum initial disturbances because of an independently self-supported structure. Some other methods were also proposed to obtain the converging shock waves. Apazidis and Lesser, and Apazidis *et al.* produced polygonal shock waves in an essentially two-dimensional cavity. Knystautas *et al.* performed a diagnostic experiment on the imploding cylindrical detonation waves. In order to avoid the initial shape imperfections and nonlinear wave interactions, a simple but effective technique for generating cylindrical converging shock waves in an ordinary horizontal shock tube was proposed based on the shock dynamics theory In particular, a curved wall profile capable of directly converting an incident planar shock wave into a cylindrical one was designed in the shock tube The detailed process of the design was provided and one case with a fixed set of controllable parameters, including the converging angle, the incident planar shock Mach number, the shock tube height, and the test section lengthwas studied numerically and experimentally Here a theoretical investigation is first carried out to reveal the effects of the geometric parameters and the incident planar shock Mach number on the shapes of the curved wall profile and the converging shock waves. Then numerical and experimental studies on the cylindrical converging shock waves are performed under different conditions. The trajectories of the incident and reflected shocks as well as the variation of the converging shock strength with position are obtained.

THEORETICAL AND NUMERICAL ANALYSIS

The generation of cylindrical converging shock waves is expected to be performed in an ordinary horizontal shock tube by designing a curved wall based on the shock dynamics theory. The shock dynamics is a simple, fast, and useful theoretical method to analyze the propagation and formation of shock waves. Particularly, when a shock moves along a tube with a small area change, the shock front will change due to the disturbances from the shock foot at the solid wall, resulting in the variation of the shock strength. The classical Chester-Chisnell-Whitham (CCW) relation gives the change of the shock Mach number M with the cross-sectional area A, which can be written as

$$\frac{2M dM}{(M^2-1)K(M)} + \frac{dA}{A} = 0,$$

where

$$K(M) = 2(2\mu + 1 + 1/M^2)^{-1}\left(1 + \frac{2}{\gamma+1}\frac{1-\mu^2}{\mu}\right)^{-1},$$

$$\mu = \left(\frac{(\gamma-1)M^2+2}{2\gamma M^2-(\gamma-1)}\right)^{1/2},$$

and γ is the specific heat ratio.

The cylindrical converging shock waves considered here are generated using the CCW relation combined with the characteristic relations. For completeness, a brief description of the method is provided here. The numerical method used here is VAS2D (2-dimensional and axisymmetric vectorized adaptive solver), which has been well-validated in simulating compressible flows such as the shockbubble and shock-body interactions and the condensation-induced waves. In the present numerical algorithm, the two-dimensional Euler equations are adopted as the governing equations which can be written in the following vectorial form:

$$\frac{\partial \mathbf{U}}{\partial t}+\frac{\partial \mathbf{F}}{\partial x}+\frac{\partial \mathbf{G}}{\partial y}=0,$$

where **U**, **F** and **G** represent the conserved variables, the convective fluxes in the x- and y-directions, respectively, i.e.,

$$\mathbf{U}=\begin{pmatrix}\rho\\ \rho u\\ \rho v\\ \rho E\end{pmatrix};\mathbf{F}=\begin{pmatrix}\rho u\\ \rho u^2+p\\ \rho uv\\ (\rho E+p)u\end{pmatrix};\mathbf{G}=\begin{pmatrix}\rho v\\ \rho uv\\ \rho v^2+p\\ (\rho E+p)v\end{pmatrix}.$$

The finite-volume method is used to discretize the conservation laws by applying them directly to each non-overlapping control volume. The MUSCL-Hancock scheme, a second-order upwind scheme, is adopted to compute the flux through the cell interface. The unstructured quadrilateral adaptive mesh is employed to refine local complex areas and could effectively capture the wave patterns.

RESULTS AND DISCUSSION

we mainly focus on the converging angle θ_0, the incident planar shock Mach number M_0, and the shock tube height h. First of all, effects of these controllable parameters on the curved wall profile are studied theoretically. The effects on the radius of the converging part, R_0, and the Mach number at point Q, M_Q, are also analyzed. Second, the change of the movement and strength of shock waves within the converging part are studied systematically. At last, the thermodynamic properties including the pressure, temperature, and density in the process of the shock convergence and reflection are further discussed. Here it is obtained that the curved wall profile changes under different conditions. It is obtained that as θ_0 increases, all the values of X_P, X_Q and Y_Q increase while with the increase of M_o, X_P increase while X_Q and Y_Q both decreases. According to the characteristic relation

we can obtain

$$\theta_Q=\theta_0=\int_{M_0}^{M_Q}\frac{dM}{cA},$$

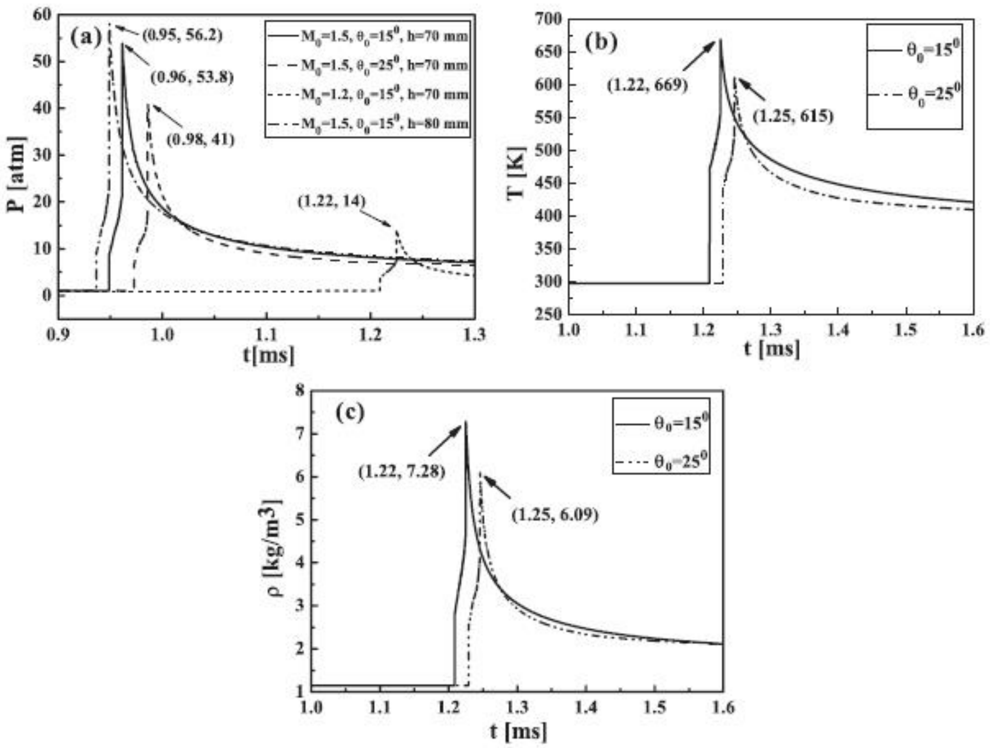
$$cA=\sqrt{(M^2-1)K(M)/2}$$

where and c is the speed of nonlinear disturbance wave on the shock
front. It can be seen that M_Q is dependent on θ_0 and M_0 but independent of h.

The results show that M_Q increases with a nearly linear tendency as a function of θ_0 or M_0, while it keeps constant when h changes. The change tendencies of R_0 with θ_0, M_0, and h are different. The value of R_0 decreases quickly when θ_0 increases. With M_0 increasing, the value of R_0 first increases quickly and then approaches a specific value. It is clear that the size of the shock tube test section is closely related to the incident shock Mach number M_0 and the geometric parameters θ_0 and h.
Note that the theoretical predictions of r-t diagrams are omitted because the flow field ahead of the reflected shock is non uniform and the shock velocities would be difficult to solve theoretically.

When the values of controllable parameters change, the strength of the converging shocks will change. Reveals the theoretical variations of the converging shock Mach number with position in the shock convergence process for different controllable parameters and some quantitative data are illustrated for comparison. It is shown that the converging angle θ_0, the initial shock Mach number, and the shock tube height have great influence on the converging shock Mach number. When θ_0 increases, the length of the converging part decreases quickly and the shock strength becomes weaker at the same position *r*. It is easy to find that M_S will increase to a higher value for a stronger incident shock wave for a fixed *r*. Also shows the change of the converging shock Mach number versus position for different shock tube heights. As mentioned above, for the same θ_0 andM_0 the variation of shock tube heights cannot change the shock Mach number M_Q where the cylindrical. shock begins to focus. However, the larger length of the converging part for a larger *h* still makes the cylindrical shock focusing happen earlier and then results in stronger shocks for the same *r*. In conclusion, a smaller θ_0 and larger values ofM_0 and *h* correspond to a larger length of the converging part and a higher velocity of the converging shock wave. The generation of converging shock waves is capable of concentrating the energy. In this section, the effects of controllable parameters on thermodynamic properties including the pressure, temperature, and density behind the cylindrical converging shock waves are analyzed. As is well known, for a shock of constant strength moving in a constant cross-sectional tube full of gas with constant properties, one-dimensional gas dynamics theory points out that the shock Mach number keeps constant and so do the ratios of the pressure p_2/p_1 and temperature T_2/T_1 between the back and front sides of the shock. While for a shock of variable strength, the case will be different.

Finally, a brief discussion about the variations of the pressure, temperature and density at a fixed position for different controllable parameters are given numerically.



(Variation of pressure, temperature & density of shock with time)

The above figures represent an example of the pressure variations with time at $r = 5$ mm. It can be easily found that the higher pressure peak corresponds to a smaller value of θ_0 and larger values of M_0 and h. Besides, highlight the effects of the converging angle on temperatures and densities behind the shock at the position of $r = 5$ mm and show that the change tendencies of the temperature and density are the same as that of the pressure. It can be concluded that with the shock moving towards the sharp point, the pressures, densities and temperatures behind the shock all increase, and when the shock reflects back the maximum values are acquired. In addition, larger values of the incident shock Mach number and the shock tube height as well as a smaller converging angle will result in higher energy at the centre of convergence.

CONCLUSION

Cylindrical converging shock waves generated in an ordinary horizontal shock tube based on the shock dynamics theory, the influences of controllable parameters including the converging angle θ_0, the incident planar shock Mach number M_0, and the shock tube height h on the curved wall profile and the resulting converging shock waves are performed using theoretical, and numerical methods. The whole process of the shock moving in the converging part is obtained and the characteristics of the cylindrical converging shock waves and the thermodynamic properties such as the pressure, temperature, and density are studied systematically. The variations of the shock Mach number with the shock position in the convergence process show that the shock strength increases quickly when the shock moves towards the sharp point and smaller θ_0 and larger M_0 and h correspond to higher velocities of the shock. Additionally, the characteristics of the thermodynamic properties indicate that higher energy can be reached for a smaller converging angle, a larger incident shock Mach number or a larger shock tube height.

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