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A Theoretical Evaluation of Low Temperature Specific Heat of High Tc Superconductor in a Magnetic Field

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ABSTRACT

Using empirical formulae, we have evaluated low temperature specific heat of high Tc superconductor in a magnetic field. Our theoretically evaluated result shows that specific heat increases with magnetic field and also with temperature which are in good agreement with the experimental data.

Keywords : Tc Superconductor, Magnetic Field, BCS Theory

I. INTRODUCTION

One of the most extensively studied properties of superconductor is the specific heat. It represents bulk measurement of the entire sample. Above the transition temperature T_c the specific heat C_n of high temperature superconductor tend to follow Debye law.[1] We know that C_n of a normal metal far below the Debye temperature Θ_D is the sum of linear term ($Ce= \gamma T$) arising from the conduction electrons, a lattice vibration or phonons term ($C_{ph}=AT^{-3}$) and sometime an additional schottky conribution[2] aT^{-2} such that

$$C_n = aT^{-2} + \gamma T + AT^3$$

In the case of high T_c superconductor one ignore the schottky term aT^{-2}. When (C_{exp}/T) verses T^2 plot was observed then it was found that high T_c superconductor obey at low temperature and deviates from it at higher temperature from Debye approximation. For most low temperature superconductor, the transition temperature T_c is sufficiently below Θ_D so that the electronic term in the specific heat is appreciable in magnitude and sometimes dominants. This is not case for high T_c superconductors. Using measured values of γ and A one has shown[3] that $AT_c^2 > \gamma$ for $(La_{0.9}\ Sr_{0.1})_2CuO_{4-\delta}$ and $YBa_2CuO_{4-\delta}$. Thus for oxide superconductor the vibrational term dominantes at T_c in agreement with the experimental data.[4] If the conduction electron have effective names m* that differ from the free electron mass m, the conduction electron specific heat coefficient γ is given by

$$\gamma = (m^*/m_0)\gamma_0$$

where γ_0 is the ordinary electron counterpart of γ. In free electron approximation we have (1a)

$$\gamma_0 = \tfrac{1}{2}\pi^2 R/T_F$$

where $R = N_A K_B$ is gs constant and N_A is avagedro's number. T_F is Fermi temperature. The effective mass ratio

$$m^* / m_0 = \gamma T_F / \tfrac{1}{2}\pi^2 R$$

Discontinuity at T_c

The transition from the normal to the superconducting state in the absence of an applied magnitude field is second order phase transition. This means that there is no latent heat but there is a discontinuity in the specific heat. The BCS theory predicts that the electronic specific heat jumps abruptly at T_c from the normal state value γ T_c to the superconducting state value C_s with ratio

$$\frac{C_S - \gamma T_C}{\gamma T_C} = 1.43$$

Now in case of high T_c superconductor the magnitude of the jump is small compared to the magnitude of the total specific heat because it is superimposed on much larger AT^3 vibrational term.[5]

Specific heat below T_c

For $T < T_c$, BCS theory predicts that the electronic contribution to the specific heat C_s depend exponentally on temperature

$$C_s = a \ \mathrm{Exp}\left(\frac{-\Delta}{k_\beta T}\right)$$

where 2Δ is the energy gap in the superconducting density of states.

The vibrational term AT^3 becomes negligible as OK is approached and other mechanism becomes important for example, antiferromagnetic ordering and nuclear hyperfine effects, two mechanisms that are utilized in cyrogenic experiments to obtain temperatures down to the microdegree Kelvin.[6]

So far, we have discussed that specific heat of superconductor in the normal and superconducting states in the absence of an applied magnetic field. When the magnetic field is present the situation is much more complicated. In this case for treating the superconducting state, one makes uses of the free energy because (i) the superconductivity state is always the state of lowest free energy at a particular temperature, (ii) the free energies of the normal and superconducting states are equal at the transition temperature. One uses Gibbs free energy G(T,B) and study the difference $[G_s(T,B)-G_n(T,B)]$ between the superconductivity and normal state. Because of the close relationship between superconductivity and magnetism, one adopts the free energy to specific heat procedure and examine the Gibbs free energy of superconductors in the presence of magnetic field. One first obtain an expression for the free energy difference $[G_s(T,B)-G_n(T,B)]$ between the superconducting and normal states. Then, one deduce the expression for entropy and enthalpy.[7-9]

In this paper, we have evaluated that low temperature specific heat of high temperature superconductors in the presence of magnetic field. In section II, we have given the mathematical formulae used in the evaluation. In the last section, we have discussed the obtained result.

II. METHODS AND MATERIAL

Mathematical formulae and in the evaluation of specific heat of a superconductor in a magnetic field

One writes down the expression for Gibbs free energy $G_S(T,B)$ of the superconducting state in the absence of applied magnetic field B.

$$G_s(T,B) = G_n(T) - \frac{1}{2}\mu_0^{-1}[B_c(T)^2 - B^2] \qquad \text{for } B < B_c(T) \qquad (2)$$

Here, $G_n(T)$ is the Gibbs free energy of the normal stage and $\frac{1}{2}\mu_0^{-1} B_c(T)^2$ is the magnetic energy density associated with critical field B_c.

$$G_n(T) = -\frac{1}{2}\gamma T^2 - \frac{1}{12}AT^4 \qquad (3)$$

Now the critical field $B_c(T)$ is given by

$$B_c(T) = B_c(0)\left[1-\left(\frac{T}{T_c}\right)^2\right] \quad (4)$$

Now at B = 0

$$G_s(T,0) = G_n(T)- \frac{1}{2}\mu_0^{-1} B_c(T)^2 \quad (5)$$

Now substituting (4) in (5) we get

$$G_s(T.0) = -\frac{1}{2}\gamma T - \frac{1}{12}AT^4 - \frac{1}{2}\mu_0^{-1}B_c(0)^2\left[1-\left(\frac{T}{T_c}\right)^2\right] \quad (6)$$

Now Gibbs free energy $G_S(T,B)$ of the superconducting state in the presence of an applied magnetic field B.

$$G_s(T,B) = -\frac{1}{2}\gamma T - \frac{1}{12}AT^4 - \frac{1}{2}\mu_0^{-1} \text{ x} \left[B_c(0)^2\left(1-\frac{T}{T_c}\right)^2 - B^2\right] \quad (7)$$

Since the applied field B does not depend on the temperature, the entropy obtained by differentiating the Gibbs free energy (7) assuming the presence of a field is the same as in the case where there is no magnetic field present

$$S_s(T) = \gamma T - \frac{1}{3}AT^3 - 2\mu_0^{-1} B_c(0)^2 \text{ x } \frac{T}{T_c^2}\left[\left(1-\frac{T^2}{T_c^2}\right)\right] \quad (8)$$

The enthalpy obtained does depend explicitly on this field

$$H_s^{'}(T, B) = \frac{1}{2}\gamma T^4 + \frac{1}{4}AT^4 - \frac{1}{2}\mu_0^{-1} B_c(0)^2 \text{ x } \left[\left(1-\frac{T^2}{T_c^2}\right)^2\right] \text{x} \left[1+4\frac{T^2}{T_c^2}\right] + \frac{1}{2}\mu_0^{-1} B^2 \quad (9)$$

and specific heat is given

$$C_s(T) = \gamma T + AT^3 + 2\mu_0^{-1} B_c(0)^2 \text{ x } \frac{T}{T_c^2}\left[3\frac{T^2}{T_c^2} - 1\right] \quad (10)$$

where Eqs. (8) and (10) are the same as their zero-field counterparts, the field dependent G_s and H'_s terms of Eqs. (7) and (9) on the other hand, differ from their zero-field counterparts by the addition of the magnetic energy density $B^2/2\mu_0$.

In a magnetic field the sample goes normal at a lower temperature than in zero field. One denotes this magnetic field transition temperature by $T_C(B) = T'_c$, where, of course, $T_c(0) = T_c$ and $T'_c < T_c$. This transition from the superconducting to the normal state occurs when the applied field H equals the critical field $B_C(T)$ at that temperature. Equation (4) may be rewritten in the form

$$T_c^{'} = T_c\left[1-\frac{B}{B_c(0)}\right]^{1/2} \quad (11)$$

to provide an explicit expression for the transition temperature T'_c in an additional field B. Now one can show that this same expression is obtained by equating the Gibbs free energies $G_S(T,B)$ and $G_n(T)$ for the superconducting and normal sates at the transition point

$$G_s(T,B) = G_n(T) \qquad T = T'_C \quad (12)$$

At the transition temperature $T_C = T_C(B)$ the superconducting and normal state entropies respectively, differ. Their difference gives the latent heat L of the transition by means of the standard thermodynamic expression

$$L = (S_n - S_s)T_c(B) \quad (13)$$

$$= 2\mu_0^{-1}B_c^2\left[\frac{T_c(B)}{T_c}\right]^2 x\left\{1-\left[\frac{T_c(B)}{T_c}\right]^2\right\} \quad (14)$$

One can show this same result can be obtained from the enthalpy difference $L = H_n^{'} - H_s^{'}$. The latent heat is a maximum a the particular transition temperature $T_C(B) = T_C/\sqrt{2}$, as may be shown by setting the derivative of Eq. (14) with respect to temperature equal to zero. One sees from this equation that there is no latent heat when the transition occurs in zero field, i.e., when $T=T_C$, or at absolute zero, T=0. In addition to the latent heat, there is also a jump in the specific heat a $T_c(B)$.

Normalized Thermodynamic Equations

The equations for $G_S(T,B)$, $S_S(T)$ and $C_S(T)$ given in the previous section, together with $H'_S(T,B)$ can be written in normalized form by defining two dimensionless independent variables,

$$t = \frac{T}{T_c} \qquad b = \frac{B}{B_c} \quad (15)$$

and two dimensionless parameter

$$a = \frac{AT_c^2}{\gamma} \qquad \alpha = \frac{B_c^2}{\mu_0\gamma T_c^2} \quad (16)$$

These expressions are valid under the conditions

$$t^2 + b < 1 \quad (17)$$

The sample becomes normal when either t or b are increased to the point where $t^2 + b = 1$, and the value of t that satisfies this expression is called t':

$$t'^2 + b = 1 \quad (18)$$

This is the normalized of Eq. (11) where $t' = T'_c/T_c$ is the normalized transition temperature in a magnetic field.

The normalized specific heat jump has the following special values :

$$\frac{\Delta C}{\gamma T_c} = 2\alpha\, t'(3t'^2 - 1)$$

$$= \begin{cases} 0 & t' = 0 \\ -\frac{4\alpha}{9} & t' = \frac{1}{3} \\ 0 & t' = \frac{1}{\sqrt{3}} \quad (\max) \\ 4\alpha & t' = 1 \end{cases} \quad (19)$$

where $4\alpha/9$ is its maximum magnitude of $\Delta C/\gamma T_c$ for reduced temperatures in the range $0 < t' < -1/\sqrt{3}$. The normalized latent heat has special values.

$$\frac{L}{\gamma T_c^2} = 2\alpha t'^2 \left(1 - t'^2\right)$$

$$= \begin{cases} 0 & t' = 0 \\ \frac{1}{2}\alpha & t' = \frac{1}{\sqrt{2}} \text{ (max)} \\ 0 & t' = 1 \end{cases} \qquad (20)$$

where its maximum ½ α is at $t' = 1/\sqrt{2}$

Normalized Equations for the thermodynamic Functions of a Superconductor in an Applied Magnetic Field B

Gibbs Free Energy $\quad g_s = \frac{G_s}{\gamma T_c^2} = -\frac{1}{2}t^2 - \frac{1}{12}at^4 - \frac{1}{2}\alpha\left[(1-t^2)^2 - b^2\right]$

Entropy $\quad s_s = \frac{S_s}{\gamma T_c} = t + \frac{1}{3}at^2 - 2\alpha\, t\left(1 - t^2\right)$

Specific Heat $\quad C_s = \frac{C_s}{\gamma T_c} = t + at^2 - 2\alpha\, t\left(3t^2 - 1\right)$

Enthalpy $\quad h_s' = \frac{H_s'}{\gamma T_c^2} = \frac{1}{2}t^2 + \frac{1}{4}at^2 - \frac{1}{2}\alpha\left[\left(1 - t^2\right)\left(1 + 3t^3 - b^2\right)\right]$

Specific Heat Jump $\quad \frac{\Delta C}{\gamma T_c} = 2\alpha t'^2\left(1 - t'^2 - 1\right)$

Latent Heat $\quad \frac{L}{\gamma T_c^2} = 2\alpha t'^2\left(1 - t'^2\right)$

Definitions of normalized variables (t,b) and parameters :

$t = \frac{T}{T_c} \qquad b = \frac{B}{B_c(0)} \qquad a = \frac{AT_c^2}{\gamma}$

$t' = \frac{T'}{T_c'} \qquad b = \frac{B_c(T')}{B_c(0)} \qquad \alpha = \frac{\left[B_c(0)\right]^2}{\mu_0 \gamma T_c^2}$

The first four expression are valid under the condition $t^2 + b < 1$ and the last two are valid at the transition point given by $t'^2 + b^2 = 1$.

Now the specific heat of high T_c superconductor in the presence of magnetic field can be studied with the use of the empirical formulae

$$\frac{C}{T} = [\gamma + \gamma'(B)] + [A - A'(B)]T^2 \qquad (21)$$

III. Discussion of Results

In this paper, we have evaluated the low temperature specific heat of high temperature superconductor in a magnetic field using an empirical formulae given in Equation (21). Theoretically evaluated results are shown in table T_2 along with the experimental result.[10-15] We have taken the values of A= 4.38 mJ/mol K^2 and

A=0.478/mol K^4 with the coefficient γ'(B) and A'(B) increasing as the applied magnetic field was increased. For magnetic field H = 3T we have put the value of

γ/γ' = 0.54, and

A/A' = 0.11

These value give close match with the experimental data. The other values like Debye temperature Θ_D and density of state $D(E_F)$ for various high T_c superconductor are given in table T_1. Our theoretical evaluated results indicates that specific heat of high temperature superconductor increases with magnetic field and also with temperature.

Table T_1

Debye Temperature Θ_D, Density of states $D(E_F)$

	Superconductors	T_C(K)	Θ_D(K)	$D(E_F)$
1.	$(La_{0.925}Sr_{0.075})_2CuO_4$	36	360	1.9
2.	$YBa_2Cu_3O_7$	92	410	2.0
3.	$YBa_2Cu_4O_{8.5}$	80	350	2.1
4.	$Bi_2Sr_2Ca_2Cu_3O_{10}$	110	260	2.5
5.	$Tl_2Ba_2Ca_2Cu_3O_{10}$	125	260	2.7
6.	$HgBa_2Ca_2Cu_3O_8$	133	280	2.2

Table T_2

Evaluated result of Low temperature specific heat of $YBa_2Cu_3O_{7-\delta}$ in a magnetic field

T(K²)	(C/T) (mJ/k² mol)			
	(H = 3T)		(H = 5T)	
	Theo.	Expt.	Theo.	Expt.
5	7.5	6.6	8.6	9.7
6	8.6	7.8	9.5	10.2
7	9.2	8.7	11.2	12.7
8	11.8	10.2	12.7	13.8
9	13.5	12.5	14.9	15.4
10	15.6	14.5	16.3	17.6
15	17.9	16.2	18.7	19.8
20	20.6	18.6	21.5	20.2
25	22.4	19.8	23.8	22.0
30	23.6	20.5	24.8	23.1
35	25.5	23.2	26.7	25.2

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