

# Separation Axioms in Bitopological Spaces



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In the present section we discuss separation axioms for bitopological spaces and study their behaviour under continuity and compactness.

First of all we give definitions of different types of pairwise  $(T_0)$ - axioms.

**Definition (2.1) :** A bitopological space  $(X, T_1, T_2)$  is said to be pairwise  $(T_0)$  – space if for every pair of distinct points of  $X$ , there exists either a  $T_1$  – open set or a  $T_2$  – open set containing one of them but not the other.

**Definition (2.2) :** A bitopological space  $(X, T_1, T_2)$  is said to be strictly pairwise  $(T_0)$  – space [stp-  $(T_0)$ ] if neither  $(X, T_1)$  nor  $(X, T_2)$  is  $T_0$  – space but for every pair of distinct points of  $X$ , there exists either a  $T_1$ - open set or a  $T_2$  – open set containing one of them but not the other.

**Definition (2.3) :** A bitopological space  $(X, T_1, T_2)$  is said to be quasi pairwise  $(T_0)$ - space if for every pair of distinct point of  $X$ , there exists a quasi open set which contains one of them but not the other.

**Definition (2.4) :** A bitopological space  $(X, T_1, T_2)$  is said to be  $*$  - pairwise  $(T_0)$  – space if for every pair of distinct point of  $X$ , there exists either a  $T_1$ - pre open set or a  $T_2$  – pre open set which contains one of them but not the other.

**NOTE (2.1) :** From the definitions it is clear that bitopological space  $(X, T_1, T_2)$  is pairwise  $(T_0)$  space if it is either Stp  $(T_0)$  – space or quasi pairwise  $(T_0)$  space or  $*$  - pairwise  $(T_0)$  space.

**NOTE (2.2) :** From examples it can be seen that definitions (2.2), (2.3) and (2.4) are independent concepts.

The concepts of different types of pairwise  $(T_1)$  spaces are given below:

**Definition (2.5) :** A bitopological space  $(X, T_1, T_2)$  is said to be pairwise  $(T_1)$  – space ( $p$  –  $(T_1)$  – space) if for every pair of distinct point  $x$  and  $y$  of  $X$ , there exist  $G \in T_1$  and  $H \in T_2$  such that

$$x \in G, y \notin G \text{ and } y \in H, x \notin H, \dots\dots\dots (2.1)$$

**Definition (2.6) :** A bitopological space  $(X, T_1, T_2)$  is said to be strictly pairwise  $(T_1)$  space (stp –  $(T_1)$  – space) if neither  $(X, T_1)$  nor  $(X, T_2)$  is  $(T_1)$  – space but the condition of definition (2.5) are satisfied.

**Definition (2.7) :** A bitopological space  $(X, T_1, T_2)$  is said to be weak pairwise  $(T_1)$  – space (wp –  $(T_1)$  – space) if for every pair of distinct points  $x$  and  $y$  of  $X$  there exist sets  $G$  and  $H$ , open either in  $T_1$  or in  $T_2$  satisfying condition (2.1).

**Definition (2.8) :** A bitopological space  $(X, T_1, T_2)$  is said to be weak\* pairwise  $(T_1)$  ( $w^* p - (T_1) - \text{space}$ ) if each singlet in  $X$  is either  $T_1 - \text{closed}$  or  $T_2 - \text{closed}$ .

**Definition (2.9) :** A bitopological space  $(X, T_1, T_1)$  is said to be quasi pairwise  $(T_1)$  space if for each pair of distinct points  $x$  and  $y$  of  $X$  there exist quasi open sets  $G$  and  $H$  satisfying condition (2.1).

**Definition (2.10) :** A bitopological space  $(X, T_1, T_2)$  is said to be quasi\* pairwise  $(T_1)$  if each singlet of  $X$  is quasi - closed.

**Definition (2.11) :** A bitopological space  $(X, P_1, P_2)$  is said to be  $ij - \text{almost } T_1$  if for any two distinct points  $x$  and  $y$  of  $X$  there exists an  $ij - \text{roset } U$  such that  $y \in U$  and  $x \notin U$ .

Now we mention an important result which appears in [Nandi, 1994].

**THEOREM (2.1) :** If any function  $f$  from  $(X, Q_1, Q_2)$  onto  $(Y, P_1, P_2)$  has  $ij \delta_2 - \text{closed graph}$  then  $(Y, P_1, P_2)$  is  $ij - \text{almost } T_1$ .

Nandi (1994) proves the following characterization of  $ij - \text{nearly compactness}$  for an  $ij - \text{almost } T_1 - \text{space}$ .

**THEOREM (2.2) :** An  $ij - \text{almost } T_1 - \text{space } (Y, P_1, P_2)$  is  $ij - \text{nearly compact}$  if each function  $f$  from any space  $(X, Q_1, Q_2)$  into  $(Y, P_1, P_2)$  with  $ij - \delta_2 - \text{closed graph}$  is  $ij - \text{almost continuous}$ .

**THEOREM (2.3) :** An  $ij - \text{almost } T_1 - \text{space } (Y, P_1, P_2)$  is  $ij - \text{nearly compact}$  if for any space  $(X, Q_1, Q_2)$  and any functions  $f, g$  from  $X$  into  $Y$  with  $if - \delta_2 - \text{closed graph}$ , the set

$$A = \{x \in X : f(x) = g(x)\} \dots\dots\dots )2.2$$

is  $Q_1 - \text{closed subset of } X$ .

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