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Pairwise Normal in Bitopological Spaces

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Now we discuss different types of pairwise normal spaces.

Definition (1) : A bitopological spaces (X, P, Q) is said to be pairwise normal if for every pair of P-closed set A and Q- closed set B with $A \cap B$, there exist as pair of disjoint sets G and H such that G is P-open, H is Q-open, and $A \supseteq G$ and $B \supseteq H$.

We prove the above definition in term of 12- pre open (pre closed) 21- pre open (pre closed) sets.

Definition (2) : A bitopological space (X, T_1, T_2) is said to be pairwise *- normal if for each pair of 12- pre closed set A and 21- pre closed set B with $A \cap B - \phi$, there exist a pair of disjoint sets G and H such that G is 12- pre open, H is 21- pre open, $A \subseteq G$ and $B \subseteq H$.

Now we obtain the following characteristic properties for pairwise * - normal spaces.

Theorem : Let (X, T_1, T_2) be a bitopological space. Then the following conditions are equivalents.

(i). (X, T_1, T_2) is pairwise *- normal.
(ii). For every 12- pre closed set A and every 21-pre open set V containing A, there exist 12- pre open set V and 21- pre closed Set L Such that $A \subset W \subset L \subset V$.
(iii). For every 21- pre closed set B and every 12- pre open set H containing B, there exist 21 – pre open set $\bigcup$ and 12- pre closed set N such that $B \subset U \subset N \subset M$.
Proof. (i) → (ii)

Suppose that (X, T_1, T_2) is pairwise *- normal bitopological space. Let A be a 12- pre closed set in X and Let V be a 21- pre open set containing A. Then V^{C} is 21- pre closed and $A \cap V^{C} = \phi$. So according to our assumption there exist 12- pre open set V and 21- pre open set $\bigcup$ Such that

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$$A \subset W,\ V^{C} \subset U \quad \text{and} \quad V \cap U = \phi .$$

This means that $A \subset W \subset U^{C} \subset V$. If we put $U^{C} = L$ then we see that L is 21- pre closed and satisfy

$A \subset W \subset L \subset V$.

(ii) $\Rightarrow$ (iii)

Suppose that (X, T_1, T_2) is a bitopological space satisfying condition (ii). Let B be a 21- pre closed set and Let N be 12- pre open set containing B, i.e., $B \subset N$. Then $M^{C} \subset B^{C}$ where N^{C} is 12- pre closed and B^{C} is 21- pre open. So according to our assumption there exist 12- pre open set W and 21- pre closed set L such that

$M^C \subset W \subset L \subset B^C$.

This gives $B \subset L^C \subset W^C \subset M$.

If we put $L^C = U$ and $W^C = N$ then

$B \subset U \subset N \subset N\text{-}M$

where U is 21- pre open and N is 12- pre closed.

(iii) $\Rightarrow$ (i)

Suppose that (X, T_1, T_2) is a bitopological space satisfying the condition (iii). Let A and B be two disjoint subsets of X such that A is 12- pre closed and B is 21- pre closed. Then A^C is 12- pre open and $B \subset A^C$. So according to our assumption there exist 21- pre open set U and 12- pre closed set N such that –

$B \subset U \subset N \subset A^C$.

This gives $B \subset U$ and $A \subset N^C$. Since N^C is 12- pre open and U is 21- pre open with $U \cap N^C = \phi$, we see that (X, T_1, T_2) is pairwise *- normal.

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