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A Three-parameter New Exponentiated Distribution for Life-time Data

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ABSTRACT

In the presented work, a continuous distribution consisting of three-parameters is proposed for life-time data called new exponentiated distribution. The discussion of some of the distribution's statistical as well as mathematical properties, including the Cumulative Distribution Function (CDF), Probability Density function (PDF), quantile function, survival function, hazard rate function, kurtosis measures and skewness, is conducted. The estimation of the presented distribution's model parameters is performed using the techniques of Cramer-Von-Mises estimation (CVME), least-square estimation (LSE), and maximum likelihood estimation (MLE). The evaluation of the proposed distribution's goodness of fit is performed through its fitting in comparison with some of the other existing life-time models with the help of a real data set.

Keywords : CVME, MLE, Model Parameter Estimation, New exponentiated distribution, Reliability function

I. INTRODUCTION

Lifetime distributions are generally used to model lifetime data of components of a system, a device, and in general, reliability and survival analysis. Often we see the use of lifetime distributions in fields like biological science, information technology, engineering, insurance, etc. For lifetime analysis, most commonly applicable lifetime distributions are exponential distribution, Weibull distribution, and Bayesian-Weibull Analysis, Cauchy and Lognormal distribution.

For a few decades, it is found that the exponential distribution is taken as base distribution to generate a new family of distribution. The modifications of the exponential distribution were introduced by different researchers, some of them are, beta exponential (Nadarajah and Kotz, 2006), Gupta and Kundu (2007) have presented the generalized exponential (GE) with some development, Abouammoh & Alshingiti (2009) has introduced the generalized inverted exponential distribution's reliability estimation, Exponential Extension (EE) distribution (Kumar, 2010), beta GE (Barreto-Souza et al., 2010), gamma EE by (Ristic and Balakrishnan, 2012), Gomez et al. (2014) have presented exponential distribution with a new extension, Kumaraswamy transmuted exponential (Afify et al., 2016) distributions and exponentiated exponential geometric (Louzada et al., 2014). Mahdavi

& Kundu (2017) have presented a new method where by an application to the exponential distribution a new distribution can be generated. Recently, the Alpha power transformed extended exponential distribution have introduced by (Hassan et al., 2018). Almarashi et al. (2019) have presented exponential distribution with a new extension along with its statistical properties. Some life-time models that are generated from exponential family of distribution and able to exhibit more flexibility are listed below

I. A life-time distribution with three-parameter with a decreasing, increasing, upside down bathtub-shaped and bathtub failure rate was presented by Dimitrakopoulou et al. (2007) This distribution's hazard function is expressed as

$$h(x)=\alpha\beta\lambda x^{\beta-1}\left(1+\lambda x^{\beta}\right)^{\alpha-1};x>0,(\alpha\beta\lambda)>0$$

As when α = 1, we can see the distribution reduces to Weibull, so the distribution is the special case of Weibull distribution.

II. Joshi (2015) has proposed another extension of exponential distribution called new extended exponential (EEN) distribution having monotonically increasing and constant hazard rate shapes. With parameters α and λ the continuous random variable X trails EEN distribution and CDF is given by

$$h(x)=\alpha\left(1+\frac{\lambda}{x}\right)e^{-\lambda/x}\ ;x>0,(\alpha,\lambda)>0$$

III. Chaudhary et al., (2020) has presented a flexible life-time model named truncated Cauchy power–exponential (TCP-E) distribution which has decreasing, increasing, constant and upside-down bathtub shaped hazard rate and the hazard function for TCP-E distribution is

$$h(x;\alpha,\beta)=\frac{4\alpha\beta}{\pi}\frac{e^{-\beta x}\left(1-e^{-\beta x}\right)^{\alpha}}{\left[1+\left(1-e^{-\beta x}\right)^{2\alpha}\right]\left[1-\frac{4}{\pi}\arctan\left[\left(1-e^{-\beta x}\right)^{\alpha}\right]\right]}$$

$;x\geq 0,\alpha>0,\beta>0$

IV. A exponential distribution's generalization was presented by Nadarajah & Haghighi (2011) and named it as an extension of exponential distribution. Its PDF exhibits unimodal and decreasing shapes, and the hazard rate shows increasing, constant and decreasing shapes.

$$h(x)=\alpha\lambda\left(1+\lambda x\right)^{\alpha-1}\ ;x>0,(\alpha,\lambda)>0$$

V. Lemonte (2013) introduced an exponential type family of distribution with three-parameter. It exhibits decreasing, increasing, constant and upside-down bathtub shaped hazard rate and the hazard function is

$$h(x)=\alpha\beta\lambda\frac{(\lambda x+1)^{\alpha-1}\exp\left[1-(\lambda x+1)^{\alpha}\right]\left\{1-\exp\left[1-(\lambda x+1)^{\alpha}\right]\right\}^{\beta-1}}{1-\left\{1-\exp\left[1-(\lambda x+1)^{\alpha}\right]\right\}^{\beta}}$$

$;x>0,(\beta,\lambda,\alpha)>0$

Introduction of a distribution with more flexibility in order to achieve better fit for lifetime data is the key aim for this paper. The different sections of the proposed study are organized as follows. In Section 2(**Three Parameter New Exponentiated Distribution**} we define a new exponentiated distribution (NED) and discuss some mathematical and statistical properties. For estimating the proposed distribution's parameters we apply the use of Maximum Likelihood Estimation (MLE), Cramer-Von-Mises estimation (CVME) methods and least-square estimation (LSE). With the help of information matrix that was observed, asymptotic confidence intervals was built for MLEs in section 3(**Methods of Parameter estimation)**. In Section 4(**Real Data Application)**, a real data set has been evaluated to explore the applications and capability of the proposed distribution. With a real data set, analysis of proposed distribution's goodness of fit is carried out by fitting it in contrast with some other existing distributions. Conclusion remarks are presented in Section 5(**CONCLUSIONS)**.

II. A THREE-PARAMETER NEW EXPONENTIATED DISTRIBUTION

Here in this section, we have defined a three-parameter new exponentiated distribution (NED). The NED is generated using exponentiated approach by taking CDF as parent distribution defined by (Dimitrakopoulou et al., 2007). A continuous nonnegative random variable X~NED(α, β, θ) if its cumulative function is in the form of

$$F(x)=\left[1-\left(1+\alpha x^{\beta}\right)e^{-\alpha x^{\beta}}\right]^{\theta} \quad ;0<x<\infty,(\alpha\beta\theta)>0 \tag{2.1}$$

The PDF is expressed as follows

$$f(x)=\alpha^{2}\beta\theta x^{2\beta-1}e^{-\alpha x^{\beta}}\left[1-\left(1+\alpha x^{\beta}\right)e^{-\alpha x^{\beta}}\right]^{\theta-1} \quad ;(\alpha\beta\theta)>0 \tag{2.2}$$

The reliability function of NED is expressed as follows,

$$R(x)=1-\left[1-\left(1+\alpha x^{\beta}\right)e^{-\alpha x^{\beta}}\right]^{\theta};(\alpha\beta\theta)>0 \tag{2.3}$$

Similarly, the hazard rate function (HRF) is

$$h(x)=\frac{\alpha^{2}\beta\theta x^{2\beta-1}e^{-\alpha x^{\beta}}\left[1-\left(1+\alpha x^{\beta}\right)e^{-\alpha x^{\beta}}\right]^{\theta-1}}{1-\left[1-\left(1+\alpha x^{\beta}\right)e^{-\alpha x^{\beta}}\right]^{\theta}} \tag{2.4}$$

2.1 Quantile and Generating Functions

Quantile Function

In the statistical literature, the quantile function related with a probability distribution of a random variable, is the distribution function's inverse and it provides a broad description of the statistical properties pertaining to the random variable. The uth quantile value can be acquired by solving the following equation

$$Q(u)=F^{-1}(u)$$

the quantile function of NED is obtained by inverting CDF (2.1) as

$$\ln\left(1+\alpha x^{\beta}\right)-\alpha x^{\beta}-\ln\left(1-u^{1/\theta}\right)=0 \quad ;0<u<1 \tag{2.1.1}$$

Generation of the random numbers:

For the random numbers generation of the NED, we simulate values of random variable X with the CDF (2.1.1). Let J denote a uniform random variable in (0, 1), then the simulated values of X can be calculated by

$$\ln\left(1+\alpha x^{\beta}\right)-\alpha x^{\beta}-\ln\left(1-j^{1/\theta}\right)=0 \quad ;0<j<1 \tag{2.1.2}$$

2.2 Skewness and Kurtosis

The Skewness measures on quantiles is Bowley's coefficient of skewness and it can be written as

$$S_{k}\left(Bowley\right)=\frac{Q(3/4)+Q(1/4)-2Q(1/2)}{Q(3/4)-Q(1/4)}, \text{ and} \tag{2.2.1}$$

The coefficient of kurtosis based on octiles was described by (Moors, 1988) which can be expressed as

$$K_{u}(M)=\frac{Q(0.875)-Q(0.625)+Q(0.375)-Q(0.125)}{Q(3/4)-Q(1/4)}, \tag{2.2.3}$$

Plots of PDF and hazard rate function of NED (α, β, θ) with various parameter value are shown in Figure 1.

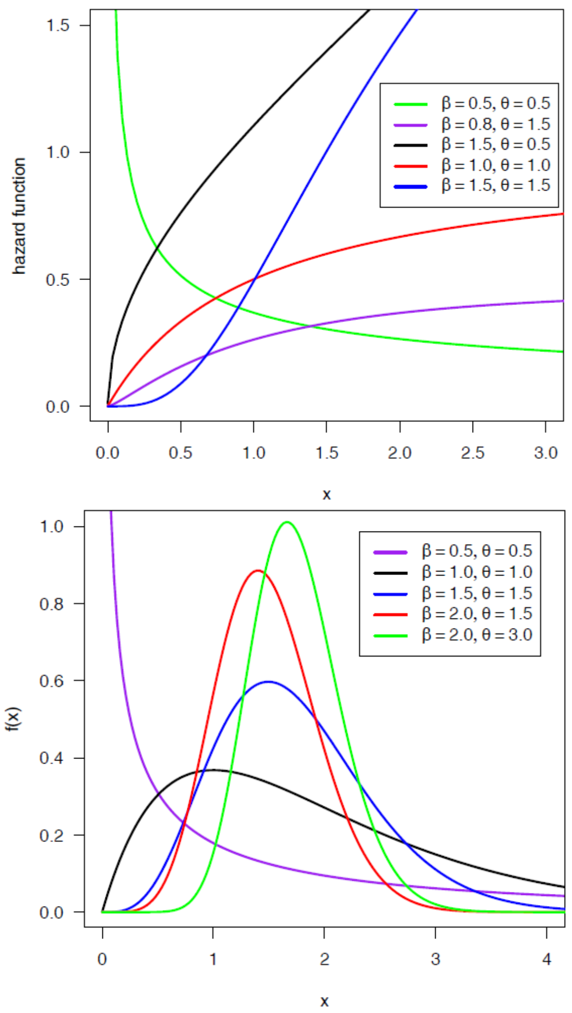


Figure 1. For different values of α, β and θ, graphs of hazard function (upper panel) and PDF (lower panel)

III.METHOD OF PARAMETER ESTIMATION

In this section, we have employing these estimation methods,

i. Cramer-Von-Mises
ii. Least square
iii. Maximum likelihood

3.1. Maximum Likelihood Estimation (MLE) method

Consider $\underline{x} = (x_1,\ldots,x_n)$ denote random sample with size 'n' in NED(α, β, θ) then the log likelihood function l(α, β, θ / x) can be expressed as,

$$l(\alpha,\beta,\theta/\underline{x}) = n\ln\left(\alpha^2\beta\theta\right) + (2\beta-1)\sum_{i=1}^{n}x_i - \alpha\sum_{i=1}^{n}x_i^{\beta} + (\theta-1)\sum_{i=1}^{n}\ln\left[1-\left(1+\alpha x_i^{\beta}\right)e^{-\alpha x_i^{\beta}}\right] \tag{3.1.1}$$

By differentiating (3.1.1) with respect to unknown parameters α, β and θ, we obtain,

$$\frac{\partial l}{\partial\alpha} = \frac{2n}{\alpha} - \sum_{i=1}^{n}x_i^{\beta} + (\theta-1)\sum_{i=1}^{n}\frac{\alpha x_i^{2\beta}e^{-\alpha x_i^{\beta}}}{\left[1-\left(1+\alpha x_i^{\beta}\right)e^{-\alpha x_i^{\beta}}\right]}$$

$$\frac{\partial l}{\partial\beta} = \frac{n}{\beta} + 2\sum_{i=1}^{n}x_i + (\theta-1)\sum_{i=1}^{n}\frac{\alpha^2 x_i^{2\beta}\ln(x)e^{-\alpha x_i^{\beta}}}{\left[1-\left(1+\alpha x_i^{\beta}\right)e^{-\alpha x_i^{\beta}}\right]}$$

$$\frac{\partial l}{\partial\theta} = \frac{n}{\theta} + \sum_{i=1}^{n}\ln\left[1-\left(1+\alpha x_i^{\beta}\right)e^{-\alpha x_i^{\beta}}\right]$$

After solving for the unknown parameters (α, β, θ) and equating these non-linear equations to zero we will obtain the ML estimators of the NED. Manually, it is difficult to solve hence by add of appropriate computer software one can solve these equations. Consider the parameter vector by $\underline{\Delta} = (\alpha,\beta,\theta)$ and the corresponding MLE of $\underline{\Delta}$ as $\hat{\underline{\Delta}} = (\hat{\alpha},\hat{\beta},\hat{\theta})$, then the asymptotic normality gives $\left(\hat{\underline{\Delta}}-\underline{\Delta}\right) \to N_3\left[0,\left(D(\underline{\Delta})\right)^{-1}\right]$ here $D(\underline{\Delta})$ is the information matrix of Fisher which is expressed as,

$$D(\underline{\Delta}) = -\begin{pmatrix} E\left(\frac{\partial^2 l}{\partial\alpha^2}\right) & E\left(\frac{\partial^2 l}{\partial\alpha\partial\beta}\right) & E\left(\frac{\partial^2 l}{\partial\alpha\partial\theta}\right) \\ E\left(\frac{\partial^2 l}{\partial\beta\partial\alpha}\right) & E\left(\frac{\partial^2 l}{\partial\beta^2}\right) & E\left(\frac{\partial^2 l}{\partial\beta\partial\theta}\right) \\ E\left(\frac{\partial^2 l}{\partial\alpha\partial\theta}\right) & E\left(\frac{\partial^2 l}{\partial\beta\partial\theta}\right) & E\left(\frac{\partial^2 l}{\partial\theta^2}\right) \end{pmatrix}$$

In practice, we don't know $\underline{\Delta}$ hence MLE having asymptotic variance $\left(D(\underline{\Delta})\right)^{-1}$ is useless. So by filling in the estimated value of the parameters we approximate the asymptotic variance. The observed fisher information matrix $O(\hat{\underline{\Delta}})$ is used as an information matrix $D(\underline{\Delta})$'s estimate which is expressed as

$$O(\hat{\underline{\Delta}}) = -\begin{pmatrix} \frac{\partial^2 l}{\partial\alpha^2} & \frac{\partial^2 l}{\partial\alpha\partial\beta} & \frac{\partial^2 l}{\partial\alpha\partial\theta} \\ \frac{\partial^2 l}{\partial\beta\partial\alpha} & \frac{\partial^2 l}{\partial\beta^2} & \frac{\partial^2 l}{\partial\beta\partial\theta} \\ \frac{\partial^2 l}{\partial\alpha\partial\theta} & \frac{\partial^2 l}{\partial\beta\partial\theta} & \frac{\partial^2 l}{\partial\theta^2} \end{pmatrix}_{|(\hat{\alpha},\hat{\beta},\hat{\theta})} = -H\left(\underline{\Delta}\right)_{|(\underline{\Delta}=\hat{\underline{\Delta}})}$$

where Hessian matrix is denoted by H

To produce the observed information matrix, we bring the use of the Newton-Raphson algorithm for likelihood's maximization. Thus, the variance-covariance matrix is expressed as,

$$\left[-H\left(\underline{\Delta}\right)_{|(\underline{\Delta}=\hat{\underline{\Delta}})}\right]^{-1} = \begin{pmatrix} \mathrm{var}(\hat{\alpha}) & \mathrm{cov}(\hat{\alpha},\hat{\beta}) & \mathrm{cov}(\hat{\alpha},\hat{\theta}) \\ \mathrm{cov}(\hat{\alpha},\hat{\beta}) & \mathrm{var}(\hat{\beta}) & \mathrm{cov}(\hat{\theta},\hat{\beta}) \\ \mathrm{cov}(\hat{\alpha},\hat{\theta}) & \mathrm{cov}(\hat{\theta},\hat{\beta}) & \mathrm{var}(\hat{\theta}) \end{pmatrix}$$

(3.1.2)

Thus for α, β and θ's approximate 100(1-α) % confidence intervals via MLEs' asymptotic normality can be constructed as,

$\hat{\beta} \pm Z_{\alpha/2}\sqrt{\mathrm{var}(\hat{\beta})}$, $\hat{\alpha} \pm Z_{\alpha/2}\sqrt{\mathrm{var}(\hat{\alpha})}$ and $\hat{\theta} \pm Z_{\alpha/2}\sqrt{\mathrm{var}(\hat{\theta})}$,

Here $Z_{\alpha/2}$ is the upper percentile of standard normal variate.

3.2. Method of Least-Square Estimation (LSE)

For the estimation of Beta distribution's parameters, we can use weighted least square estimators and ordinary least square estimators as given by Swain et al. (1988). In this section NED parameter estimation can be done by the same method. The unknown parameters α, β and θ of NED's least-square estimators can be obtained with minimization of

$$B(X;\alpha,\beta,\theta)=\sum_{i=1}^{n}\left[F(X_i)-\frac{i}{n+1}\right]^2 \qquad (3.2.1)$$

with respect to unknown parameters α, β and θ.

From a distribution function F(.), let $F(X_i)$ represent the distribution function of the ordered random variables $X_{(1)} < X_{(2)} < \ldots < X_{(n)}$ and $\{X_1, X_2, \ldots, X_n\}$ denotes random sample of size n. The unknown parameters' (α, β and θ) least-square estimators represented as $\hat{\alpha}, \hat{\beta}, and\ \hat{\theta}$, can be acquired with minimization of,

$$B(X;\alpha,\beta,\theta)=\sum_{i=1}^{n}\left[\left[1-\left(1+\alpha x_i^{\beta}\right)e^{-\alpha x_i^{\beta}}\right]^{\theta}-\frac{i}{n+1}\right]^2$$

$$;x>0,(\alpha\beta\theta)>0. \qquad (3.2.2)$$

with respect to α, β and θ.

By differentiation (3.2.2) with respect to α, β and θ the following can be obtained,

$$\frac{\partial B}{\partial \alpha}=2\theta\alpha\sum_{i=1}^{n}\left[\{A(x_i)\}^{\theta}-\frac{i}{n+1}\right]x_i^{2\beta}e^{-\alpha x_i^{\beta}}\{A(x_i)\}^{\theta-1}$$

$$\frac{\partial B}{\partial \beta}=2\alpha^2\theta\sum_{i=1}^{n}\{A(x_i)\}^{\theta-1}x_i^{2\beta}e^{-\alpha x_i^{\beta}}\ln x_i\left[\{A(x_i)\}^{\theta}-\frac{i}{n+1}\right]$$

$$\frac{\partial B}{\partial \theta}=2\theta\sum_{i=1}^{n}x_i^{\beta}\left[\{A(x_i)\}^{\theta}-\frac{i}{n+1}\right]\{A(x_i)\}^{\theta}\ln\left[A(x_i)\right]$$

Where $A(x_i)=1-\left(1+\alpha x_i^{\beta}\right)e^{-\alpha x_i^{\beta}}$

Likewise we can get the weighted least square estimators with minimization of,

$$B(X;\alpha,\beta,\theta)=\sum_{i=1}^{n}w_i\left[F(X_{(i)})-\frac{i}{n+1}\right]$$

with respect to α, β and θ. The weights w_i are

$$w_i=\frac{(n+2)(n+1)^2}{i(n-i+1)}=\frac{1}{Var(X_{(i)})}$$

Similarly we can get weighted least square estimators of α, β and θ correspondingly with minimization of,

$$B(X;\theta,\beta,\alpha)=\sum_{i=1}^{n}\frac{(n+2)(n+1)^2}{i(n+1-i)}\left[\left[1-\left(\alpha x_i^{\beta}+1\right)e^{-\alpha x_i^{\beta}}\right]^{\theta}-\frac{i}{n+1}\right]^2 \qquad (3.2.3)$$

with respect to α, β and θ.

3.3. Cramer-Von-Mises estimation (CVME) Method

We can get Cramer-Von-Mises estimators of α, β and θ with minimization of function

$$C(X;\alpha,\beta,\theta)=\frac{1}{12n}+\sum_{i=1}^{n}\left[F\left(x_{i:n}\mid\alpha,\beta,\theta\right)-\frac{2i-1}{2n}\right]^2$$

$$=\frac{1}{12n}+\sum_{i=1}^{n}\left[\left[1-\left(1+\alpha x_i^{\beta}\right)e^{-\alpha x_i^{\beta}}\right]^{\theta}-\frac{2i-1}{2n}\right]^2 \qquad (3.3.1)$$

By differentiation of (3.3.1) with respect to α, β and θ following can be obtained,

$$\frac{\partial C}{\partial \alpha}=2\theta\alpha\sum_{i=1}^{n}\left[\{A(x_i)\}^{\theta}-\frac{2i-1}{2n}\right]x_i^{2\beta}e^{-\alpha x_i^{\beta}}\{A(x_i)\}^{\theta-1}$$

$$\frac{\partial C}{\partial \beta}=2\alpha^2\theta\sum_{i=1}^{n}\{A(x_i)\}^{\theta-1}x_i^{2\beta}e^{-\alpha x_i^{\beta}}\ln x_i\left[\{A(x_i)\}^{\theta}-\frac{2i-1}{2n}\right]$$

$$\frac{\partial C}{\partial \theta}=2\theta\sum_{i=1}^{n}x_i^{\beta}\left[\{A(x_i)\}^{\theta}-\frac{2i-1}{2n}\right]\{A(x_i)\}^{\theta}\ln\left[A(x_i)\right]$$

Where $A(x_i)=1-\left(1+\alpha x_i^{\beta}\right)e^{-\alpha x_i^{\beta}}$

We will get the CVM estimators by solving

$\frac{\partial C}{\partial \alpha}=0, \frac{\partial C}{\partial \beta}=0\ and\ \frac{\partial C}{\partial \theta}=0$ simultaneously

IV.REAL DATA APPLICATION

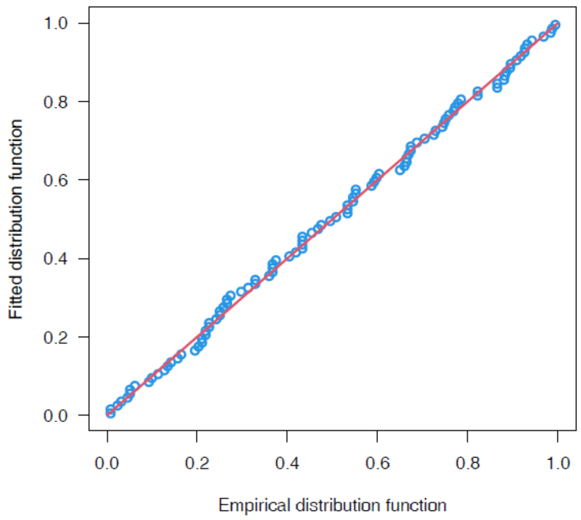
In this portion, we demonstrate NED application with the help of a real dataset. The data represent the waiting times (in minute) of 100 bank customers (Ghitany et al., 2008). The data are presented below, 0.8, 0.8, 1.3, 1.5, 1.8, 1.9, 1.9, 2.1, 2.6, 2.7, 2.9, 3.1, 3.2, 3.3, 3.5, 3.6, 4.0, 4.1, 4.2, 4.2, 4.3, 4.3, 4.4, 4.4, 4.6, 4.7, 4.7, 4.8, 4.9, 4.9, 5.0, 5.3, 5.5, 5.7, 5.7, 6.1, 6.2, 6.2, 6.2, 6.3, 6.7, 6.9, 7.1, 7.1, 7.1, 7.1, 7.4, 7.6, 7.7, 8.0, 8.2, 8.6, 8.6, 8.6, 8.8, 8.8, 8.9, 8.9, 9.5, 9.6, 9.7, 9.8, 10.7, 10.9, 11.0, 11.0, 11.1, 11.2, 11.2, 11.5, 11.9, 12.4, 12.5, 12.9, 13.0, 13.1, 13.3, 13.6, 13.7, 13.9, 14.1, 15.4, 15.4, 17.3, 17.3, 18.1, 18.2, 18.4, 18.9, 19.0, 19.9, 20.6, 21.3, 21.4, 21.9, 23.0, 27.0, 31.6, 33.1, 38.5

The MLEs of NED are calculated by employing the R software' s optim() function (R Core Team, 2020) and (Mailund, 2017) with maximization of the likelihood function (3.1.1). We have obtained the value of Log-Likelihood is l = -317.0262. Illustration of the MLE's along with standard errors (SE) for α, β, and θ is shown in Table 1.

Table 1. MLE and SE of NED for α, β and θ

Parameter	SE	MLE
alpha	0.3430	0.3998
beta	0.2374	0.8107
theta	0.9847	1.5490

In Figure 2 we have plotted the Q-Q plot and P-P plot and it is observed that proposed distribution fits the data very well.



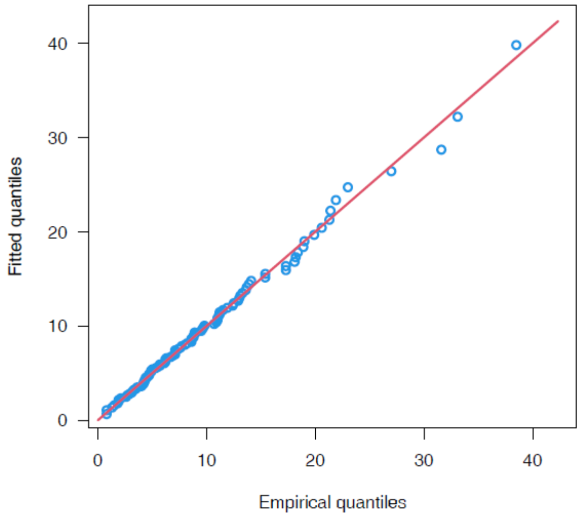
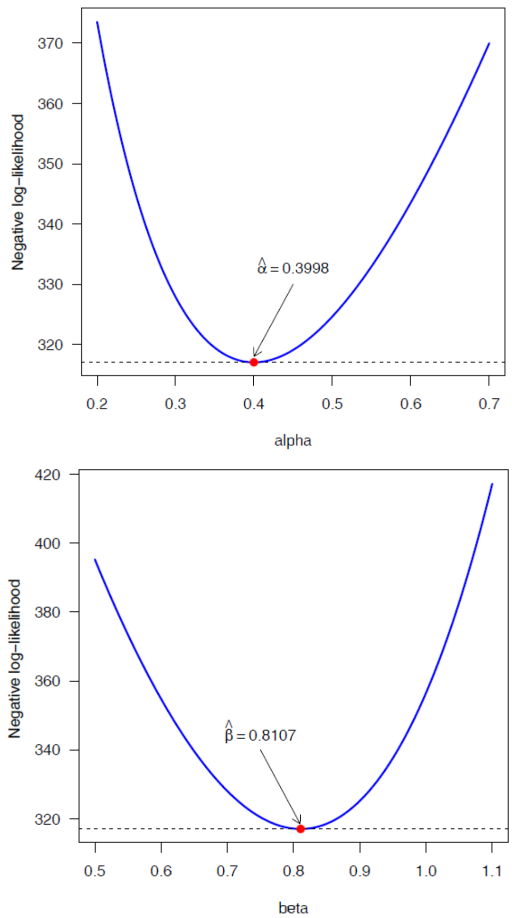


Figure 2. The plots of P-P (upper panel) and Q-Q (lower panel) of the NED.

Illustration of the plot of the profile log-likelihood function of α, β, and θ is done in Figure 3 (Kumar & Ligges, 2011) and it is seen that the MLEs are uniquely determined.



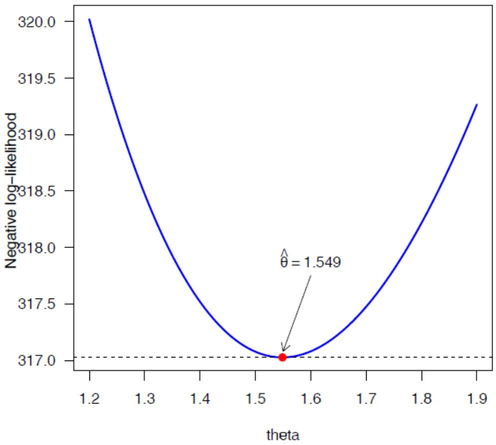


Figure 3. Plots of profile log-likelihood function of α, β, and θ.

In Table 2 Illustration of the NED parameters' estimated value obtained from LSE, MLE and CVE method and their respective negative log-likelihood, and AIC criterion.

Table 2

Estimated parameters, log-likelihood, and AIC

Method of Estimation	$\hat{\alpha}$	$\hat{\beta}$	$\hat{\theta}$	-LL	AIC
MLE	0.3998	0.8107	1.5490	317.0262	640.0525
LSE	0.5694	0.7067	2.0105	317.1670	640.3340
CVE	0.4542	0.7741	1.6975	317.0399	640.0799

In Table 3 we have presented The KS, W and A² statistics with their corresponding p-value of MLE, LSE and CVE estimates.

Table 3

The KS, W and A² statistics with a p-value

Method of Estimation	*KS(p-value)*	*W(p-value)*	*A²(p-value)*
MLE	0.0361(0.9995)	0.0176(0.9988)	0.1280(0.9996)
LSE	0.0388(0.9982)	0.0171(0.9990)	0.1348(0.9994)
CVE	0.0363(0.9994)	0.0169(0.9991)	0.1255(0.9997)

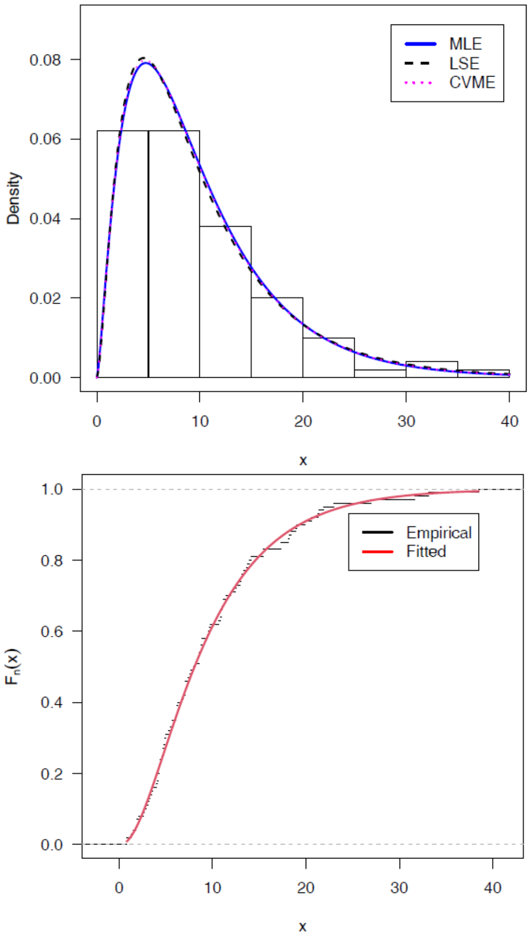


Fig 4. The Histogram and the density function of fitted distributions of estimation methods MLE, LSE and CVM (upper panel) and KS plot of NED (lower panel).

For demonstration of NED's goodness-of-fit, the following distributions are used for providing comparison

A. Flexible Weibull (FW) distribution:

The density of Flexible Weibull (FW) distribution (Bebbington, 2007) with parameters α and β is

$$f_{FW}(x)=\left(\alpha+\frac{\beta}{x^2}\right)\exp\left(\alpha x-\frac{\beta}{x}\right)\exp\left\{-\exp\left(\alpha x-\frac{\beta}{x}\right)\right\}$$

$;x\geq 0,\ \alpha>0,\ \beta>0.$

B. Generalized Rayleigh distribution

The PDF of Generalized Rayleigh (GR) distribution (Kundu & Raqab, 2005) is

$$f_{GR}\ (x;\ \alpha,\ \lambda)=2\ \alpha\ \lambda^2\ x\ e^{-(\lambda x)^2}\left\{1-e^{-(\lambda x)^2}\right\}^{\alpha-1}$$

$;(\alpha,\lambda)>0,\ x>0$

Where λ and α are the scale and shape parameters correspondingly

C. Exponential Extension (EE) distribution

The density of exponential extension (EE) distribution as given by (Nadarajah & Haghighi, 2011) with parameters α and λ is

$$f_{EE}(x)=\alpha\,\lambda\,(1+\lambda x)^{\alpha-1}\exp\left\{1-(1+\lambda x)^{\alpha}\right\}$$

$;x\geq 0,\ \alpha>0,\ \lambda>0.$

D. Extended Exponential New distribution

The density of EEN distribution (Joshi, 2015) with parameters α and λ is

$$f_{EEN}(x)=\alpha\left(1+\frac{\lambda}{x}\right)e^{-\lambda/x}\exp(-\alpha x e^{-\lambda/x});\ x>0$$

For the judgment of potentiality of the proposed model we have calculated the Bayesian information criterion (BIC), Akaike information criterion (AIC), Corrected Akaike information criterion (CAIC) and Hannan-Quinn information criterion (HQIC) which are displayed in Table 4.

Table 4

AIC, Log-likelihood (LL), CAIC, BIC and HQIC

Distribution	AIC	-LL	CAIC	BIC	HQIC
NED	640.0525	317.0262	640.3025	647.8680	643.2156
EEN	639.6955	317.8478	639.8192	644.9058	641.8042
FW	646.5363	321.2682	646.6600	651.7467	648.6450
GR	647.0364	321.5182	647.1601	652.2467	649.1451
EE	650.8973	323.4487	651.0185	656.1077	653.0060

Figure 5 demonstrates the comparison made between the density function of selected distributions.

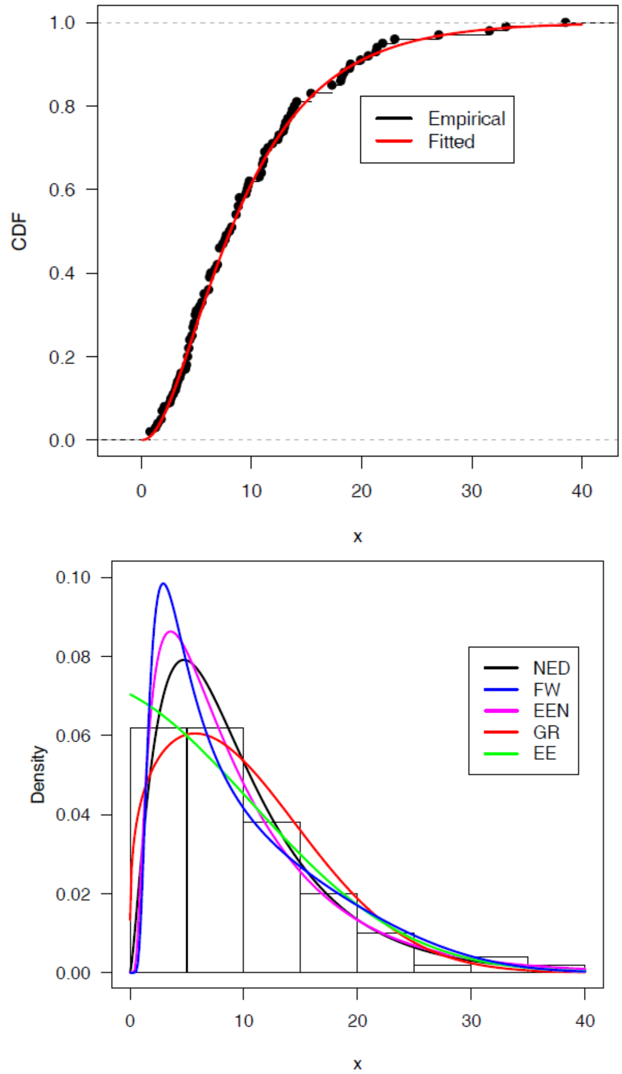


Fig. 5. Estimated distribution function with Empirical distribution function (upper panel) and The Histogram and the density function of fitted distributions (lower panel).

To evaluate proposed distribution's goodness-of-fit, the values of the Cramer-Von Mises (A^2) , Kolmogorov-Simnorov (KS), and the Anderson-Darling (W) statistics for the different selected distribution in Table 5. It is observed that the NED's test statistic value is the minimum and with higher *p*-value thus we can derive the conclusion that NED is more consistent while giving better fit for the lifetime data also producing consistent results than others taken for comparision

Table 5

The goodness-of-fit statistics and their corresponding p-value

Distribution	*A^2(p-value)*	*KS(p-value)*	*W(p-value)*
NED	0.1280(0.9996)	0.0361(0.9995)	0.0176(0.9988)
EEN	0.3465(0.8993)	0.0639(0.8093)	0.0520(0.8650)
FW	0.7710(0.5021)	0.0849(0.4717)	0.1116(0.5316)
GR	1.0911(0.3126)	0.0945(0.3337)	0.2043(0.2595)
EE	1.5539(0.1642)	0.1069(0.2028)	0.2096(0.2499)

V. CONCLUSION

We have introduced a new three parameter exponentiated distribution along with description of some of the statistical properties. Also kurtosis and skewness measures along with quantile function are derived. For estimation of model parameters, we have used LSE, MLE and CVME methods where we found that MLE gives better estimation. A real dataset is taken to demonstrate proposed distribution's application and flexibility and we observed that the NED is more consistent while giving better fit for the lifetime data also producing consistent results than other lifetime distribution used for comparison. In the domain of applied statistics and survival analysis, we hope that this distribution can be a substitute for the distribution in common use.

VI. REFERENCES

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