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Nonlinear Characteristics Analysis of Eccentric Rotor System Based on FEM

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ABSTRACT

In this paper, based on the theory of rotor dynamics, the nonlinear dynamics of a rotor system with an eccentric rotor is analyzed. Taking the double-support rotor system with a single turntable as the object, the mathematical modeling method of the rotor system with an eccentric rotor and the rotation analysis of the turntable are analyzed in detail. According to ANSYS software, the influence of eccentricity on the rotor system and the relationship between eccentricity and nonlinearity are analyzed.

Keywords : Finite Element Method, Nonlinear Rotor Dynamic, Simulation Analysis, Eccentric Rotor

I. INTRODUCTION

Rotor dynamics are very important for studying the rotor system, that is, the relationship between the rotor and the support. Rotor dynamics mainly studies the critical speed, stability, vibration, and dynamic balance of the rotor system and a new dynamic modeling method for ball-bearing rotor systems is proposed based on rigid body elements [1]. The general dynamic modeling of ball bearing-rotor system by using a finite element method was analyzed concretely [2]. A coupled dynamic model of the rotor blade is established, and the effectiveness of the model is verified by the finite element model and the natural frequency determined by the experiment [3,4]. Based on ANSYS software, a finite element model of the rotor-disk coupling system is established, and the vibration response of the system caused by the blade casing rubbing is analyzed [5]. Literature [6] studied the influence of shaft diameter, shaft length, and disc parameters on the first-order natural frequency of rotor under different speed by using hybrid modeling technology. Based on an 8-degree-of-freedom double-disk rotor system, the influence of the gyro effect on the critical speed and unbalanced response of the rotor under the action of different supporting positions and supporting stiffnesses was studied [7]. By analyzing the simple disc model, the influence law of disc physical parameters on the equivalent stiffness is summarized and the equivalent calculation formula is given [8, 9]. In modeling the support of sliding bearings, the nonlinear oil slick of sliding bearings is often linearized, and then the dynamic factor of sliding bearings is obtained [10]. This paper mentions the theoretical basis of rotor dynamics, and then investigates various nonlinear factors in the rotor dynamics system, and uses the finite element method to analyze the rotor dynamics system.

1. Theoretical basis of rotor dynamics

1.1 Rotor dynamics analysis method

In the rotor dynamics analysis, the dynamic model is used for analysis. There are continuous mass models and discrete mass models. In the continuous mass model, the rotor is considered as an elastic body with continuous mass distribution. In mathematical modeling, the continuous quality model is close to the actual situation. The discrete mass model discretizes the actual structure, transforming a continuous infinite degree of freedom model into a discrete finite degree of freedom model, and ordinary differential equations are often used to describe its motion. Discrete quality models can be classified into concentrated quality models and finite element models. The finite element model divides the continuum into sufficiently small elements. According to the simple functions of the nodes and boundaries of these elements, it can approximately represent the distribution or change of the physical quantities of the divided elements, thereby obtaining method of distribution and change of the physical quantities of the entire continuum. The segments are connected by mass nodes. The segments have no mass and are regarded as rigid bodies or elastic body.

1.2 Calculation method

The calculation and analysis methods include the transfer matrix method and the direct stiffness method (finite element method, modal synthesis method, etc.). The transfer matrix method can better deal with the dynamics of the multi-disk rotor system. The transfer matrix method is used to solve the torsional vibration problem of the disc rotor, the vibration problem of the beam, etc., especially for the dynamic solution of the rotor dynamic system. The transfer matrix method can be combined with the numerical integration method, such as the transfer matrix-direct integration method to solve the dynamic problems of the rotor system.

II. Investigation of Nonlinear Factors in Rotor Dynamics System

2.1 Investigation of the non-linear factor of the state including contact

Many general structures including change in contact state exhibit state-related nonlinear behavior. For example, a cable that can only be stretched may be loose or taut; the bearing sleeve may be in contact, or it may not be in contact. The stiffness of these systems changes suddenly due to changes in the state of the system. The state change may be directly related to the load, or it may be caused by some external reason. Contact is a very common nonlinear behavior, and it is a special and important subset of the nonlinear type of state change.

2.2 Investigation of nonlinear factors of structural materials

There are many nonlinear factors of structural materials. The important thing is that there is a nonlinear element in the rotating structure itself. Due to the manufacturing process, precision requirements, material imbalance, etc., eccentricity occurs, which will cause the product to be asymmetric in structure. Therefore, the longer the turbine is used, the easier it is to wear and even crack the shaft. This is an important factor that causes nonlinear interference and must also be considered in the research. The general structural nonlinearity is the eccentric rotor model. When referring to an eccentric rotor, it is generally considered to be the eccentricity of the turntable.

2.3 Investigation on Nonlinear Factors of Bearing Oil Film Force

The bearing can not only support the rotor system and reduce the movement friction, but also ensure the rotation accuracy of the rotor system. There are two types of bearings; rolling bearings and sliding bearings. Oil film bearings are generally used in hydraulic turbines and steam turbines. The bearing capacity of the oil film bearing is expressed as the resultant force of the pressure at all points in the oil film. The rotating shaft is relied on by the bearing. In the rotor system, one of the nonlinear excitation sources is the nonlinear oil film force. However, in the case of small disturbances, the bearing oil film force is weakly nonlinear and can be investigated with a linear oil film force model. When there is a large disturbance, the bearing oil film force is strongly nonlinear, which indicates that the change of the oil film thickness in the bearing clearance is very important and cannot be ignored.

III. Dynamic equation and model establishment of rotor system

3.1 Dynamic equation of rotor system and analysis of turntable rotation

3.1.1 Dynamic equations of the rotor system

The rotor system consists of a turntable and two supports, and a single disc with eccentric mass is placed in the center of the shaft.

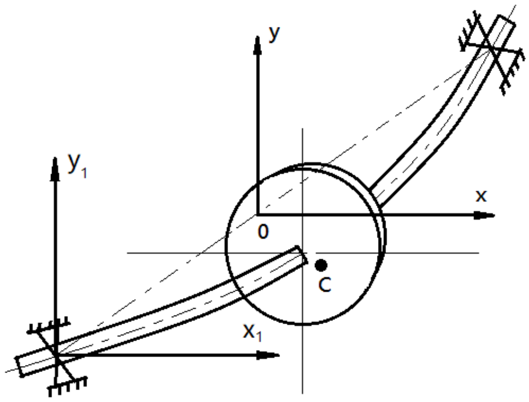

Figure 1. Single-disk rotor system model

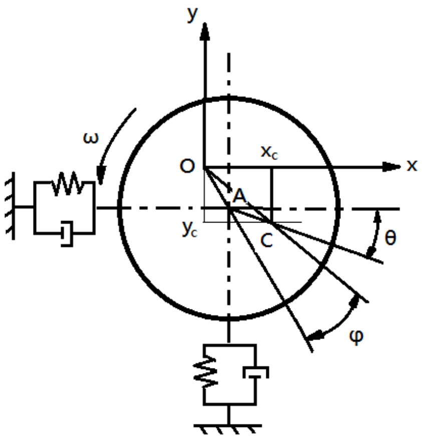

Figure 2. Rotor eccentric model

In the figure 2, C is the center of mass of the turntable, A is the center of the turntable, xc, yc are the coordinates of the center of mass of the turntable, CA is the mass eccentricity (CA=Le), and ω is the rotation angular velocity. The relationship of the centroid coordinates of the turntable (x_c, y_c) and the fixed coordinates(x, y) is;

$$\begin{cases} x_c = x + L_e\cos\theta \\ y_c = y + L_e\sin\theta \end{cases} \tag{1}$$

The finite element method divides the rotor structure into a finite number of elements, converts the equation of the entire structure into a single parallel linear algebraic equation, and applies various methods to obtain the solution. The general dynamic equation is as follows:

$$[M]\{\ddot{U}\} + [C]\{\dot{U}\} + [K]\{U\} = \{F\} \tag{2}$$

Where [M] is the mass, [C] is the damping matrix, [K] is the stiffness matrix, {U} is the displacement matrix, and {F} is the external force matrix. Since then, the motion equation of the rotor system in the fixed coordinate system is;

$$\begin{cases} m\ddot{x} + c\dot{x} + kx = mL_e(\omega^2\cos\theta + \beta\sin\theta) \\ m\ddot{y} + c\dot{y} + ky = mL_e(-\omega^2\sin\theta + \beta\cos\theta) \end{cases} \tag{3}$$

Where m, c and k are the mass of the turntable, the external damping of the system and the stiffness of the shaft respectively; $\theta=\omega t$, $\beta = \dot{\omega}$ are the angular acceleration; Le is the eccentric distance.

Written in the form of standard equation;

$$\begin{cases} \ddot{x} + 2\xi\omega_n\dot{x} + \omega_n^2 x = L_e(\omega^2\cos\theta + \beta\sin\theta) \\ \ddot{y} + 2\xi\omega_n\dot{y} + \omega_n^2 y = L_e(-\omega^2\sin\theta + \beta\cos\theta) \end{cases} \tag{4}$$

Where ω_n is the lateral undamped natural frequency of the system, $\omega_n = \sqrt{\frac{k}{m}}$,ξis the damping ratio of the system, $\xi = \frac{c}{2m\omega_n}$

Take the single turntable and dual-support rotor system model as a matrix to express as follows;

$$\begin{bmatrix} m & 0 & 0 & 0 \\ 0 & m & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} \ddot{x} \\ \ddot{y} \\ \ddot{x}_1 \\ \ddot{y}_1 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & c_{xx} & c_{xy} \\ 0 & 0 & c_{yx} & c_{yy} \end{bmatrix}\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{x}_1 \\ \dot{y}_1 \end{bmatrix} + \begin{bmatrix} k & 0 & k & 0 \\ 0 & k & 0 & -k \\ 0 & -{}^{k}/_{2} & k_{yx} & k_{yy} + {}^{k}/_{2} \\ -{}^{k}/_{2} & 0 & k_{xx} + {}^{k}/_{2} & k_{xy} \end{bmatrix}\begin{bmatrix} x \\ y \\ x_1 \\ y_1 \end{bmatrix} = \begin{bmatrix} mL_e\omega^2 \\ 0 \\ 0 \\ 0 \end{bmatrix}\cos\omega t + \begin{bmatrix} 0 \\ mL_e\omega^2 \\ 0 \\ 0 \end{bmatrix}\sin\omega t \tag{5}$$

Where m is the unbalanced mass, k is the elastic coefficient of the rotating shaft, c_{xx}, c_{yx}, c_{yy}, c_{xy} are the system support damping characteristic coefficients, and k_{xx}, k_{yx}, k_{yy}, k_{xy} are the system support stiffness characteristic coefficients. If the turntable is not in the middle of the two supports, considering the gyroscopic effect, the differential equation of the rotor system motion at that time is:

$$[M]\{\ddot{U}\} + ([C] + [G])\{\dot{U}\} + [K]\{U\} = \{F\} \tag{6}$$

Where [G] is the gyroscopic effect matrix of the rotor system.

3.1.2 Turntable rotation analysis

For the rotor system, the turntable is usually regarded as a mass point in finite element analysis.

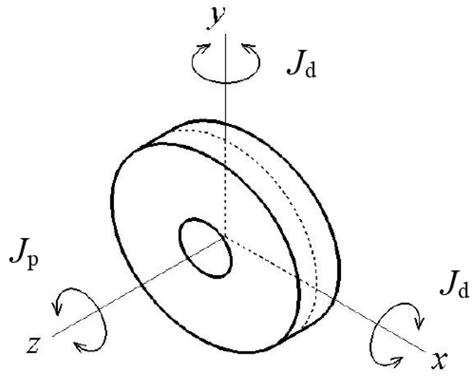

Figure 3. Moment of inertia of turntable

The turntable has dimensions, mass, and moment of inertia. The turntable on the rotor system generally has two moments of inertia, namely the central diameter moment of inertia of the x and y coordinate axes and the central pole moment of inertia of the z coordinate axis. The calculation formula is:

$$\begin{cases} J_{xy} = \frac{1}{12} m\left[3\left(D_{\text{out}}^2 + D_{in}^2\right) + H^2\right] \\ J_z = \frac{1}{2} m(D_{in}^2 + D_{out}^2) \end{cases} \tag{7}$$

Where m is the mass of the turntable, D_{in}, D_{out} is the inner diameter and outer diameter of the turntable, H is the width of the turntable. If the critical speed is solved on the rotor system, the gyroscopic torque should be considered. The equation of the undamped whirl frequency ω_c when the rotational inertia of the turntable is J is:

$$\left|-\left(M - \frac{\omega}{\omega_c}\right)\omega_c^2 + k\right| = 0 \tag{8}$$

3.2 Dynamic modelling of rotor system

3.2.1 Geometric modelling

The rotor system consists of two bearings, a turntable and a shaft. The geometric model of the rotor system to be simulated is as follows.

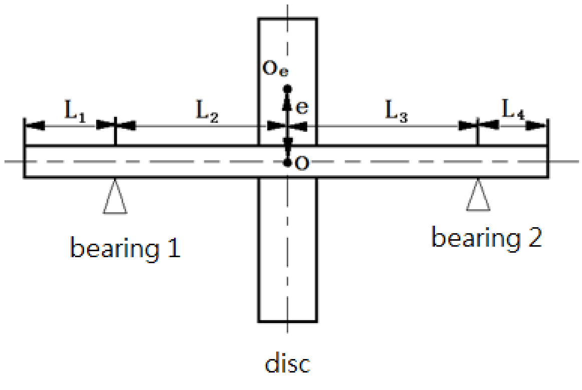

Figure 4. Geometric model of the rotor system

3.2.2 Finite element model of the rotor system

The turntable is simulated with Mass21 mass element, and the shaft is treated as a beam element with BEAM188 element for simulation. The bearing is simulated with COMBIN14 element.

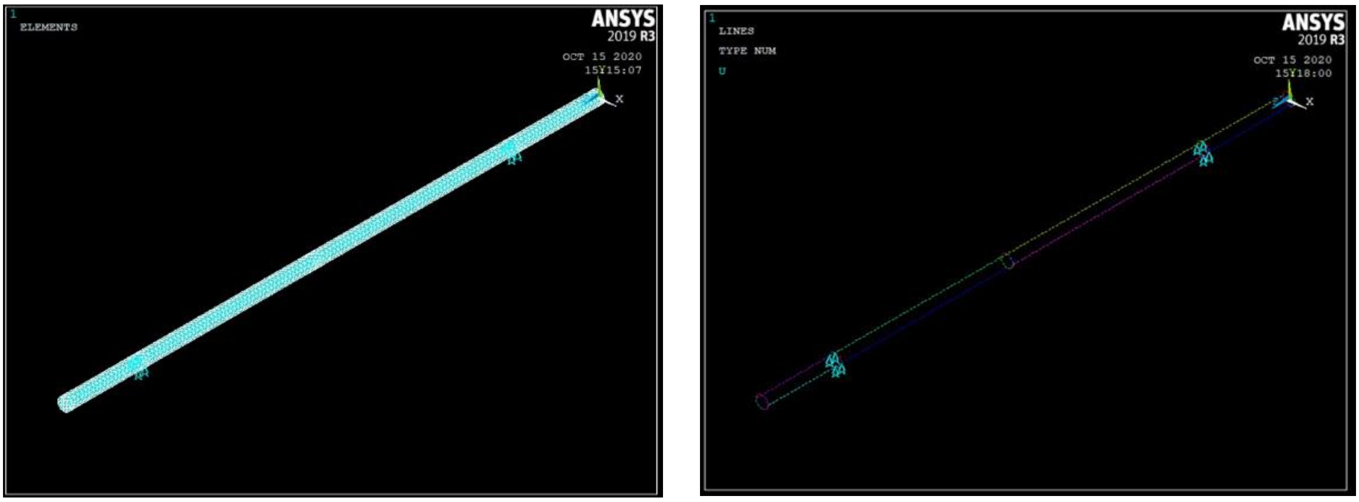

Figure 5. Finite element model

IV. SIMULATION ANALYSIS

4.1 Structure and material parameters

Table 1. Structural parameters

	length (m)	radius (m)	mass(kg)	Eccentric mass(kg)	Eccentric distance (m)
Shaft	L_1=0.4,L_2=0.7, L_3=0.75.L_4=0.35	R =0.02			
Turntable		R_1=0.45	m =25		
Rotor system				m_e=0.0025	L_e=0.1

Table 2. Material parameters

	Elastic Modulus (Pa)	Poisson's ratio	density (kg/m³)	Supporting stiffness (N/m)	Damping N/(m/s)
BearingI	2.1×10^{11}	0.3	7800	k_x=k_y=$1.1x10^8$	C_x=C_y=100
BearingII	2.1×10^{11}	0.3	7800	k_x=k_y=$1.2x10^8$	C_x=C_y=100

4.2 Result

4.2.1 Critical speed analysis

The modal analysis of the model was performed with ANSYS software, and the critical speed value was calculated without considering the eccentricity. The results can be seen in Table 3 and Figure 3.

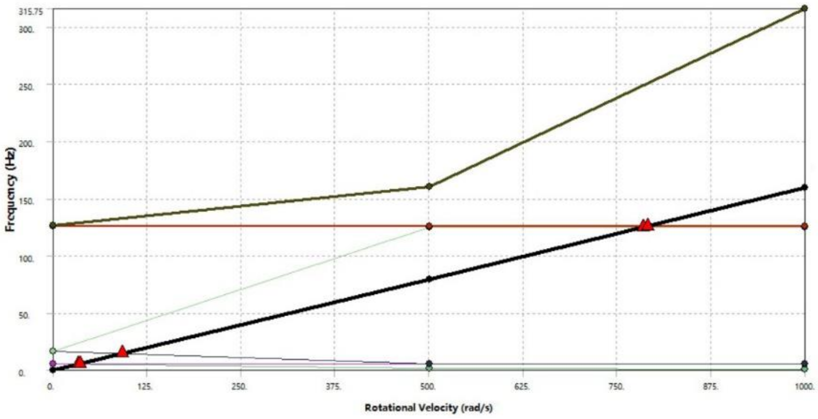

Figure 6. Campbell with critical speed

Table 3. Critical speed value of rotor system model (without considering eccentricity)

	First-order		Second-order		Third-order	
Critical speed	16.01	16.23	40.52	40.69	102.34	102.45

It can be seen from Table 3 that each frequency has two critical speed values. This is because after considering the elastic support of the bearing, the critical speed in the x direction and y direction of the rotor is no longer consistent, and its trajectory will also become Ellipse, this result is consistent with the theory of rotor dynamics. The eccentric force Fe can be used to find the critical speed when the rotor is eccentric. The eccentric force F_e can be marked as follows:

$$\mathrm{F}_e = mr\omega^2 = m_e L_e \omega^2 \tag{9}$$

When the eccentric force is added to the model, the horizontal and vertical component forces continue to change due to the rotating force, so the force must be separated into two forces, namely the real and imaginary forces, to be simulated. The expression is as follows:

$$\begin{cases} \mathrm{F}_{ex} = \mathrm{F}_e \cdot [\cos(\omega_t t) - \sin(\omega_t t)] \\ \mathrm{F}_{ey} = \mathrm{F}_e \cdot [\sin(\omega_t t) + \cos(\omega_t t)] \end{cases} \tag{10}$$

Let the rotor system accelerate from 0 to 5000rpm within 5s. At this time, the instant speed and simulation results are as follows:

$$\omega_t = \frac{5000 \cdot 2\pi}{5 \cdot 60} t \tag{11}$$

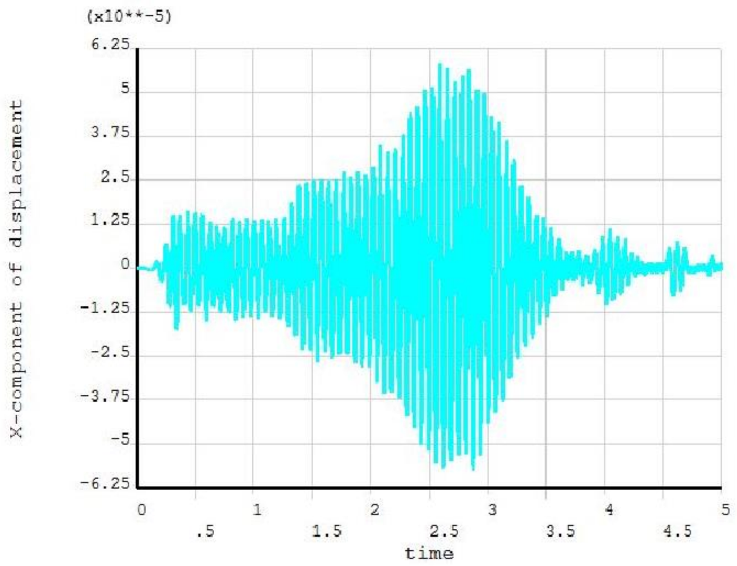

Figure 7. the x-direction displacement of the axis changes with time

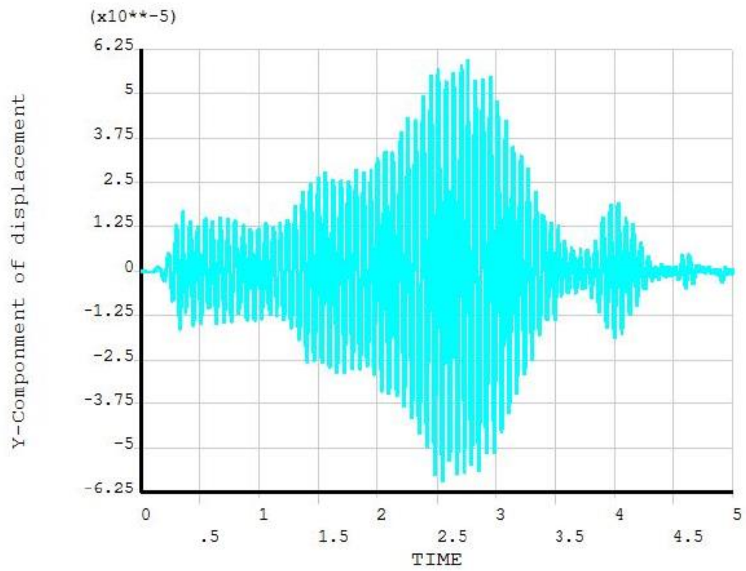

Figure 8. the y-direction displacement of the axis changes with time

As can be seen from the above figure, the abscissa of the maximum displacement is for the x direction, when the time is 2.535s, the corresponding speed is 73.6rps and for the y direction, when the time is 2.594s, the corresponding speed is 84.7rps.

Table 4. Critical speed value considering eccentricity

	Rotation speed when the center of mass of the turntable reaches the maximum amplitude	
Rotating speed（rps）	73.6	84.7

Through the comparison of Table 3 and Table 4, it can be known that the critical speed value of the rotor system considering eccentricity is significantly reduced than that of the rotor system without eccentricity, which proves that the eccentric effect will reduce the critical speed value of the rotor system.

4.2.2 Analysis of Vibration Amplitude of Disc Center under Different Eccentric Distances

Table 5. Disc center vibration amplitude under different eccentric distances

Eccentric distance（m）	0.05	0.1	0.15	0.2	0.25	0.3
The maximum amplitude of disc center（m）	2.8125 $\times10^{-5}$	5.62535 $\times10^{-5}$	8.2356 $\times10^{-5}$	1.0369 $\times10^{-5}$	1.2896 $\times10^{-5}$	1.4891 $\times10^{-5}$

It can be seen from Table 5 that the relationship between the rotor and the eccentricity is nonlinear, and it is demonstrated that the eccentricity is nonlinear in the rotor system. When eccentric factors such as eccentric distance and eccentric mass are small, the influence of eccentric force on the rotor system is approximately linear, and when the value of eccentric factor is large, the nonlinear influence of eccentric force must be considered.

V. CONCLUSION

In this paper, a nonlinear dynamic analysis of a rotor system with an eccentric rotor is carried out. On the basis of the rotor dynamics theory, taking the single turntable double support rotor system as the object, the mathematical model making method of the rotor system with eccentric rotor and the turntable rotation analysis are analyzed in detail. According to ANSYS software, the influence of eccentricity on the rotor system and the relationship between eccentricity and nonlinearity are analyzed.

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