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The ArcTan Lomax Distribution with Properties and Applications

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ABSTRACT

Here, in this paper, a continuous distribution called ArcTan Lomax distribution with three-parameter has been introduced along with some relevant properties of statistics and mathematics pertaining to the distribution. With the help of three established estimations methods including maximum likelihood estimation (MLE), estimation of the presented distribution's model parameters is done. Also with the help of a real set of data, the distribution's goodness-of-fit is examined in contrast to some established models in survival analysis.

Keywords: ArcTan distribution, Estimation, Goodness-of-fit, Lomax distribution, MLE

I. INTRODUCTION

Life-time models like Cauchy, Gamma, Weibull, exponential are generally used in survival analysis for evaluating lifespan of a system in areas such as engineering, biology, medicine etc. For a few years, most of the researchers are attracted towards modified distribution generated from a parent distribution in modeling life-time data as excellent performance by these generated distribution has been observed. We aim to introduce a distribution with more flexibility from the parent distribution adding an extra parameter to it to achieve good fit to the data encountered in survival analysis.

In this study we have taken Lomax distribution, introduced by Lomax (1954), as parent distribution which is heavy-tailed shaped distribution with applications in many areas like life science, medicine, engineering, and many more. The model's CDF and PDF can be written as

$$F\left(x,\alpha,\beta\right)=1+\left(1+\beta x\right)^{-\alpha}\,;\left(\alpha,\beta\right)>0 \qquad (1.1)$$

and

$$f\left(x,\alpha,\beta\right)=\alpha\beta\left(1+\beta x\right)^{-(\alpha+1)}\,;\left(\alpha,\beta\right)>0 \qquad (1.2)$$

Using the Lomax distribution, Chaudhary and Kumar (2020) have introduced Logistic Lomax distribution. The inverse Lomax distribution was introduced by

(Kleiber, 2004) and among ordered statistics he applied the IL distribution to obtain Lorenz ordering relationship. Application of IL model in actuarial sciences and economics was demonstrated by Kleiber and Kotz (2003). The Poisson inverted Lomax distribution was introduced by Joshi and Kumar (2020) using inverse Lomax distribution. Joshi and Kumar (2021) also introduced the Logistic inverse Lomax distribution using inverse Lomax distribution.

In this paper we have used arc tan- G family of distribution which was introduced by (Gómez-Déniz & Calderín-Ojeda, 2015) which was used in modeling insurance data of Norwegian fire and the new distribution was called Pareto arc tan distribution and found that this distribution provide a good fit as compared to some well-known distributions. The CDF and PDF of arc tan family of distribution with support [a, b] is

$$F(x)=1-\frac{\arctan[\alpha\{1-G(x)\}]}{\arctan(\alpha)};x\geq 0,\alpha>0;\alpha\in[a,b] \quad (1.3)$$

$$f(x)=\frac{1}{\arctan(\alpha)}\frac{\alpha g(x)}{1+[\alpha\{1-G(x)\}]^2};x\geq 0,\alpha>0 \quad (1.4)$$

Here, $G(x)$ *and* $g(x)$ are the CDF and PDF

Structure of the article is explained as follows. we present the continuous distribution namely ArcTan Lomax distribution with three-parameter has been introduced along with some relevant properties of statistics and mathematics pertaining to the distribution in Section 2. Along with brief explanation of the estimation methods, estimation of the unknown parameters is done in Section 3. A real data set has been analysed exploring the applications and capability of the distribution introduced in Section 4. Then conclusion has been presented in Section 5.

II. THE ARCTAN LOMAX(ATL) DISTRIBUTION

Here a novel distribution called ArcTan Lomax (ATL) distribution is presented. In this study, we have taken the Lomax as baseline distribution. Considering a non-negative random variable be denoted by X with parameters α, β and λ then CDF of distribution is defined by using equations (1.1) and (1.3) as

$$F(x)=1-\frac{\arctan\{\alpha(1+\beta x)^{-\lambda}\}}{\arctan(\alpha)};x\geq 0,(\alpha,\beta,\lambda)>0 \quad (2.1)$$

And the PDF is

$$f(x)=\frac{\alpha\beta\lambda}{\arctan(\alpha)}\frac{(1+\beta x)^{-(\lambda+1)}}{1+\{\alpha(1+\beta x)^{-\lambda}\}^2};x\geq 0,(\alpha,\beta,\lambda)>0 \quad (2.2)$$

Reliability function

$$R(x)=1-F(x)=\frac{\arctan\{\alpha(1+\beta x)^{-\lambda}\}}{\arctan(\alpha)} \quad (2.3)$$

Hazard function

$$h(x)=\frac{f(x)}{R(x)}$$

$$=\frac{\alpha\beta\lambda(1+\beta x)^{-(\lambda+1)}}{\arctan\{\alpha(1+\beta x)^{-\lambda}\}[1+\{\alpha(1+\beta x)^{-\lambda}\}^2]} \quad (2.4)$$

$;x\geq 0,(\alpha,\beta,\lambda)>0$

Reversed Hazard function

$$Rh(x)=\frac{f(x)}{F(x)}$$

$$=\frac{\alpha\beta\lambda(1+\beta x)^{-(\lambda+1)}}{[\arctan(\alpha)-\arctan\{\alpha(1+\beta x)^{-\lambda}\}][1+\{\alpha(1+\beta x)^{-\lambda}\}^2]} \quad (2.5)$$

For varying values of α, β and λ, in Figure 1, PDF and hazard rate function plots of ATL distribution has been demonstrated

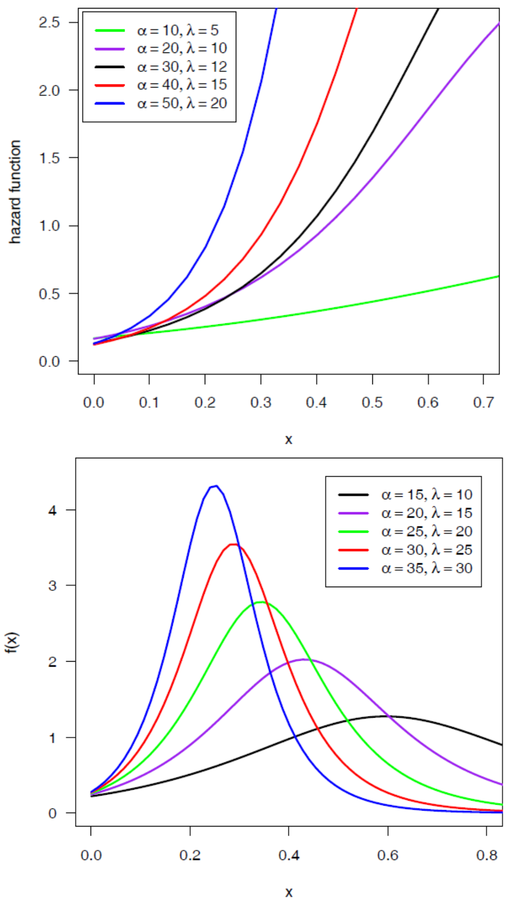

Figure 1. For varying values of α, β and λ, hazard function plot (upper panel) and PDF plot (lower panel)

Quantile function

$$Q(p)=\frac{1}{\beta}\left[\left\{\frac{1}{\alpha}\tan\left\{(1-p)\arctan\alpha\right\}\right\}^{-1/\lambda}-1\right];0<p<1 \qquad (2.6)$$

Random deviate generation:

$$x=\frac{1}{\beta}\left[\left\{\frac{1}{\alpha}\tan\left\{(1-u)\arctan\alpha\right\}\right\}^{-1/\lambda}-1\right];0<u<1 \qquad (2.7)$$

Skewness and Kurtosis:

The measures of Skewness based on quantiles is Bowley's coefficient of skewness and it can be expressed as

$$Skewness=\frac{Q(0.75)+Q(0.25)-2Q(0.5)}{Q(0.75)-Q(0.25)}$$ and

Coefficient of kurtosis based on octiles (Moors, 1988) is

$$K_u(M)=\frac{Q(0.875)-Q(0.625)+Q(0.375)-Q(0.125)}{Q(3/4)-Q(1/4)}, \qquad (2.9)$$

III.ESTIMATION METHODS

Here in this section, estimation of the ATL distribution's unknown parameters is done

3.1. Maximum Likelihood Estimates

From $ATL(\alpha,\beta,\lambda)$, consider, $x_1,x_2,...,x_n$ be a random sample and the likelihood function, $L(\alpha,\beta,\lambda)$ is,

$$L(\psi;x_1,x_2...x_n)=f(x_1,x_2,...x_n/\psi)=\prod_{i=1}^{n}f(x_i/\psi)$$

Now log-likelihood density is

$$l=n\ln\alpha+n\ln\beta+n\ln\lambda-n\ln\{\arctan(\alpha)\}$$
$$-(\lambda+1)\sum_{i=1}^{n}\ln(1+\beta x_i)-\sum_{i=1}^{n}\ln[1+\{\alpha(1+\beta x_i)^{-\lambda}\}^2] \qquad (3.1.1)$$

Differentiation of (3.1.1) w.r.t. unknown parameters α, β and λ we obtain,

$$\frac{\partial l}{\partial\alpha}=\frac{n}{\alpha}-\frac{n}{\arctan(\alpha)[1+\alpha^2]}-\sum_{i=1}^{n}\frac{2\alpha(1+\beta x_i)^{-2\lambda}}{[1+\{\alpha(1+\beta x_i)^{-\lambda}\}^2]}$$

$$\frac{\partial l}{\partial\beta}=\frac{n}{\beta}-(\lambda+1)\sum_{i=1}^{n}\frac{x_i}{(1+\beta x_i)}+2\alpha\lambda\sum_{i=1}^{n}\frac{x_i(1+\beta x_i)^{-2\lambda-1}}{[1+\{\alpha(1+\beta x_i)^{-\lambda}\}^2]}$$

$$\frac{\partial l}{\partial\lambda}=\frac{n}{\lambda}-\sum_{i=1}^{n}\ln(1+\beta x_i)+2\alpha\sum_{i=1}^{n}\frac{(1+\beta x_i)^{-2\lambda}\ln(1+\beta x_i)}{[1+\{\alpha(1+\beta x_i)^{-\lambda}\}^2]}$$

Solving for α, β and λ after equating above equations to zero we get the MLEs $\hat{\alpha},\ \hat{\beta}\ and\ \hat{\lambda}$. Maximizing (3.1.1) with software platform like Mathematica, R platform etc. estimated value of α, β and λ can be acquired. The parameters' confidence interval estimation along with testing of hypothesis can be done with observed information matrix for the parameter which can be obtained as,

(2.8)

$$U = \begin{bmatrix} U_{11} & U_{12} & U_{13} \\ U_{21} & U_{22} & U_{23} \\ U_{31} & U_{32} & U_{33} \end{bmatrix}$$

Where

$$U_{11} = \frac{\partial^2 l}{\partial \alpha^2},\ U_{12} = \frac{\partial^2 l}{\partial \alpha \partial \beta},\ U_{13} = \frac{\partial^2 l}{\partial \alpha \lambda}$$

$$U_{21} = \frac{\partial^2 l}{\partial \beta \partial \alpha},\ U_{22} = \frac{\partial^2 l}{\partial \beta^2},\ U_{23} = \frac{\partial^2 l}{\partial \beta \partial \lambda}$$

$$U_{31} = \frac{\partial^2 l}{\partial \lambda \partial \alpha},\ U_{32} = \frac{\partial^2 l}{\partial \beta \partial \lambda},\ U_{33} = \frac{\partial^2 l}{\partial \lambda^2}$$

Let $\Omega = (\alpha, \beta, \lambda)$ represent the parameter space and the MLE of Ω be represented as $\hat{\Omega} = (\hat{\alpha}, \hat{\beta}, \hat{\lambda})$, then $\left(\hat{\Omega} - \Omega\right) \to N_3\left[0, \left(U(\Omega)\right)^{-1}\right]$ where $U(\Omega)$ denotes information matrix of Fisher. With the help of Newton-Raphson algorithm maximizing likelihood gives observed information matrix and thus variance-covariance matrix which is

$$\left[U(\Omega)\right]^{-1} = \begin{pmatrix} \mathrm{var}(\hat{\alpha}) & \mathrm{cov}(\hat{\alpha}, \hat{\beta}) & \mathrm{cov}(\hat{\alpha}, \hat{\lambda}) \\ \mathrm{cov}(\hat{\alpha}, \hat{\beta}) & \mathrm{var}(\hat{\beta}) & \mathrm{cov}(\hat{\beta}, \hat{\lambda}) \\ \mathrm{cov}(\hat{\alpha}, \hat{\lambda}) & \mathrm{cov}(\hat{\beta}, \hat{\lambda}) & \mathrm{var}(\hat{\lambda}) \end{pmatrix} \quad (3.1.2)$$

Therefore, via MLEs' asymptotic normality, approximate 100(1-α) % confidence intervals for the unknown parameters is constructed as $\hat{\alpha} \pm Z_{\alpha/2} SE(\hat{\alpha})$, $\hat{\beta} \pm Z_{\alpha/2} SE(\hat{\beta})$ and, $\hat{\lambda} \pm Z_{\alpha/2} SE(\hat{\lambda})$ where the standard normal variate's upper percentile is denoted by $Z_{\alpha/2}$

3.2. Least-Square Estimation (LSE)

The unknown parameters (α, β and λ)'s least-square estimators (Swain et al., 1988) of ATL distribution is obtained with minimization of

$$M\left(X; \alpha, \beta, \lambda\right) = \sum_{i=1}^{n} \left[F(X_i) - \frac{i}{n+1} \right]^2 \quad (3.2.1)$$

w.r.t. α, β and λ.

From a distribution function F(.), consider $F(X_i)$ represent the distribution function of the ordered random variables $X_{(1)} < X_{(2)} < \ldots < X_{(n)}$ where $\{X_1, X_2, \ldots, X_n\}$ is a random sample of size n. The parameters' least-square estimators denoted by $\hat{\alpha}, \hat{\beta}, and\, \hat{\lambda}$, is calculated with minimization of

$$M\left(X; \alpha, \beta, \lambda\right) = \sum_{i=1}^{n} \left[1 - \frac{\arctan\{\alpha(1+\beta x_i)^{-\lambda}\}}{\arctan(\alpha)} - \frac{i}{n+1} \right]^2$$
$$; x \ge 0, \left(\alpha, \beta, \lambda\right) > 0 \quad (3.2.2)$$

w.r.t. α, β and λ.

Differentiation of (3.2.2) w.r.t. α, β and λ we obtain,

$$\frac{\partial M}{\partial \alpha} = -2\sum_{i=1}^{n}\left[1 - \frac{\arctan\{\alpha(1+\beta x_i)^{-\lambda}\}}{\arctan(\alpha)} - \frac{i}{n+1}\right]\left[\frac{(1+\beta x_i)^{-\lambda}}{(1+\alpha^2(1+\beta x_i)^{-2\lambda})\arctan(\alpha)} - \frac{\arctan\{\alpha(1+\beta x_i)^{-\lambda}\}}{\left(1+\alpha^2\right)\arctan^2\{\alpha\}}\right]$$

$$\frac{\partial M}{\partial \beta} = 2\alpha\lambda\sum_{i=1}^{n}\left[1 - \frac{\arctan\{\alpha(1+\beta x_i)^{-\lambda}\}}{\arctan(\alpha)} - \frac{i}{n+1}\right]\left[\frac{x_i(1+\beta x_i)^{-\lambda-1}\left[1+\{\alpha(1+\beta x_i)^{-\lambda}\}^{-2}\right]}{\arctan(\alpha)}\right]$$

$$\frac{\partial M}{\partial \lambda} = -2\alpha\sum_{i=1}^{n}\left[1 - \frac{\arctan\{\alpha(1+\beta x_i)^{-\lambda}\}}{\arctan(\alpha)} - \frac{i}{n+1}\right]\left[\frac{(1+\beta x_i)^{-\lambda}\left[1+\{\alpha(1+\beta x_i)^{-\lambda}\}^{-2}\right]\ln(1+\beta x_i)}{\arctan(\alpha)}\right]$$

Likewise the weighted least square estimators (Swain et al., 1988) is calculated with minimization of

$$M\left(X; \alpha, \beta, \lambda\right) = \sum_{i=1}^{n} w_i \left[F(X_{(i)}) - \frac{i}{n+1} \right]^2$$

w.r.t. α, β and λ. The weights *wi* are

$$w_i = \frac{1}{Var(X_{(i)})} = \frac{\left(n+1\right)^2 \left(n+2\right)}{i\left(n-i+1\right)}$$

Thus, using the weighted least square estimators, the unknown parameters is calculated with minimization of

$$M\left(X; \alpha, \beta, \lambda\right) = \sum_{i=1}^{n} \frac{\left(n+1\right)^2 \left(n+2\right)}{i\left(n-i+1\right)} \left[1 - \frac{\arctan\{\alpha(1+\beta x_i)^{-\lambda}\}}{\arctan(\alpha)} - \frac{i}{n+1} \right]^2 \quad (3.2.3)$$

w.r.t α, β and λ.

3.3. Cramer-Von-Mises estimation (CVME)

The Cramer-Von-Mises estimators of α, β and λ is calculation with minimization of the following function

$$C(X;\alpha,\beta,\lambda)=\frac{1}{12n}+\sum_{i=1}^{n}\left[F\left(x_{i:n}\mid\alpha,\beta,\lambda\right)-\frac{2i-1}{2n}\right]^2$$

$$=\frac{1}{12n}+\sum_{i=1}^{n}\left[1-\frac{\arctan\{\alpha(1+\beta x_i)^{-\lambda}\}}{\arctan(\alpha)}-\frac{2i-1}{2n}\right]^2 \qquad (3.3.1)$$

Differentiation of (3.3.1) w.r.t. α, β and λ we obtain,

$$\frac{\partial C}{\partial\alpha}=-2\sum_{i=1}^{n}\left[1-\frac{\arctan\{\alpha(1+\beta x_i)^{-\lambda}\}}{\arctan(\alpha)}-\frac{2i-1}{2n}\right]\left[\frac{(1+\beta x_i)^{-\lambda}}{(1+\alpha^2(1+\beta x_i)^{-2\lambda})\arctan(\alpha)}-\frac{\arctan\{\alpha(1+\beta x_i)^{-\lambda}\}}{\left(1+\alpha^2\right)\arctan^2\{\alpha\}}\right]$$

$$\frac{\partial C}{\partial\beta}=2\alpha\lambda\sum_{i=1}^{n}\left[1-\frac{\arctan\{\alpha(1+\beta x_i)^{-\lambda}\}}{\arctan(\alpha)}-\frac{2i-1}{2n}\right]\left[\frac{x_i(1+\beta x_i)^{-\lambda-1}\left[1+\{\alpha(1+\beta x_i)^{-\lambda}\}^{-2}\right]}{\arctan(\alpha)}\right]$$

$$\frac{\partial C}{\partial\lambda}=-2\alpha\sum_{i=1}^{n}\left[1-\frac{\arctan\{\alpha(1+\beta x_i)^{-\lambda}\}}{\arctan(\alpha)}-\frac{2i-1}{2n}\right]\left[\frac{(1+\beta x_i)^{-\lambda}\left[1+\{\alpha(1+\beta x_i)^{-\lambda}\}^{-2}\right]\ln(1+\beta x_i)}{\arctan(\alpha)}\right]$$

By solving $\frac{\partial C}{\partial\alpha}=0, \frac{\partial C}{\partial\beta}=0$ *and* $\frac{\partial C}{\partial\lambda}=0$ CVM estimators is obtained

IV.APPLICATION TO A REAL DATASET

Here, we demonstrate the applicability of arctan Lomax distribution's applicability using a real dataset used (Oguntunde et al., 2015). The dataset is on the breaking stress of carbon fibres of 50 mm length (GPa). The dataset is as follows:

0.39, 0.85, 1.08, 1.25, 1.47, 1.57, 1.61, 1.61, 1.69, 1.80, 1.84, 1.87, 1.89, 2.03, 2.03, 2.05, 2.12, 2.35, 2.41, 2.43, 2.48, 2.50, 2.53, 2.55, 2.55, 2.56, 2.59, 2.67, 2.73, 2.74, 2.79, 2.81, 2.82, 2.85, 2.87, 2.88, 2.93, 2.95, 2.96, 2.97, 3.09, 3.11, 3.11, 3.15, 3.15, 3.19, 3.22, 3.22, 3.27, 3.28, 3.31, 3.31, 3.33, 3.39, 3.39, 3.56, 3.60, 3.65, 3.68, 3.70, 3.75, 4.20, 4.38, 4.42, 4.70, 4.90

The MLEs are calculated using the platform of R (R Core Team, 2020) and (Mailund, 2017) with the help of optim() function for maximization of the likelihood function (3.1.1). We have obtained Log-Likelihood value is l = -85.3094 and the MLE's with their standard errors (SE) and 95% confidence interval for α, β, and λ are presented in Table 1.

Table 1

MLE and SE and 95% confidence interval for α, β and λ

Parameter	MLE	SE	95% ACI
alpha	119.4960	3.043	(113.5357, 125.4643)
beta	0.0192	0.00077	(0.0176, 0.0207)
lambda	91.8188	0.4096	(91.0172, 92.6228)

.

Illustration of the plots of the profile log-likelihood function of the parameters α, β, and λ is done in Figure 2 (Kumar & Ligges, 2011) where observation that MLEs are unique can be made

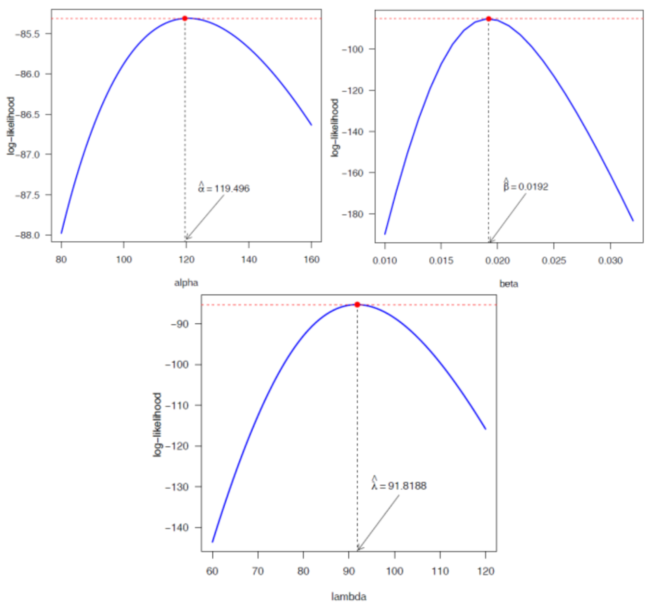

Figure 2. Graph of profile log-likelihood function of α, β, and λ.

In Figure 3, Q-Q plot and P-P plot of ATL distribution has been illustrated.

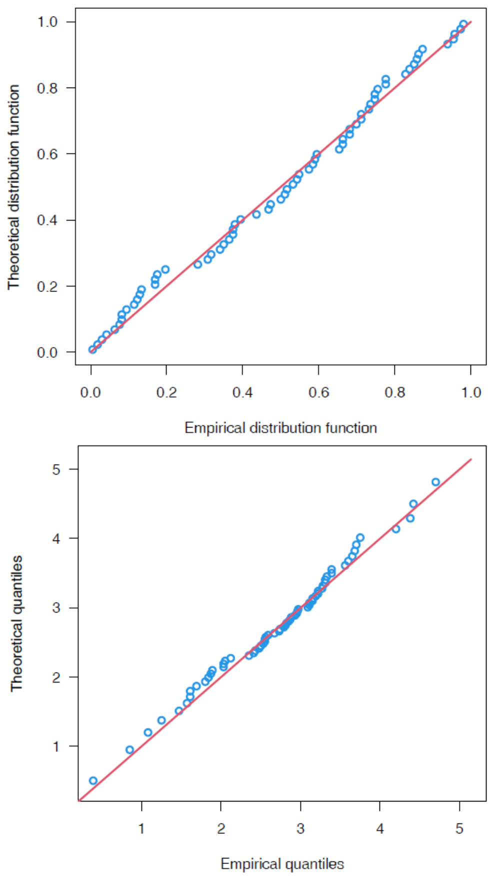

Figure 3. The plots of P-P (left panel) and Q-Q (right panel) of ATL distribution

In Table 2 we have presented the parameters' estimated value from the different estimation methods and their corresponding negative log-likelihood, AIC and KS criterion.

Table 2

Estimated parameters, log-likelihood, and AIC

Estimation method	MLE	LSE	CVE
$\hat{\alpha}$	119.4960	101.5352	114.4477
$\hat{\beta}$	0.0192	0.0169	0.0155
$\hat{\lambda}$	91.8188	100.3071	111.9520
-LL	85.3094	85.3322	85.2889
AIC	176.6188	176.6645	176.5778
KS(p-value)	0.0673(0.9264)	0.0659(0.9363)	0.0674(0.9254)

In Figure 3 we have plotted the Q-Q plot of estimation methods MLE, LSE and CVM.

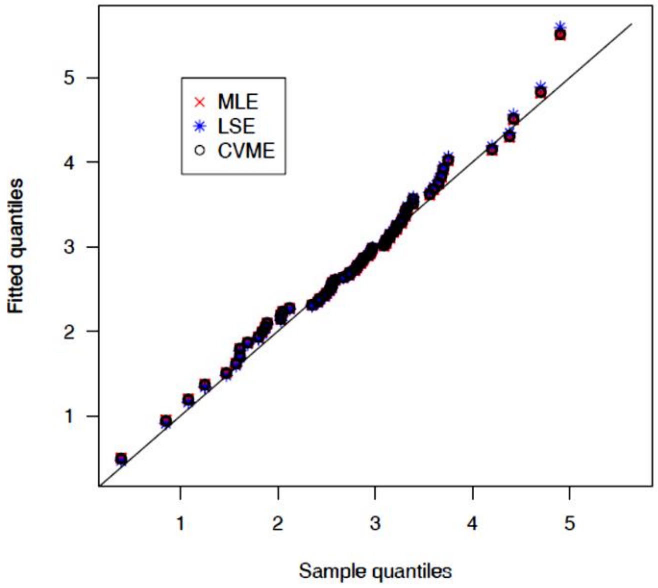

Figure 3. The fitted quantiles and sample quantiles of MLE, LSE and CVM of ATL distribution.

For evaluating goodness of fit of the ATL distribution, following lifetime models are taken for comparision

A. Generalized Gompertz (GG)distribution

The GG distribution's PDF (El-Gohary et al., 2013) with parameters α, λ and θ is

$$f_{GGZ}(x)=\theta\lambda e^{\alpha x} e^{-\frac{\lambda}{\alpha}\left(e^{\alpha x}-1\right)}\left[1-\exp\left(-\frac{\lambda}{\alpha}\left(e^{\alpha x}-1\right)\right)\right]^{\theta-1}$$

; $\lambda,\theta>0,\alpha\geq 0, x\geq 0$

B. Exponentiated Exponential Poisson (EEP):

The probability density function of EEP (Ristić & Nadarajah, 2014) can be expressed as

$$f(x)=\frac{\alpha\beta\lambda}{\left(1-e^{-\lambda}\right)}e^{-\beta x}\left(1-e^{-\beta x}\right)^{\alpha-1}\exp\left\{-\lambda\left(1-e^{-\beta x}\right)^{\alpha}\right\}$$

; $x>0, \alpha>0, \lambda>0$

C. Generalized Exponential Extension (GEE) distribution:

The PDF of GEE (Lemonte, 2013) with parameters α, β and λ is

$$f_{GEE}\left(x;\alpha,\beta,\lambda\right)=\alpha\beta\lambda\left(1+\lambda x\right)^{\alpha-1} exp\left\{1-\left(1+\lambda x\right)^{\alpha}\right\} \left[1-exp\left\{1-\left(1+\lambda x\right)^{\alpha}\right\}\right]^{\beta-1}$$

$; \; x \geq 0.$

D. Exponential power (EP) distribution:

The EP distribution's PDF (Smith & Bain, 1975) is

$$f_{EP}(x)=\alpha\,\lambda^{\alpha}\,x^{\alpha-1}\,e^{(\lambda\,x)^{\alpha}}\,\exp\left\{1-e^{(\lambda\,x)^{\alpha}}\right\}.$$

$;(\alpha,\lambda)>0, \quad x\geq 0$

where α and λ are the shape and scale parameters, respectively.

E. Generalized *Rayleigh distribution*

The Generalized Rayleigh (GR) distribution's PDF (Kundu & Raqab, 2005) is

$$f_{GR}\;(\text{x};\,\alpha,\,\lambda) = 2\,\alpha\,\lambda^{2}\;\text{x}\;\text{e}^{-(\lambda x)^{2}}\left\{1-\text{e}^{-(\lambda x)^{2}}\right\}^{\alpha-1}$$

$;(\alpha,\lambda)>0, x>0$

Here α and λ are the shape and scale parameters respectively.

For the assessment of potentiality of the proposed model we have calculated the Bayesian information criterion (BIC), Hannan-Quinn information criterion (HQIC), Akaike information criterion (AIC) and Corrected Akaike information criterion (CAIC) which are presented in Table 3.

Table 3

Log-likelihood (LL), AIC, BIC, CAIC and HQIC

Distribution	LL	AIC	BIC	CAIC	HQIC
ATL	-85.3094	176.6188	183.1878	177.0059	179.2146
GGZ	-85.6858	177.3716	183.9406	177.7587	179.9673
EEP	-86.7885	179.5770	186.1460	179.9641	182.1727
GEE	-87.2705	180.5409	187.1099	180.9280	183.1366
EP	-87.3974	178.7949	183.1742	178.9795	180.5254
GR	-88.6368	181.2735	185.6528	181.4640	183.0040

The distribution's Histogram and the density function and Empirical distribution function with the estimated distribution function of ATL are presented in Figure 4.

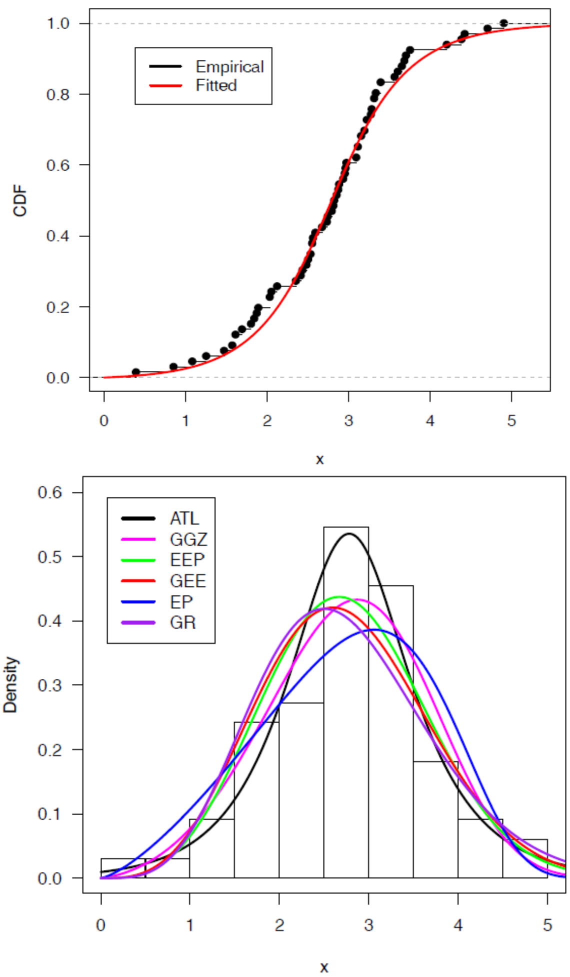

Figure 4. Empirical distribution function with estimated distribution function (Left panel) and The Histogram and the density function of fitted distributions (Right panel)

To compare the ATL distribution's goodness-of-fit against the distributions taken we have given the value of the Anderson-Darling (W), Kolmogorov-Simnorov (KS), and the Cramer-Von Mises (A^2) statistics in Table 4 where we observe that the ATL distribution has the test statistic with the minimum value and higher *p*-value. Therefore we can derive the conclusion that ATL distribution gets quite better fit with more consistency and reliability in outcomes from others taken for comparison

Table 4. The goodness-of-fit statistics and their corresponding p-value

Distribution	*KS(p-value)*	*W(p-value)*	*A^2(p-value)*
ATL	0.0673(0.9264)	0.0492(0.8827)	0.3446(0.9010)
GGZ	0.0833(0.7498)	0.0715(0.7443)	0.4457(0.8020)
EEP	0.0931(0.6160)	0.1098(0.5403)	0.5980(0.6492)
GEE	0.1099(0.4021)	0.1541(0.3780)	0.7853(0.4913)
EP	0.1126(0.3729)	0.1434(0.4111)	0.8796(0.4266)
GR	0.1205(0.2931)	0.1948(0.2781)	1.0049(0.3544)

V. CONCLUSION

In this article, ArcTan Lomax distribution has been presented along with relevant distributional properties where flexibility of the model was observed and the model had inverted bathtub shaped hazard function. Estimation of the unknown parameters is carried out with the help of established methods which are CVME, LSE and MLE where we observed that the MLEs are quite better. Using a real set of data, suitability and applicability was determined and we observed that the ATL distribution has the test statistic with the minimum value and higher *p*-value. Thus we derived the conclusion that ATL distribution gets quite better fit with more consistency and reliability in results from others taken for comparison

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