



## Parameter Space of a PWM Chopper Controlled DC Series Drive System

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### ABSTRACT

In this paper a study is made on the bifurcation behavior for speed response of a chopper fed dc motor drive system. The key idea is to identify the operating conditions over the allowable range of load torque for stable operation of the motor drive system. With only speed loop operative investigation is made by numerically simulating the drive system and also performing laboratory experiment on a 0.5 hp, 220V and 1800 rpm dc series motor.

**Keywords :** Chopper, Pwm, Bifurcation, Chaos, Chopper, Period One.

### I. INTRODUCTION

Since last few decades investigations on bifurcation behavior in nonlinear systems [1-6] have taken a lead almost in every sector of research activities. The reason behind this enhanced research activity is to explore wide diversion in dynamic behavior that ranges from periodic to sub harmonic and even chaotic responses. Advances in the field of power electronics and the unique advantage of high starting torque have made the dc series motors widely acceptable in traction, hoisting and different variable speed applications. All power electronic systems are inherently nonlinear in nature. It is therefore highly desirable to investigate dynamic behavior of this high torque dc motor drive system for identifying the optimum range of operating conditions to avoid occurrence of undesirable chaotic oscillation. However, although chaos is unwanted in a practical dc series motor drive system, but there are also different promising applications of chaos [7]. These

include compaction, mixing and grinding, washing etc.

In the existing literature for power electronics and drives, several investigations are present exploring bifurcation behavior of dc drive systems [8-19]. Most of these publications have considered brushless dc motor, PMDC or shunt motor as the electromechanical equipment. For example, Chau et al in [20] had described bifurcation and chaos in a chopper fed PMDC motor-load system. Responses of a dc drive system operated from buck converter [21-22] against variations in input voltage, controller gain, and load torque are also present. These investigations are pre requisite for designing a motor drive system in order to avoid chaos or enforce chaos. Interestingly very few studies [22-24] are available in the literature for dc series motor drives which are largely used in traction and different modern industrial and domestic appliances.

This paper therefore presents a detailed study on a pwm controlled buck converter fed dc series motor drive system. This investigation with source side ac

bus voltage is important for domestic and small scale industrial applications of dc series motor where the cost for installing a dc bus is not economical. Behavior of the drive system is mainly described with low and high speed operation against variation of load torque and controller gain. These observations are necessary for selecting the most suitable operating conditions to ensure stable operation of the drive system in different sections of the load characteristic, like low speed-high torque or vice versa or low speed low torque etc.

Organization of the paper is as follows: section 2 describes the operating modes of a buck converter fed dc series motor drive system with source side ac bus voltage. Section 3 is presented a short note on the different nonlinear behaviors of the drive system and their identification for proper design of the drive system. Section 4 explores bifurcation phenomena of the drive system for variation in load torque and controller gain with low speed and high speed operation. In section 5 results for laboratory experiment showing performance of the control circuit for period-1, period-2 and chaotic response of drive system are presented.

## II. SYSTEM DESCRIPTION

Fig. 1 shows schematic of the pwm buck converter fed dc series motor drive system. It is observed from Fig.1, that device switching pulse is generated by comparing the actuating signal  $v_{con}(t)$  with an externally applied sawtooth waveform  $v_{tr}(t)$ . The  $v_{con}(t)$  is produced by a PI controller to which is fed the speed error signal. If  $v_{con}(t)$  be more than sawtooth waveform, switch  $SW$  becomes on and  $D_F$  remains in off state. Otherwise  $SW$  remains off and  $D_F$  starts conducting to free wheel the energy of capacitance and motor inductance. Thus the drive system has two switching modes. These are STATE-1, when  $SW$  is on and  $D_F$  is off and STATE-2 when  $SW$

is off and  $D_F$  becomes turned on. Interestingly each switching state has two operating modes depending on the state of charging or discharging of the filter capacitor  $C$ .

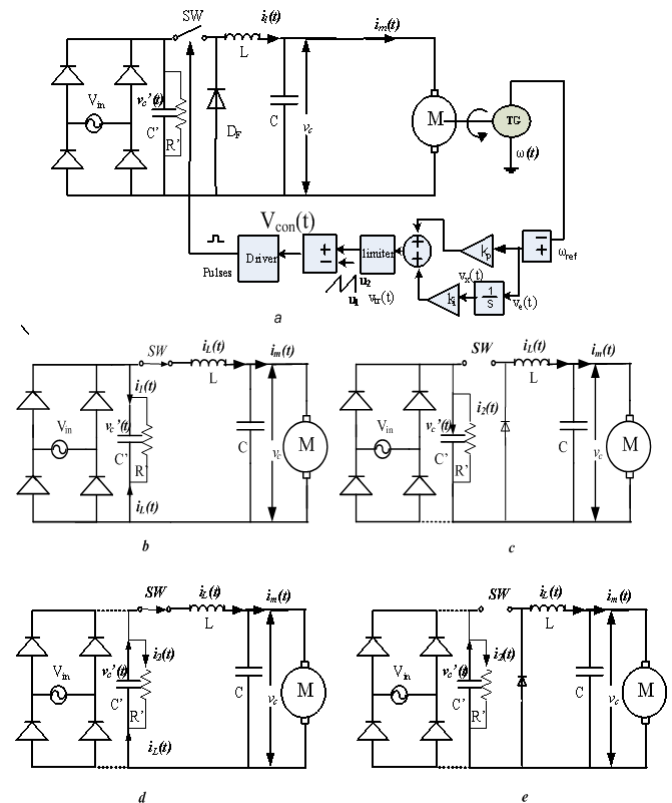


Fig.1: Schematic of PWM buck converter fed dc series motor drive system.

a,b: state of charging and discharging of  $C'$  during state1 respectively.

c,d: state of charging and discharging of  $C'$  during state2 respectively

Systems of differential equations governing the two operating modes during each switching state are stated in 1.1-1.4 with state vector as in 1.0

$$x = [i_L(t) \quad i_m(t) \quad \omega(t) \quad v_c'(t) \quad v_c(t) \quad v_e(t)] \quad 1.0$$

In 1.0, the phase variables are chosen as the inductor current  $i_L(t)$ , motor current  $i_m(t)$ , speed  $\omega(t)$ , voltage of filter capacitor ( $C'$ ) or  $v_c'(t)$ , voltage input to motor armature  $v_c(t)$  and input to the integral controller  $v_e(t)$ . The operating modes are briefly described in the following section.

Mode 1(State-1):  $V_{in}(t) > v_c'(t)$ . During this mode capacitor  $C'$  takes charging current  $i_l(t)$  from the ac supply and simultaneously also discharges by causing  $i_L(t)$  to flow through the LC section as shown in Fig. 1(b). So,  $i_L(t)$  continues to charge the capacitor  $C$  with upper plate positive. ' $C$ ' however discharges through motor armature by sending current  $i_m(t)$ , provided back emf of motor will remain always less than  $v_c(t)$ . This condition is also necessary to ensure motoring mode of the drive system, which demands careful selection of  $C$ .

$$\begin{bmatrix} \frac{di_L(t)}{dt} \\ \frac{di_m(t)}{dt} \\ \frac{d\omega(t)}{dt} \\ \frac{dv_c(t)}{dt} \\ \frac{dv_c(t)}{dt} \\ \frac{dv_c(t)}{dt} \end{bmatrix} = \begin{bmatrix} -\frac{R}{L} & 0 & 0 & \frac{1}{L} & 0 & 0 \\ 0 & -\frac{r}{l} & -\frac{k_e * i_m(t)}{l} & 0 & \frac{1}{l} & 0 \\ 0 & \frac{k_e}{J} i_m(t) & -\frac{B}{J} & 0 & 0 & 0 \\ -\frac{1}{C'} & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{C} & -\frac{1}{C} & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} i_L(t) \\ i_m(t) \\ \omega(t) \\ v_c'(t) \\ v_c(t) \\ v_e(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ -\frac{T_L}{J} \\ \frac{V_{in}(t) * (2 * \pi * 50 * C')}{C'} \\ 0 \\ \omega_{ref}(t) \end{bmatrix} \quad 1.1$$

Mode2 (state-1):  $V_{in}(t) < v_c'(t)$ . In this mode capacitor  $C'$  stops charging from the supply and starts discharging either through the LC section and resistor  $R'$  or only through  $R'$ . However  $C'$  can continue discharge through LC section if  $v_{C'}(t)$  remains more than  $v_c(t)$ . Otherwise SW becomes reverse biased and hence turned off, although gate pulse still exists. Therefore while selecting circuit components value of the capacitors must be properly selected so that never  $v_{C'}(t)$  becomes less than  $v_c(t)$ . The situations are shown in Fig.1. (c)

$$\begin{bmatrix} \frac{di_L(t)}{dt} \\ \frac{di_m(t)}{dt} \\ \frac{d\omega(t)}{dt} \\ \frac{dv_c(t)}{dt} \\ \frac{dv_c(t)}{dt} \\ \frac{dv_c(t)}{dt} \end{bmatrix} = \begin{bmatrix} -\frac{R}{L} & 0 & 0 & \frac{1}{L} & 0 & 0 \\ 0 & -\frac{r}{l} & -\frac{k_e * i_m(t)}{l} & 0 & \frac{1}{l} & 0 \\ 0 & \frac{k_e}{J} i_m(t) & -\frac{B}{J} & 0 & 0 & 0 \\ -\frac{1}{C'} & 0 & 0 & e^{-\left(\frac{t}{R'C}\right)} & 0 & 0 \\ \frac{1}{C} & -\frac{1}{C} & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} i_L(t) \\ i_m(t) \\ \omega(t) \\ v_c'(t) \\ v_c(t) \\ v_e(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ -\frac{T_L}{J} \\ 0 \\ 0 \\ \omega_{ref}(t) \end{bmatrix} \quad 1.2$$

State 2 (mode-1):  $V_{in}(t) > v_c'(t)$  As the switch remains in off state, so  $C'$  can't discharge through LC section and only continues to charge from the supply. The inductor discharges through the freewheeling diode  $D_F$  and  $C$  simultaneously continues to charge by  $i_L(t)$  and discharge through the motor armature.

$$\begin{bmatrix} \frac{di_L(t)}{dt} \\ \frac{di_m(t)}{dt} \\ \frac{d\omega(t)}{dt} \\ \frac{dv_c(t)}{dt} \\ \frac{dv_c(t)}{dt} \\ \frac{dv_c(t)}{dt} \end{bmatrix} = \begin{bmatrix} -\frac{R}{L} & 0 & 0 & 0 & 0 & 0 \\ 0 & -\frac{r}{l} & -\frac{k_e * i_m(t)}{l} & 0 & \frac{1}{l} & 0 \\ 0 & \frac{k_e}{J} i_m(t) & -\frac{B}{J} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{C} & -\frac{1}{C} & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} i_L(t) \\ i_m(t) \\ \omega(t) \\ v_c'(t) \\ v_c(t) \\ v_e(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ -\frac{T_L}{J} \\ \frac{V_{in}(t) * (2 * \pi * 50 * C')}{C'} \\ 0 \\ \omega_{ref}(t) \end{bmatrix} \quad 1.3$$

State 2(mode-2):  $V_{in}(t) < v_c'(t)$ . During this switching state  $C'$  simply discharges through  $R'$  and rest of the circuit operation is same as in mode-1 of state 2.

$$\begin{bmatrix} \frac{di_L(t)}{dt} \\ \frac{di_m(t)}{dt} \\ \frac{d\omega(t)}{dt} \\ \frac{dv_c(t)}{dt} \\ \frac{dv_c(t)}{dt} \\ \frac{dv_c(t)}{dt} \end{bmatrix} = \begin{bmatrix} -\frac{R}{L} & 0 & 0 & \frac{1}{L} & 0 & 0 \\ 0 & -\frac{r}{l} & -\frac{k_e * i_m(t)}{l} & 0 & \frac{1}{l} & 0 \\ 0 & \frac{k_e}{J} i_m(t) & -\frac{B}{J} & 0 & 0 & 0 \\ 0 & 0 & 0 & e^{-\left(\frac{t}{R'C}\right)} & 0 & 0 \\ \frac{1}{C} & -\frac{1}{C} & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} i_L(t) \\ i_m(t) \\ \omega(t) \\ v_c'(t) \\ v_c(t) \\ v_e(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ -\frac{T_L}{J} \\ 0 \\ 0 \\ \omega_{ref}(t) \end{bmatrix} \quad 1.4$$

Thus, order of the drive system is higher than the previous and therefore complexity in system behaviour will be more.

### III. BRIEF NOTE ON SYSTEM BEHAVIOR

For the motor drive system as considered in this paper, gate pulses for the switch SW are generated by comparing the actuating signal with an externally applied sawtooth waveform. Therefore, different dynamic behaviours are described by scaling the ripple frequency in response of motor speed or the switching frequency with that of sawtooth waveform,

$v_{tr}(t)$ . When switching frequency is same as that of  $v_{tr}(t)$ , the drive operates in period-1 mode and values of motor current or speed sampled at starting instant of every ramp cycle will have same value. In the bifurcation diagram, the sampled values plotted along y-axis show a single point or very close distributed points at the corresponding parameter value plotted along the x-axis. This is basically the stroboscopic sampling technique and samples should be taken only after steady state has reached. Similarly higher periodic behaviour means values of stroboscopic samples will be repeated after two (period-2) or three (period-3) or some higher number of consecutive ramp cycles. But chaotic or quasiperiodic evolution implies aperiodic switching or switching frequency is incommensurable from the sawtooth frequency. Stroboscopic samples in either case will be a collection of different values of the samples and such behaviours in bifurcation diagram will be presented as dense collection of points spreaded over a length along the y-axis. No measure in this investigation is taken to differentiate whether the response is chaotic or quasiperiodic, as it aims only to locate the area of load characteristic, where the desired period-1 behaviour is possible.

#### Bifurcation Behavior

A dc series motor drive system is nonlinear in nature and therefore very much sensitive to changes in operating conditions. So variations in the user defined parameters, like

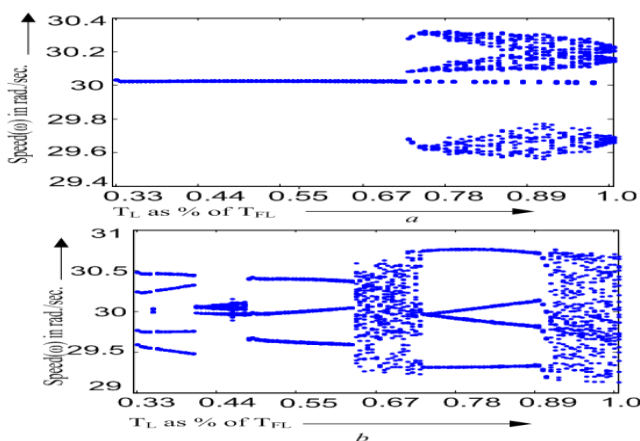


Fig.2: bifurcation in speed response of the drive systems against variations in load torque with  $\omega_{ref} = 30$  rad./sec and  $k_p = 2.1$  (a),  $k_p = 4.1$  (b)

load torque, controller gain, input voltage or speed command results a change in qualitative behaviour of drive system along with average value of motor speed or current. Thus for presenting a study on the load characteristics of the dc series motor drive system, bifurcation behaviours for motor speed are reported with low speed and high speed operation against variations in load torque and controller gain with parameter values as listed in table 1. Numerical simulation is performed on the drive system and bifurcation diagrams are prepared by using stroboscopic sampling method. This investigation aims to find the zones in load characteristics where the desired nominal period-1 behaviour is possible.

Table1: Different parameters of the motor drive system

Parameters of the drive system		
Symbol	Quantity	Value
$R_m$	Total resistance of dc series motor	29.2 $\Omega$
$L_m$	Total inductance of dc series motor	0.0736 H
$k_e$	Back-emf constant	0.039 V/rad./s
$k_t$	Torque constant	0.039 N-m/A
$J$	Moment of inertia	0.015 kg-m <sup>2</sup>
$B$	Viscous friction coefficient	0.001 N-m/rad/s
$C'$	Capacitor filter	0.0022 F
$R$	Resistor for discharging of $C'$	220 $\Omega$
$C$	Power circuit capacitance	0.0001 F
$L$	Power circuit inductance	370mH

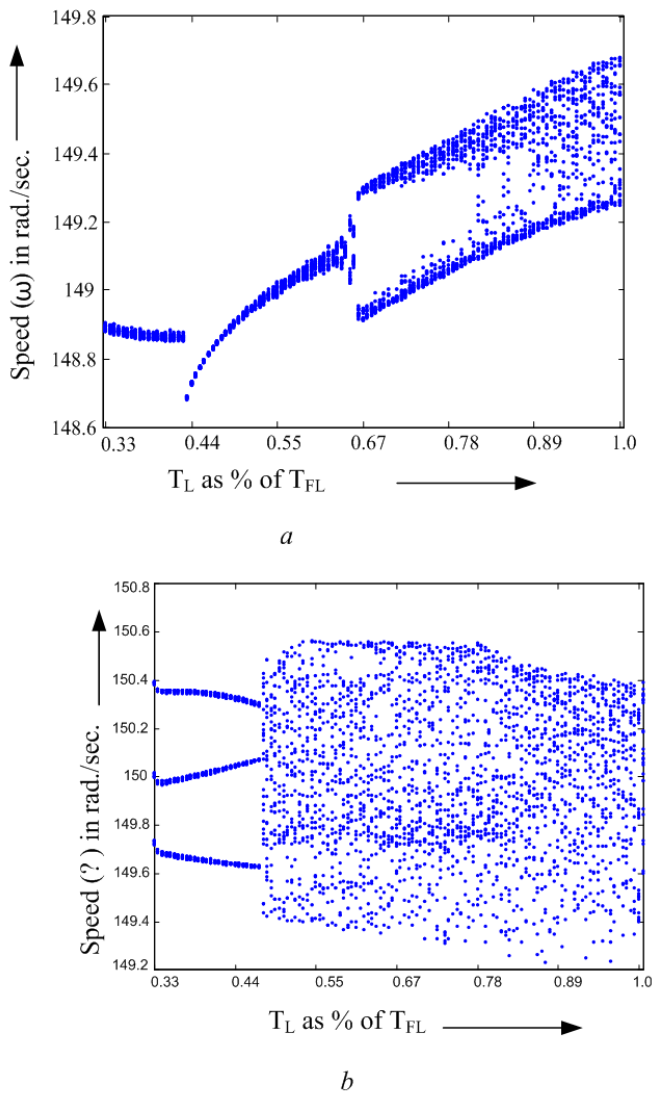


Fig.3: Bifurcation in motor speed with high speed operation

### A. Low Speed Operation

It is observed from Fig. 2, that no period-1 behaviour is found to occur when the gain value for the proportional controller is increased from 2.1 to 4.1 with  $\omega_{ref} = 30 \text{ rad./sec.}$  and  $k_i = 1.1$ . In fact intermittent chaotic windows and higher periodic behaviours are observed with  $k_p = 4.1$  as present in Fig. 2(b). However, Fig. 2(a) shows that, with  $k_p = 2.1$  period 1 response occurs when load torque is increased from 33% to 70% of full load torque,  $T_{FL}$ . Also with further increase in load torque period-1 response is found to occur only at some values of  $T_L$

and at other values the drive system has two band chaotic response

### B. High Speed Operation

With  $k_p = 2.1$  it is observed from Fig. 3(a), that speed response is period-1 for almost the same range of load torque variation as noted with low speed operation. With further increase in load torque motor speed bifurcates to period-2 behavior and finally at higher values of load torque behaves chaotically. However, the period-1 behaviour has ripples and the speed is observed to decrease with  $T_L$ . However in SYSTEM 2 when  $k_p$  is increased from 2.1 to 4.1, the desired period-1 behaviour doesn't occur as shown in Fig. 7(d).

## IV. CONCLUSION

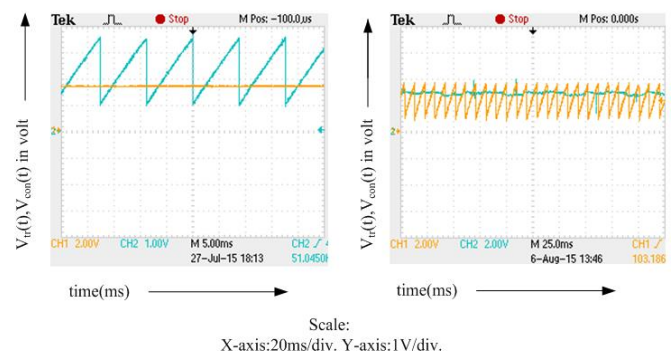


Fig.4:

In this investigation a report is presented on comparative study of the speed response in a dc series motor drive system when operated from two different configurations of dc/dc converters, named as SYSTEM-1 and SYSTEM-2. From the results of numerical simulation and laboratory experiment it has been observed that, over the entire range of operating speed, the speed response in both drive systems are very much sensitive with the gain value of proportional controller. SYSTEM-1 gives period-1 behaviour with very low gain value of proportional controller for a lower range of load torque. However, to have period-1 response in SYSTEM-2 the upper



limit for gain value of proportional controller is relatively wide as compared to SYSTEM-1. Similar to SYSTEM-1, the period1 behaviour in SYSTEM-2 becomes unstable with high values of load torque. This paper also presents an expression for load characteristic in both the drive systems and shows that with increase in load torque, decrease in speed is comparatively more in SYSTEM-2 than in SYSTEM-1.

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