



A Study of Energy of Graphs Using Multiple Regression Technique

S. Perumal¹, Dr. Kavitha², J, Dr. J.Vijayarangam³

¹Assistant Professor, Department of Mathematics, RMK College of Engineering & Technology, Puduvoyal, Tamil Nadu, India

²Associate Professor, Department of Mathematics, New Horizon College Of Engineering , Bangalore, Karnataka, India

³Guest Faculty, BITS Pilani WILP, Chennai, Tamil Nadu, India

ABSTRACT

Energy of a graph is an important aspect of a graph and its applications are growing in a fast pace over the years. Results regarding energy of a graph and its components are one of the happening areas in research. This paper is about one such result using the established and stabilized line of statistical regression. It considers a dataset of five compact graphs K_2 , K_3 , K_4 , K_5 , and their line graphs, collecting a few components of them and regressing their energy using the components.

Keywords : Energy of graph, Line graph, Multiple regression

I. INTRODUCTION

The energy of a graph is the sum of modulus values of the eigen values of its Adjacency matrix. Study of the energy of a graph is an important one in application domains.

It is one of the most referred articles in energy of graphs as it gives you a nice and good discussion about the topic.

A complete graph is a graph in which every possible combination of vertices has an edge connecting them. The line graph $L(G)$ of a graph G is a graph such that there is a one to one correspondence between edges in G and vertices in $L(G)$ and two vertices in $L(G)$ are adjacent if and only if their respective edges share a vertex in G .

It also discusses about energy of graphs. It studies energies of Laplacian graphs and line graphs. It has many results regarding the energy of graphs. It has discussions about hyper energetic and equi-energetic graphs. This is a master of science thesis of a IIT, Bhubaneswar student which has nice discussions about energy of different types of transformed complete graphs.

There are many lines of discussions in literature regarding the energy of a graph but not much yet using a statistical regression one. This paper is addressing that line of discussion. For simplicity sake, we consider complete graphs K_2 , K_3 , K_4 , K_5 and their line graphs, obtain data from them like number of vertices, number of edges and their degrees, the order of its Adjacency matrix, positive inertia, negative and its energy, Fig1.

Table 1 -Components of a graph

	vertices	Edges	Degree of edges	nu+	nu-	o(A(G))	Energy
K2	2	1	1	1	1	2	2
L(K2)	1	0	0	0	0	1	0
K3	3	3	2	3	0	3	4
L(K3)	3	3	2	3	0	3	4
K4	4	6	3	1	3	4	6
L(K4)	6	12	4	1	2	6	8
K5	5	10	4	1	4	5	8
L(K5)	10	30	6	5	5	10	20.1065

Fig 1

Then , use multiple regression technique to first find out which of the selected variables are significant ones to explain the energy of a graph and once it is found out, use multiple regression techniques once again to obtain a relation between the energy of a graph and the significant variables which can explain that better than others.

II. METHODOLOGY

Prepare a dataset consisting of details of a graph like, number of vertices, number of edges and their

degrees, the order of its Adjacency matrix, positive inertia, negative and its energy

Run a multiple regression with energy as dependent variable and all other variables as independent ones and identify the significant variables.

Run a regression or multiple regressions for energy with one or more significant variables from step2 to obtain a relation for energy.

III. RESULT AND ANALYSIS:

The regression run with energy as dependent variable and all others as independent ones yields the result,

Table 2 - Multiple Regression 1 for Energy of graphs

Regression Statistics	
Multiple R	0.999899
R Square	0.999798
Adjusted R Square	0.499293
Standard Error	0.163547
Observations	8

ANOVA					
	<i>df</i>	<i>SS</i>	<i>MS</i>	<i>F</i>	<i>Significance F</i>
Regression	6	264.8319	44.13865	1980.223	0.0171999
Residual	2	0.053495	0.026748		
Total	8	264.8854			

	<i>Coefficient s</i>	<i>Standard Error</i>	<i>t Stat</i>	<i>P-value</i>	<i>Lower 95%</i>	<i>Upper 95%</i>	<i>Lower 95.0%</i>	<i>Upper 95.0%</i>
Intercept	-0.811868995	0.591209202	1.37323	0.303364	-3.35563688	1.73189889	-3.35563688	1.73189889
vertices	0	0	65535	#NUM!	0	0	0	0
Edges	0.254430834	0.096124826	2.64688	0.117999	0.159160912	0.66802258	0.159160912	0.66802258
Degree of edges	0.195749098	0.399651084	0.4898	0.672732	1.915308924	1.523810728	1.915308924	1.523810728
nu+	0.724797924	0.069360038	10.44979	0.009034	0.426365768	1.023230081	0.426365768	1.023230081
nu-	0.660915254	0.126527607	5.223487	0.034751	0.116510899	1.205319609	0.116510899	1.205319609
o(A(G))	0.752477359	0.512098086	1.469401	0.279492	1.450902866	2.955857585	1.450902866	2.955857585

Fig 2

From the Fig2 it is clear that only Positive inertia and negative inertia are found as significant independent variables (bolded P values in the table) for understanding the energy of a graph. The other notable thing from the Fig-2 is that the number of vertices and the number of edges turn out

to be insignificant variables for predicting the energy of a graph which was not in the expected lines. Then, using the two significant independent variables, again run a multiple regression for the energy to obtain,

Table 3 - Multiple Regression 2 for Energy of graphs

SUMMARY OUTPUT	
<i>Regression Statistics</i>	
Multiple R	0.969961182
R Square	0.940824695
Adjusted R Square	0.917154574
Standard Error	1.770574803
Observations	8

ANOVA					
	Df	SS	MS	F	Significance F
Regression	2	249.2107488	124.6054	39.74735	0.000851827
Residual	5	15.67467566	3.134935		
Total	7	264.8854245			

	Coefficients	Standard Error	t Stat	P-value	Lower 95%	Upper 95%	Lower 95.0%	Upper 95.0%
Intercept	-1.095315217	1.076365636	-1.01761	0.35555	-3.86220117	1.671570735	-3.86220117	1.671570735
nu+	1.911209239	0.427966328	4.465793	0.006605	0.811086772	3.011331707	0.811086772	3.011331707
nu-	2.146725543	0.358656614	5.985462	0.001866	1.224769367	3.06868172	1.224769367	3.06868172

Fig 3

From Fig-3, we obtain the regression equation for the energy of a graph in terms of the most significant predictors, the positive inertia and the negative inertia as,

Energy of graph = $-1.09532 + 1.9112 \times \text{Positive inertia} + 2.1467 \times \text{Negative inertia}$

IV. DISCUSSION:

The study has resulted in a concrete mathematical equation connecting the energy of a graph and its inertial values. But, the immediate reaction possibly could be ,

- Is it applicable for a random graph as the graphs employed are complete graphs and their line graphs.
- Can it be used to obtain a much easier way of obtaining energy of a graph using other variables.

V. REFERENCES

- [1]. Statistical Methods, S.P.Gupta, Sultan Chand &sons, 1976
- [2]. Rangaswami Balakrishnan and K Ranganathan. A textbook of graph theory. Springer Science & Business Media, 2012.
- [3]. I. Gutman, The energy of a graph, Ber. Math.-Statist. Sect. Forschungsz. Graz 103 (1978) 1–22.
- [4]. The energy of a graph, Master of Science Dissertation, IIT Bhubaneshwar, 2017
- [5]. Kinkar Ch.Das, Seyed Ahmed Mojallal, Ivan Gutman, On energy of line graphs, Linear Algebra and its applications, 499 (2016) 79-89.
- [6]. R Balakrishnan. The energy of a graph. Linear Algebra and its Applications, 387:287{295, 2004.
- [7]. Ivan Gutman. The energy of a graph: old and new results. In Algebraic combinatorics and applications, pages 196{211. Springer, 2001.
- [8]. I Gutman, Y Hou, HB Walikar, HS Ramane, and PR Hampiholi. No huckel graph is hyperenergetic. Journal of the Serbian Chemical Society, 65(11):799{801, 2000.