# Controlling Two-Legged Mobile Robot <br> Nguyen Xuan Hong 

School of Mechanical Engineering, Hanoi University of Science and Technology, Hanoi, Vietnam

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#### Abstract

Since the appearance of robots, they have brought many benefits, for example: they can work continuously; they can work in harsh and dangerous environments that cannot be accessed by humans. Thanks to their mobility, mobile robots have a wide and flexible working area, especially two-legged mobile robots that can move in bumpy terrains, go up and down stairs or step over obstacles easily. Nowadays, with the increasing development of science, more and more mobile robots are applied and participated in human activities not only in service activities but also in direct coordination with humans. Robot control methods usually come from robot dynamic model and robot motion differential equation, thereby, calculating driving forces based on the deviation of input and output signals to drive motors on joints in order to ensure that robots moves in the desired trajectory. Two-legged mobile robots have a structure of many phases and joints connected together, besides, due to a large number of degrees of freedom, this type of robot is able to operate flexibly and move easily, however, it has a difficulty in dynamic and kinematic modeling, and robot control. Normally, the differential equation of robot motion will have complex quantities and massive formulas. In order to improve the walk of this robot, this study focuses on researching and surveying the problem of kinetics and dynamics and using a control method to control a specific two-legged mobile robot that moves in a cycle of walking.


Keywords : Mobile Robot, Kinematic Robot, Dynamic Robot, Control Robot

## I. INTRODUCTION

Two-legged mobile robots, especially human-like bipedal mobile robots, have been studied and applied for many decades [1-16]. Versions of robots are increasingly developed, with more complex and diverse form. However, there are still many issues that need to be studied further towards creating more
human-like robots, not only in appearance, but also movement, manipulation, communication and intelligence. In addition to creating the right robot structure, robot operation modeling, including motion trajectory design, dynamic and kinematic survey, motion control for smooth and flexible operation, and energy saving is still a topical issue.

Mobile robot researched in this article is bipedal and humanoid type as shown in Figure 1. Each leg of the robot consists of six degrees of freedom for six rotational motions of the joints. The article focuses on surveying control problems with PD control law when there is an assumption that some dynamic parameters are known. The article uses modern software tools such as Matlap, Maple and programming tools to automatically establish dynamic and kinematic equations and control equations for quick results and simulations.

The article presents the next sections with the following layout: Section 2 - Robot kinetics presents the analysis and calculation of kinetics to obtain kinematic equations as well as the motion law of robot, the results are used for further research content. Section 3 - Robot dynamics, presents the construction of robot dynamics models. Section 4 shows a PD + Force control method. Section 5 describes the application of PD + Force controller in controlling robot to perform moving motion. Section 6 presents some simulation results obtained. Section 7 presents conclusion, evaluation of applicability of the proposed controller, and proposals for the next research direction.


Figure 1: Two-legged mobile robot model

## II. ROBOT KINEMATICS

Robot kinematics solves problems of position, velocity, and acceleration, finding relationships between kinematic quantities and robot parameters. Firstly, it is essential to choose the method of dynamic analysis, then, to base on the analytical method to establish equations describing kinematic relations. Then, it is necessary to base on each specific dynamic problem such as forward problem or inverse problem for different algorithms.

With the aim of improving the robot's walking, this study will focus on researching the lower body of the robot, including two legs; and the upper body consisting of the body, head, and arms is considered as a phase, because there is no relative motion to each other among them.

The Denavit - Hetenberg dynamic analysis method is used to analyze the kinetics of the bipedal mobile robot. Assuming that only the moving part of the legs is surveyed, the coordinate systems attached to the robot's leg joints are shown in the figure below.


Figure 2: Diagram of attaching coordinate systems to robot's joints

The coordinate system Ooxoyozo is a fixed coordinate system attached to the ground, the coordinate system Oooxooyoozoo is a coordinate system attached to the body of the robot. On the left leg, the coordinate
 on the dynamic joint axis respectively, the coordinate system $\mathrm{O}_{16 \times \mathrm{X} 16 \mathrm{y} 16 \mathrm{Z} 16}$ is attached to the left foot. Similarly, on the right leg, coordinate systems from O20X20y20Z20 to O25X25y25Z25 are attached on the dynamic joint axis respectively, the coordinate system $\mathrm{O}_{26 \times 26 \mathrm{y} 26 \mathrm{Z} 26}$ is attached to the right foot.

Throught attaching coordinate systems to the joints as shown above, we obtain a table of dynamic parameter corresponding to the two legs of the robot as follows.

TABLE I. Kinematics Parameters of Left Leg

| Link | Frame | $\theta_{i}$ | $\mathrm{d}_{\mathrm{i}}$ | ai | $\alpha_{i}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1-0 | O10x10y $10 z 10$ | 0 | 0 | -a10 | 0 |
| 1-1 | $\mathrm{O}_{11 \mathrm{X} 11 \mathrm{y} 11 \mathrm{z} 11}$ | $\theta_{11}$ | $\mathrm{d}_{11}$ | 0 | $90^{\circ}$ |
| 1-2 | $\mathrm{O}_{12 \mathrm{X} 12 \mathrm{y}{ }_{12 \mathrm{Z}}^{12} \text { }}$ | $\theta_{12}$ | 0 | 0 | $90^{\circ}$ |
| 1-3 | $\mathrm{O}_{13 \mathrm{X} 13 \mathrm{y} 13 \mathrm{z} 13}$ | $\theta_{13}$ | 0 | a13 | 0 |
| 1-4 | $\mathrm{O}_{14 \mathrm{X} 14 \mathrm{y} 14 \mathrm{Z} 14}$ | $\theta_{14}$ | 0 | a14 | 0 |
| 1-5 | O15x15y 15 Z 15 | $\theta_{15}$ | 0 | 0 | $90^{\circ}$ |
| 1-6 | $\mathrm{O}_{16 \mathrm{X} 16 \mathrm{y} 16 \mathrm{Z} 16}$ | $\theta_{16}$ | 0 | a16 | $90^{\circ}$ |

TABLE II. Kinematics Parameters of Left Leg

| Link | Frame | $\theta_{\mathrm{i}}$ | $\mathrm{d}_{\mathrm{i}}$ | $\mathrm{ai}^{\text {i }}$ | $\alpha^{\text {i }}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 2-0 | O20X20y20Z20 | 0 | 0 | a20 | 0 |
| 2-1 | $\mathrm{O}_{21 \mathrm{X} 21 \mathrm{y} 21 \mathrm{Z} 21}$ | $\theta_{21}$ | -d21 | 0 | $90^{\circ}$ |
| 2-2 | $\mathrm{O}_{22 \mathrm{X} 22 \mathrm{y} 22 \mathrm{Z} 22}$ | $\theta_{22}$ | 0 | 0 | $90^{\circ}$ |
| 2-3 | O23x23y23z23 | $\theta_{23}$ | 0 | a23 | 0 |
| 2-4 | $\mathrm{O}_{24 \mathrm{X} 24 \mathrm{y} 24 \mathrm{Z} 24}$ | $\theta_{24}$ | 0 | a24 | 0 |
| 2-5 | O25X25y25Z25 | $\theta_{25}$ | 0 | 0 | $90^{\circ}$ |
| 2-6 | $\mathrm{O}_{26 \mathrm{X} 26 \mathrm{y} 26 \mathrm{Z} 26}$ | $\theta_{26}$ | 0 | a26 | $90^{\circ}$ |

Positions of the robot's body, left foot and right foot $\operatorname{are}\left(x_{B}, y_{B}, z_{B}, \alpha_{B}, \beta_{B}, \eta_{B}\right),\left(x_{L}, y_{L}, z_{L}, \alpha_{L}, \beta_{L}, \eta_{L}\right)$, and ( $x_{R}$, $y \mathrm{y}, \mathrm{zR}, \alpha_{\mathrm{R}}, \beta_{\mathrm{R}}, \eta_{\mathrm{R}}$ ) in which ( $\mathrm{x}, \mathrm{yb}, \mathrm{z}_{\mathrm{B}}$ ) are the coordinates of the body, ( $\mathrm{xL}, \mathrm{yL}_{\mathrm{L}}, \mathrm{zL}^{\mathrm{L}}$ ) are the coordinates of the left foot and ( $\mathrm{xr}, \mathrm{yR}, \mathrm{zR}$ ) are the coordinates of the foot the right in the fixed coordinate system $O_{0} X_{0} Y_{0} Z_{0}$, meanwhile, ( $\alpha_{B}, \beta_{B}, \eta_{B}$ ), ( $\alpha_{L}, \beta_{L}, \eta_{L}$ ), and ( $\alpha_{R}$, $\beta_{\mathrm{R}}, \eta_{\mathrm{R}}$ ) are the Cardan angles that determine the direction of the robot's body, left foot and right foot.

Therefore, the matrix determining the position and direction of the body, left foot, right foot as ${ }^{0} \mathrm{~A}_{00},{ }^{0} \mathrm{~A}_{16}$, ${ }^{0} \mathrm{~A}_{26}$, respectively has the following formula.
${ }^{0} \mathrm{~A}_{00}=\left[\begin{array}{cccc}c_{\mathrm{B}}\left(\alpha_{\mathrm{B}}, \beta_{\mathrm{B}}, \eta_{\mathrm{B}}\right) & c_{\mathrm{B} 12}\left(\alpha_{\mathrm{B}}, \beta_{\mathrm{B}}, \eta_{\mathrm{B}}\right) & c_{\mathrm{B} 13}\left(\alpha_{\mathrm{B}}, \beta_{\mathrm{B}}, \eta_{\mathrm{B}}\right) & \mathrm{x}_{\mathrm{B}} \\ c_{\mathrm{B} 21}\left(\alpha_{\mathrm{B}}, \beta_{\mathrm{B}}, \eta_{\mathrm{B}}\right) & c_{\mathrm{B} 22}\left(\alpha_{B}, \beta_{\mathrm{B}}, \eta_{\mathrm{B}}\right) & c_{\mathrm{B} 23}\left(\alpha_{\mathrm{B}}, \beta_{\mathrm{B}}, \eta_{\mathrm{B}}\right) & \mathrm{y}_{\mathrm{B}} \\ \mathrm{c}_{\mathrm{B} 31}\left(\alpha_{\mathrm{B}}, \beta_{\mathrm{B}}, \eta_{\mathrm{B}}\right) & \mathrm{c}_{\mathrm{B} 32}\left(\alpha_{B}, \beta_{\mathrm{B}}, \eta_{\mathrm{B}}\right) & c_{\mathrm{B} 33}\left(\alpha_{\mathrm{B}}, \beta_{\mathrm{B}}, \eta_{\mathrm{B}}\right) & z_{\mathrm{B}} \\ 0 & 0 & 0 & 1\end{array}\right]$
${ }^{0} \mathrm{~A}_{16}=\left[\begin{array}{cccc}c_{\mathrm{L} 11}\left(\alpha_{\mathrm{L}}, \beta_{\mathrm{L}}, \eta_{\mathrm{L}}\right) & c_{\mathrm{L} 12}\left(\alpha_{\mathrm{L}}, \beta_{\mathrm{L}}, \eta_{\mathrm{L}}\right) & c_{\mathrm{L} 13}\left(\alpha_{\mathrm{L}}, \beta_{\mathrm{L}}, \eta_{\mathrm{L}}\right) & \mathrm{x}_{\mathrm{L}} \\ c_{\mathrm{L} 21}\left(\alpha_{\mathrm{L}}, \beta_{\mathrm{L}}, \eta_{\mathrm{L}}\right) & c_{\mathrm{L} 22}\left(\alpha_{\mathrm{L}}, \beta_{\mathrm{L}}, \eta_{\mathrm{L}}\right) & c_{\mathrm{L}}\left(\alpha_{\mathrm{L}}, \beta_{\mathrm{L}}, \eta_{\mathrm{L}}\right) & \mathrm{y}_{\mathrm{L}} \\ \mathrm{c}_{\mathrm{L} 31}\left(\alpha_{\mathrm{L}}, \beta_{\mathrm{L}}, \eta_{\mathrm{L}}\right) & c_{\mathrm{L} 232}\left(\alpha_{\mathrm{L}}, \beta_{\mathrm{L}}, \eta_{\mathrm{L}}\right) & c_{\mathrm{L} 33}\left(\alpha_{\mathrm{L}}, \beta_{\mathrm{L}}, \eta_{\mathrm{L}}\right) & \mathrm{z}_{\mathrm{L}} \\ 0 & 0 & 0 & 1\end{array}\right]$
${ }^{0} A_{26}=\left[\begin{array}{cccc}c_{\mathrm{R} 11}\left(\alpha_{R}, \beta_{R}, \eta_{R}\right) & c_{R 12}\left(\alpha_{R}, \beta_{R}, \eta_{R}\right) & c_{R 13}\left(\alpha_{R}, \beta_{R}, \eta_{R}\right) & x_{R} \\ c_{R 21}\left(\alpha_{R}, \beta_{R}, \eta_{R}\right) & c_{R 22}\left(\alpha_{R}, \beta_{R}, \eta_{R}\right) & c_{R 23}\left(\alpha_{R}, \beta_{R}, \eta_{R}\right) & y_{R} \\ c_{R 31}\left(\alpha_{R}, \beta_{R}, \eta_{R}\right) & c_{R 22}\left(\alpha_{R}, \beta_{R}, \eta_{R}\right) & c_{R 33}\left(\alpha_{R}, \beta_{R}, \eta_{R}\right) & z_{R} \\ 0 & 0 & 0 & 1\end{array}\right]$

Based on the diagram attaching the above coordinate systems, for each robot leg, there will be a corresponding kinematic sequence. With the left leg, the kinematic sequence will come from the coordinate axis system Oooxooyoozoo and pass
 foot sequence also comes from Oooxooyoozoo and passes from $\mathrm{O}_{20 \mathrm{X} 20 \mathrm{y} 20 \mathrm{Zz} 20}$ to $\mathrm{O}_{26 \mathrm{X} 26 \mathrm{y} 26 \mathrm{Z} 26 \text {. Therefore, the matrix }}$ determining the position and direction of the left and right foot in turn is determined through the corresponding kinematic sequences as follows.

$$
\begin{align*}
& { }^{0} \mathrm{~A}_{16}={ }^{0} \mathrm{~A}_{00}{ }^{00} \mathrm{~A}_{10}{ }^{10} \mathrm{~A}_{11}{ }^{11} \mathrm{~A}_{12}{ }^{12} \mathrm{~A}_{13}{ }^{13} \mathrm{~A}_{14}{ }^{14} \mathrm{~A}_{15}{ }^{15} \mathrm{~A}_{16}={ }^{0} \mathrm{~A}_{00}{ }^{00} \mathrm{~A}_{16}(\mathrm{q})  \tag{4}\\
& { }^{0} \mathrm{~A}_{26}={ }^{0} \mathrm{~A}_{00}{ }^{00} \mathrm{~A}_{20}{ }^{20} \mathrm{~A}_{21}{ }^{21} \mathrm{~A}_{22}{ }^{22} \mathrm{~A}_{23}{ }^{23} \mathrm{~A}_{24}{ }^{24} \mathrm{~A}_{25}{ }^{25} \mathrm{~A}_{26}={ }^{0} \mathrm{~A}_{00}{ }^{00} \mathrm{~A}_{26}(\mathrm{q}) \tag{5}
\end{align*}
$$

In which
${ }^{00} \mathrm{~A}_{10}$ : the transfrom matrix from the coordinate system attached to the body $\mathrm{O}_{00 \mathrm{x} 00 \mathrm{y} 00 \mathrm{zon}}$ to the coordinate system O10X10y10z10.
${ }^{10} \mathrm{~A}_{11}$ : the transfrom matrix from the coordinate system $\mathrm{O}_{10 \mathrm{x} 10 \mathrm{y} 10 \mathrm{Z} 10}$ to the coordinate system O11x11y11z11.
${ }^{11} \mathrm{~A}_{12}$ : the transfrom matrix from the coordinate system $\mathrm{O}_{11 \mathrm{X} 11 \mathrm{y}{ }_{11 \mathrm{Z}} 11}$ to the coordinate system $\mathrm{O}_{12 \mathrm{X} 12 \mathrm{y}}{ }^{12 \mathrm{Z}} 12$.
${ }^{13} \mathrm{~A}_{13}$ : the transfrom matrix from the coordinate system $\mathrm{O}_{12 \mathrm{X} 12 \mathrm{y} 12 \mathrm{Z} 12}$ to the coordinate system $\mathrm{O}_{13 \mathrm{X} 13 \mathrm{y}} 13 \mathrm{Z} 13$.
${ }^{13} \mathrm{~A}_{14}$ : the transfrom matrix from the coordinate system $\mathrm{O}_{13 \mathrm{X} 13 \mathrm{y}{ }_{13 \mathrm{Z}} 13}$ to the coordinate system $\mathrm{O}_{14 \mathrm{X} 14 \mathrm{y}} 14 \mathrm{Z} 14$.
${ }^{14} \mathrm{~A}_{15}$ : the transfrom matrix from the coordinate system $\mathrm{O}_{14 \mathrm{X} 14 \mathrm{y}{ }_{14 \mathrm{Z}} 14}$ to the coordinate system O 15 X 15 y 15 Z 15.
${ }^{15} \mathrm{~A}_{16}$ : the transfrom matrix from the coordinate system $\mathrm{O}_{15 \mathrm{X} 15 \mathrm{y} 15 \mathrm{Z} 15}$ to the coordinate system O16X16y 16 Z 16.
${ }^{00} \mathrm{~A}_{20}$ : là ma trận chuyển từ hệ tọa độ gắn với thân O00x00y00zoo to the coordinate system O20x20y20z20.
${ }^{20} \mathrm{~A}_{21}$ : the transfrom matrix from the coordinate system $\mathrm{O}_{20 \mathrm{X} 20 \mathrm{y} 20 \mathrm{z} 20}$ to the coordinate system $\mathrm{O}_{21 \mathrm{X} 21 \mathrm{y}} \mathbf{2 1 z 2 1}$.
${ }^{21} \mathrm{~A}_{22}$ : the transfrom matrix from the coordinate system $\mathrm{O}_{21 \mathrm{X} 21 \mathrm{y} 21 \mathrm{Z} 21}$ to the coordinate system $\mathrm{O}_{22 \mathrm{X} 22 \mathrm{y} 22 \mathrm{Z} 22 \text {. }}$
${ }^{22} \mathrm{~A}_{23}$ : the transfrom matrix from the coordinate system $\mathrm{O}_{22 \mathrm{X} 22 \mathrm{y} 22 \mathrm{Z} 22}$ to the coordinate system $\mathrm{O}_{23 \mathrm{X} 23 \mathrm{y} 23 \mathrm{Z} 23 \text {. }}$
${ }^{23} \mathrm{~A}_{24}$ : the transfrom matrix from the coordinate system $\mathrm{O}_{23 \mathrm{X} 23 \mathrm{y} 23 \mathrm{Z} 23}$ to the coordinate system $\mathrm{O}_{24 \mathrm{X} 24 \mathrm{y} 24 \mathrm{Z} 24 .}$
${ }^{24} \mathrm{~A}_{24}$ : the transfrom matrix from the coordinate system $\mathrm{O}_{24 \mathrm{X} 24 \mathrm{y} 24 \mathrm{Z} 24}$ to the coordinate system O25X25y25Z25.
${ }^{25} \mathrm{~A}_{26}$ : the transfrom matrix from the coordinate system $\mathrm{O}_{25 \mathrm{X} 25 \mathrm{y} 25 \mathrm{Z} 25}$ to the coordinate system O26X26y26Z26.

Matrices ${ }^{00} \mathrm{~A}_{16}$ and ${ }^{00} \mathrm{~A}_{26}$ are matrices that converted from the coordinate system attached to the robot body to the coordinate systems attached to the left and right foot.

We obtain the system of robot kinematic equations in the matrix form as follows.

$$
\left\{\begin{array}{l}
{ }^{0} A_{16}\left(x_{L}, y_{L}, z_{L}, \alpha_{L}, \beta_{L}, \eta_{L}\right)={ }^{0} A_{00}\left(x_{B}, y_{B}, z_{B}, \alpha_{B}, \beta_{B}, \eta_{B}\right)^{00} A_{16}(q)  \tag{6}\\
{ }^{0} A_{26}\left(x_{R}, y_{R}, z_{R}, \alpha_{R}, \beta_{R}, \eta_{R}\right)={ }^{0} A_{00}\left(x_{B}, y_{B}, z_{B}, \alpha_{B}, \beta_{B}, \eta_{B}\right){ }^{00} A_{26}(q)
\end{array}\right.
$$

It can be abbreviated as follows.

$$
\begin{equation*}
f(p, q)=0 \tag{7}
\end{equation*}
$$

In which

$$
\begin{align*}
& q=\left[q_{11}, q_{12}, q_{13}, q_{14}, q_{15}, q_{16}, q_{21}, q_{22}, q_{23}, q_{24}, q_{25}, q_{26}\right]^{T} \\
& p=\left[x_{B}, y_{B}, z_{B}, \alpha_{B}, \beta_{B}, \eta_{B}, x_{L}, y_{L}, z_{L}, \alpha_{L}, \beta_{L}, \eta_{L}, x_{R}, y_{R}, z_{R}, \alpha_{R}, \beta_{R}, \eta_{R}\right]^{T} \tag{9}
\end{align*}
$$

From the system of kinetic equations (6) or (7), we can determine the position and direction of the left foot and right foot if we know the motion laws of the leg joints and the body; or, if we know the position and direction of the feet and the motion laws of the body, we can find the motion laws of the legs so that the robot body moves according to the given law.

The motion laws of the robots body and feet can be interpolated through real human movements by measurement with using modern image processing methods.

In this article, the motion laws of the robot's body and feet are given to solve the problems of kinetics, dynamics and control. The study surveys the
horizontal motion of the body and the hip joint, so the body motion is considered as the motion of the hip joint because they have the same linear motion and are similar to the body motion. This article surveys the case where the robot moves in the first step when the legs are upright, then the right leg is the pillar and the left foot steps up and moves until the moment just before touching the ground.


Figure 3: Footsteps positions for the first step

The motion laws of the hip joint and foot are shown in Figure 4, the motion trajectory, velocity, and acceleration in time of the hip joint and foot are shown in Figure 5.


Figure 4: Motion trajectory of the robot's hip joint and foot in one step


Figure 5 : Motion trajectory of the hip joint and foot over time

## III.ROBOT DYNAMICS MODELLING

Robot dynamics helps to find the relationship between the applied force or control torque with the characteristic quantities of position and direction of operation phases along with their derivatives such as velocity, acceleration, angle velocity, angle acceleration through dynamical equations.

Robot dynamics equation is written in the matrix form as follows (10).

$$
\begin{equation*}
\mathrm{M}(\mathrm{q}) \ddot{\mathrm{q}}+\mathrm{C}(\mathrm{q}, \dot{\mathrm{q}}) \dot{\mathrm{q}}+\mathrm{G}(\mathrm{q})+\mathrm{Q}=\mathrm{U} \tag{10}
\end{equation*}
$$

In which:
$\mathrm{M}(\mathrm{q})$ is the mass matrix of the robot, calculated according to the following formula.

$$
\begin{equation*}
\mathrm{M}(\mathrm{q})=\left[\sum_{\mathrm{i}=1}^{12}\left(\mathrm{~J}_{\mathrm{Ti}}^{\mathrm{T}} \mathrm{~m}_{\mathrm{i}} \mathrm{~J}_{\mathrm{Ti}}++^{\mathrm{ci}} \mathrm{~J}_{\mathrm{Ri}}^{\mathrm{T}}{ }^{\mathrm{ci}} \Theta_{\mathrm{ci}}{ }^{\mathrm{ci}} \mathrm{~J}_{\mathrm{Ri}}\right)\right]_{(12 \times 12)} \tag{11}
\end{equation*}
$$

$\mathrm{m}_{\mathrm{i}}, \mathrm{JTi}_{\mathrm{Ti}},{ }^{\mathrm{ci}} \mathrm{J}_{\mathrm{Ri}},{ }^{\mathrm{C}} \Theta_{\mathrm{Ci}}$ are the mass, the translational Jacobi matrix, the rotational Jacobi matrix and the inertial matrix of the ith phase, respectively.
$\mathrm{C}(\mathrm{q}, \dot{\mathrm{q}}) \dot{\mathrm{q}}$ : the generalized force vector of the Coriolis and centrifugal inertia forces, calculated by Equation (12).

$$
\begin{align*}
& \mathrm{C}(\mathrm{q}, \dot{\mathrm{q}}) \dot{\mathrm{q}}=\left[\mathrm{c}_{1}, \mathrm{c}_{2}, \ldots, \mathrm{c}_{12}\right]^{\mathrm{T}} ; \quad \mathrm{c}_{\mathrm{j}}=\sum_{\mathrm{k}, \mathrm{l}=1}^{12}(\mathrm{k}, \mathrm{l} ; \mathrm{j}) \dot{\mathrm{q}}_{\mathrm{k}} \dot{\mathrm{q}}_{\mathrm{l}} ; \\
& (\mathrm{k}, \mathrm{l} ; \mathrm{j})=\frac{1}{2}\left(\frac{\partial \mathrm{~m}_{\mathrm{kj}}}{\partial \mathrm{q}_{\mathrm{l}}}+\frac{\partial \mathrm{m}_{\mathrm{lj}}}{\partial \mathrm{q}_{\mathrm{k}}}-\frac{\partial \mathrm{m}_{\mathrm{k} 1}}{\partial \mathrm{q}_{\mathrm{j}}}\right) \tag{12}
\end{align*}
$$

Here, ( $\mathrm{k}, 1 ; \mathrm{j}$ ) the 3 -index Christoffel notation of type 1 ; $\mathrm{m}_{\mathrm{kl}}(\mathrm{k}, \mathrm{l}=1, \ldots, 12)$ are the elements of the matrix $\mathrm{M}(\mathrm{q})$.
$G(q)$ : the generalized force vector of conservative forces.

$$
\begin{equation*}
\mathrm{G}(\mathrm{q})=\left[\mathrm{G}_{1}, \mathrm{G}_{2}, . ., \mathrm{G}_{12}\right]^{\mathrm{T}} ; \quad \mathrm{G}_{\mathrm{j}}=\frac{\partial \Pi}{\partial \mathrm{q}_{\mathrm{j}}} \tag{13}
\end{equation*}
$$

$\Pi$ is the potential energy of the system.
Q: the generalized force vector of the nonconservative forces.

$$
\begin{equation*}
\mathrm{Q}=\left[\mathrm{Q}_{1}, \mathrm{Q}_{2}, . ., \mathrm{Q}_{12}\right]^{\mathrm{T}} \tag{14}
\end{equation*}
$$

U : the generalized force vector of the driving forces/ torques.

$$
\begin{equation*}
\mathrm{U}=\left[\mathrm{U}_{1}, \mathrm{U}_{2}, \ldots, \mathrm{U}_{12}\right]^{\mathrm{T}} ; \quad \mathrm{U}_{\mathrm{i}}=\tau_{\mathrm{i}} \tag{15}
\end{equation*}
$$

Here, $\tau_{\mathrm{i}}$ - the driving force at the joints, with the translational joint $\tau_{i}$ as the force, the torque with the rotational joint.

The non-conservative forces include viscous drag in the dynamic joints, as well as friction at the contact between the robot's feet and the ground below. There are also abnormal disturbances from the environment, etc.

This study will focus on researching the case that the mobile robot moves without being affected by external forces, that is, the generalized force components of impossible forces such as viscous drag or friction are degraded.

$$
\begin{equation*}
\mathrm{Q}=0 \tag{16}
\end{equation*}
$$

Therefore, the robot's dynamical equation is written as (17).

$$
\begin{equation*}
\mathrm{M}(\mathrm{q}) \ddot{\mathrm{q}}+\mathrm{C}(\mathrm{q}, \dot{\mathrm{q}})+\mathrm{G}(\mathrm{q})=\mathrm{U} \tag{17}
\end{equation*}
$$

With assuming that the motion of the hip joint and body is the same, the phases 1-0, 2-0 and the body are considered one phase, and assuming that the robot has straight motion, so the robot's motion is on the vertical plane.

Therefore, the phases $1-1,2-1$ and the body will not have relative motion to each other, so it is considered
as one phase. The pair of phase 1-1 and 1-3, 2-2 and $2-3,1-5$ and 1-6, 2-5 and 2-6 also do not have relative motion to each other, so each pair is considered one phase. Then the coordinates $\mathrm{q}_{11}, \mathrm{q}_{12}, \mathrm{q}_{16}, \mathrm{q}_{21}, \mathrm{q}_{22}, \mathrm{q}_{26}$, and their derivatives of levels are zero. Therefore, the robot's motion is given by the coordinates $\mathrm{q}_{13}, \mathrm{q}_{14}, \mathrm{q}_{15}$, $\mathrm{q}_{23}, \mathrm{q}_{24}, \mathrm{q}_{25}$ and the coordinates of the robot's body.

In order to facilitate the calculation of formulas, we reset the robot's coordinates as follows.
$q=\left[q_{1}, q_{2}, q_{3}, q_{4}, q_{5}, q_{6}, q_{7}\right]$

With:
$\mathrm{q}_{1}$ - angle of rotation of the right foot to the ground $\alpha_{R}$
$\mathrm{q}_{2}$ - angle of rotation of the right leg to the right foot

$$
-q_{13}
$$

$\mathrm{q}_{3}$ - angle of rotation of the right thigh to the right leg

$$
-q_{14}
$$

$\mathrm{q}_{4}$ - angle of rotation of the body to the right thigh q15
$\mathrm{q}_{5}$ - angle of rotation of the left thigh to the body - $\mathrm{q}_{23}$
$\mathrm{q}_{6}$ - angle of rotation of the left leg to the left thigh q2
$\mathrm{q}_{7}$ - angle of rotation of the left foot to the left leg q25

The next section shows the application of the dynamic model established above to the PD + Force controller.

## IV. PD + FORCE CONTROLLER

In this section, the author uses PD + Force control method when assuming that all dynamic quantities in the motion differential equation of the robot's motion are known.

Here, we choose the control law of the form (18):

$$
\begin{equation*}
\mathrm{u}=\mathrm{M}(\mathrm{q}) \tau+\mathrm{C}(\mathrm{q}, \dot{\mathrm{q}}) \dot{\mathrm{q}}+\mathrm{G}(\mathrm{q}) \tag{18}
\end{equation*}
$$

$\tau$ - the selected control signal with a PD structure as shown in (19).

$$
\begin{equation*}
\tau=\ddot{\mathrm{q}}_{\mathrm{d}}-\mathrm{K}_{\mathrm{P}} \mathrm{e}-\mathrm{K}_{\mathrm{D}} \dot{\mathrm{e}} \tag{19}
\end{equation*}
$$

In which:
$\mathrm{e}, \dot{\mathrm{e}}$ - the vectors representing the coordinate deviations and the velocity of the joints at each time the robot moves.

$$
\left\{\begin{array}{l}
\mathrm{e}=\mathrm{q}_{\mathrm{d}}-\mathrm{q}  \tag{20}\\
\dot{\mathrm{e}}=\dot{\mathrm{q}}_{\mathrm{d}}-\dot{\mathrm{q}}
\end{array}\right.
$$

$\mathrm{q}_{\mathrm{d}}, \dot{\mathrm{q}}_{\mathrm{d}}, \ddot{\mathrm{q}}_{\mathrm{d}}$ - vectors that define the desired laws of motion in terms of joint coordinates, velocity and acceleration as required.
$\mathrm{q}, \dot{\mathrm{q}}, \mathrm{q}-$ vectors that define the actually received laws of motion of the robot system in terms of joint coordinates, velocity and acceleration.
$K_{p}, K_{v}$ - diagonal matrix respectively with the elements on the diagonal as positive constants.

$$
\begin{array}{ll}
\mathrm{K}_{\mathrm{P}}=\operatorname{diag}\left\{\mathrm{k}_{\mathrm{P} 1}, \mathrm{k}_{\mathrm{P} 2}, \ldots, \mathrm{k}_{\mathrm{P} 7}\right\} ; & \mathrm{k}_{\mathrm{Pi}}>0 \\
\mathrm{~K}_{\mathrm{v}}=\operatorname{diag}\left\{\mathrm{k}_{\mathrm{v} 1}, \mathrm{k}_{\mathrm{V} 2}, \ldots, \mathrm{k}_{\mathrm{v} 7}\right\} ; & \mathrm{k}_{\mathrm{vi}}>0 \tag{22}
\end{array}
$$

The PD control diagram is shown in the figure below.

The SIMULINK model of the PD controller in the Matlab software is shown in the figure below.


Figure 7: Control model in SIMULINK
Details of blocks inside the SIMULINK control model are described in the following figures.


Figure 8: Input Data block


Figure 9: PD Control block


Figure 10: Subroutine block - Control PD

Figure 6: Diagram of PD control



Figure 11: Subroutine block - Torque


Figure 12: Actuator block - Robot


Figure 13: Results display block - Graph

The results of PD control will be shown in the simulation results in the next section.

## V. SIMULATION RESULTS

Surveying the mobile robot has the following kinematic and dynamic parameters.

TABLE III
Dynamics Parameter of the robot

| Link | Mass <br> $(\mathrm{kg})$ | Length <br> $(\mathrm{m})$ | Possition of the center <br> of gravity in the link <br> coordinate system |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  | $\mathbf{x c i}^{\text {ci }}$ | $\mathbf{y c i}$ | $\mathbf{z}_{\text {ci }}$ |
| Body | 38.8 |  | 0.368695 | 0 | 0 |
| $1-2$ | 1.23 | 0 |  |  |  |
| $1-3$ | 4.63020 | 0.38 | - | 0 | 0.000968 |


|  | 8 |  | 0.217258 |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $1-4$ | 3.25665 <br> 0 | 0.38 | - <br> 0.196234 | 0 | 0.000833 |
| $1-5$ | 0.42 | 0 |  |  |  |
| $1-6$ | 0.80803 <br> 3 | 0.08 | 0.053502 | 0.046994 | 0.001880 |
| $2-2$ | 1.23 | 0 |  |  |  |
| $2-3$ | 4.63020 <br> 8 | 0.38 | - <br> 0.162742 | 0 | 0.000968 |
| $2-4$ | 3.25665 <br> 0 | 0.38 | - <br> 0.183766 | 0 | 0.000833 |
| $2-5$ | 0.42 | 0 |  |  |  |
| $2-6$ | 0.80803 <br> 3 | 0.08 | - | 0.021398 | 0.001880 |
|  |  |  |  |  |  |

TABLE IV
Dynamics Parameter of the robot

| $\begin{gathered} \mathrm{Lin} \\ \mathbf{k} \end{gathered}$ | Ixx | Iyy | Izz | Ixy | Iyz | Izx |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\begin{gathered} \text { Bod } \\ \mathrm{y} \end{gathered}$ | $\begin{array}{\|c} \hline 0.243 \\ 899 \end{array}$ | $\begin{gathered} 2.2114 \\ 55 \end{gathered}$ | $\begin{gathered} 2.0294 \\ 78 \end{gathered}$ | 0 | 0 | 0 |
| 1-2 | $\begin{array}{\|c\|c} \hline 0.002 \\ 785 \\ \hline \end{array}$ | $\begin{gathered} 0.1041 \\ 70 \end{gathered}$ | $\begin{gathered} 0.1046 \\ 67 \end{gathered}$ | 0 | $\begin{gathered} 0.00237 \\ 8 \end{gathered}$ | 0 |
| 1-4 | $\begin{gathered} \hline 0.001 \\ 220 \end{gathered}$ | $\begin{gathered} 0.0469 \\ 51 \end{gathered}$ | $\begin{gathered} 0.0471 \\ 41 \end{gathered}$ | 0 | $\begin{gathered} 0.00166 \\ 8 \end{gathered}$ | 0 |
| 1-5 | $\begin{array}{\|c\|} \hline 0.004 \\ 905 \end{array}$ | $\begin{gathered} 0.0011 \\ 11 \end{gathered}$ | $\begin{gathered} 0.0054 \\ 97 \end{gathered}$ | $0.00085$ | $\begin{gathered} 0.00012 \\ 4 \end{gathered}$ | $\begin{gathered} 0.0001 \\ 09 \end{gathered}$ |
| 2-2 | $\begin{array}{\|c\|} \hline 0.002 \\ 785 \\ \hline \end{array}$ | $\begin{gathered} 0.1041 \\ 70 \end{gathered}$ | $\begin{gathered} 0.1046 \\ 67 \end{gathered}$ | 0 | $\begin{gathered} 0.00237 \\ 8 \end{gathered}$ | 0 |
| 2-4 | $\begin{gathered} \hline 0.001 \\ 220 \end{gathered}$ | $\begin{gathered} 0.0469 \\ 51 \end{gathered}$ | $\begin{gathered} 0.0471 \\ 41 \end{gathered}$ | 0 | $\begin{gathered} 0.00166 \\ 8 \end{gathered}$ | 0 |
| 2-5 | 0.001 | 0.0046 | 0.0054 | 0.00128 | 0.00015 | 0.0000 |
| 2-6 | 376 | 40 | 97 | 7 | 7 | 49 |

Calculation results for controlling the robot to move in the first step are shown in the figure below.


Figure 14: Diagram of the position and the position error of q1, q2


Figure 15: Diagram of the position and the position error of q3, q4


Figure 16: Diagram of the position and the position error of q5, q6


Figure 17: Diagram of the position and the position error of q7

## VI.CONCLUSION

Research object and research objectives are in line with current research trends. The article shows the robot kinetics and dynamics models, the results of the kinematic and dynamical problems have been applied to simulate common controllers for two-legged mobile robots.

The results show that the PD controller has coordinate error asymptotic to zero, so that the PD controller can meet the requirements for accuracy and reliability.

The study of robot motion control in the first step can easily be deployed to the next steps and considered as a premise for other control methods such as intelligent control or optimized control to control mobile robots.

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