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Heat Transfer Flow Over a Heated Stretching Sheet in the Presence of a Magnetic Field

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ABSTRACT

In the present paper on steady flow of a viscous incompressible fluid past a heated stretching sheet with thermal radiation is studied. Magnetic field is applied normal to the flow. With suitable similarity transformations, the momentum and energy equations are reduced to ordinary differential equations. The governing differential equations with corresponding boundary conditions are solved numerically using MATLAB inbuilt solver bvp4c,. Graphical results for velocity and temperature fields, tabular values of Skin-friction and Nusselt numbers are presented and discussed. It is found the temperature of the fluid increases for the increasing values of radiation parameter.

Keywords : Thermal radiation, Steady flow, heated stretching sheet, MATLAB inbuilt solver bvp4c.

I. INTRODUCTION

Most of the problems in scientific phenomena such as heat transfer and diffusion ones functions are non-linear. It is known that except a limited number of these problems, most of them do not have analytical solutions. So, these nonlinear equations should be solved using numerical methods or other analytical methods. There are some restrictions to solve this problem, first we encountered with the nonlinearity of system, and on the other hand this problem is a boundary value problem with infinite boundary values. In this study Homotopy Analysis method (HAM) which was expected by Liao [1-3] has been successfully applied as an analytical method to solve the non-linear problem. This method has been successfully applied to solve many types of nonlinear problems [4-6].

The flow over a heated stretching surface has received enormous attention during the last decades because of several applications in geophysics and energy related engineering problems that includes both metal and polymer sheets. For instance, it happens in the aerodynamic extrusion of polymer sheets, thermal energy storage and recoverable systems, petroleum reservoirs, continuous filament extrusion from a dye, cooling of an infinite metallic plate in a cooling bath and during cooling reduction in both the thickness and width takes place because these strips are sometimes stretched. The temperature distribution, thickness and width reduction are function of draw ratio and stretching distance. In all these technologies, the quality of the final product depends on the rate of heat and mass transfer at the stretching surface. Sakiadis [7] studied first the boundary layer flow over a continuous solid surface moving in its own plane with constant speed. He has proven that in this case the characteristics of the boundary layer is quite different from that of the blassius flow due to entrainment of the ambient fluid.

II. LITERATURE SURVEY

Erickson et al [8] investigated a similar problem in which the transverse velocity at the moving surface is non-zero, taking account of the heat and mass transfer in the boundary layer. Investigations of this type are significant due to their relevance to the problem of a polymer sheet extruded continuously from a dye. A tacit assumption is being made that the sheet is inextensible. In polymer industry it is essential to tackle the boundary layer flow over a stretching sheet, McCormack and Crane [14]. Gupta and Gupta [11] carried out the analysis of momentum, heat and mass transfer in the boundary layer over a stretching sheet, subjected to suction or blowing. Radwan *et al* [15] examined the mass transfer over a stretching surface with variable concentration in a transverse magnetic field. In all the above mentioned cases the viscosity of the fluid was assumed uniform in the flow region. Jang *et al* [13] studied the rate of temperature dependent viscosity in the flow and vortex instability of a heated horizontal free convection boundary layer flow. Ioan Pop *et al* [*12*] analyzed the effect of variable viscosity on flow and heat transfer to a continuous moving flat plate. Lai *et al* [9] studied the effect of variable viscosity on convective heat transfer along a vertical surface in saturated porous medium.

Several investigators have studied different dimensions of the boundary-layer flow of electrically conducting fluid and heat transfer due to stretching sheet in the presence of a transverse magnetic field. The flow of an electrically conducting fluid past stretching sheet under the effect of a magnetic field has drawn the attention of many researchers in view of its broad applications in many engineering problems such as magneto hydrodynamic (MHD) generator, plasma studies, nuclear reactors, oil exploration, geothermal energy extraction, and the boundary layer control in the field of aerodynamics

Samad and Mohebujjaman [17] studied a steady-state two dimensional magneto hydrodynamic heat and mass transfer free convective flow along a vertical stretching sheet in the presence of a magnetic field with heat generation. Fadzilah et al. [18] discussed the steady magneto-hydrodynamic boundary-layer flow and heat transfer of a viscous and electrically conducting fluid over a stretching sheet with an induced magnetic field. Ishak [19,20] studied the steady MHD boundary-layer flow and heat transfer due to a stretching sheet. Mixed convection boundary layer in the stagnation point flow to-wards stretching sheet was studied by Ishak et al. [21]. Lahiri *et al* [16] analyzed the effects of transverse magnetic field on the momentum and heat transfer characteristics in the boundary layer of an incompressible fluid flow over a stretching sheet when viscosity of the fluid depends on temperature.

Actually, many processes in engineering areas occur at high temperature and knowledge of radiation heat transfer becomes very important for the design of the pertinent equipment, Nuclear power plants, and the various propulsion devices for air craft, missiles, satellites and space vehicles are examples. In such cases one has to take into account the effects of radiation. In most of the previous stated studies analytical or perturbation methods were applied to obtain the solution of the problem.

So in the present paper the effect of radiation on steady flow of a viscous incompressible fluid past a heated stretching sheet with temperature dependent viscosity is.studied Using appropriate similarity transformations, the momentum and energy equations are reduced to ordinary differential equations. To obtain the solution, the governing differential equations with corresponding boundary conditions are solved numerically using MATLAB inbuilt solver bvp4c

III. MATHEMATICAL FORMULATION

Steady two-dimensional flow of a viscous incompressible, electrically conducting fluid over a heated stretching sheet is considered. The motion of the fluid is being caused solely by the surface` which is moving horizontally with a speed proportional to the distance from the origin (x=0). Additionally, the viscosity of the fluid is assumed to be dependent on the temperature. A magnetic field of uniform strength is applied normal to the flow. The continuity, momentum and energy equations governing such a flow in the boundary layer when subjected to an magnetic field of strength β_0 (Ferraro *et al* [10]) are written, as

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\frac{1}{\rho}\frac{\partial \mu}{\partial T}\frac{\partial T}{\partial y}\frac{\partial u}{\partial y}+\frac{\mu}{\rho}\frac{\partial^2 u}{\partial y^2}-\frac{\sigma B_0^2}{\rho}u \tag{2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2}-\frac{1}{\rho c_p}\frac{\partial q_r}{\partial y} \tag{3}$$

The radiative flux q_r by using the Rosseland approximation [22], is given by

$$q_r=-\frac{4\sigma^*}{3a_R}\frac{\partial T^4}{\partial y'} \tag{4}$$

It has been assumed that the temperature differences within the flow are sufficiently small. So T^4 may be expressed as a linear function of the temperature *T*. This can be accomplished by expanding T^4 in a Taylor series about T_∞, as follows.

$$\text{Let } f(T)=f(T_\infty)+(T-T_\infty)f'(T_\infty)+\frac{(T-T_\infty)^2}{2!}f''(T_\infty)+\ldots$$

$$\text{where, } f(T)=T^4, \text{then } f'(T)=4T^3,\ f''(T)=12T^2 \tag{5}$$

Simplification of (5), gives

$$T^4=T_\infty^4+4(T-T_\infty)T_\infty^3+12\frac{(T-T_\infty)^2}{2!}T_\infty^2+\ldots$$

In the above expansion, neglecting the higher order terms, we have

$$T^4\approx 4T_\infty^3T-3T_\infty^4 \tag{6}$$

Substituting (6) in (4) and then (4) in (3), we get

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2}+\frac{16\sigma^*T_\infty^3}{3\rho c_p a_R}\frac{\partial^2 T}{\partial y^2} \tag{7}$$

Where u and v are the components of velocity, respectively in the x and y directions, T is the temperature, α is the coefficient of thermal diffusivity, ρ is the fluid density, σ is the conductivity of the fluid and μ is the coefficient of fluid viscosity. The boundary conditions are given by

$$u = cx, \quad v = 0, \quad T = T_{wt} \quad at \quad y = 0 \tag{8}$$

$$u \to 0, \quad T \to T_{\infty} \quad as \ y \to \infty \tag{9}$$

Hence $c(>0)$ is a constant, T_w is the uniform wall temperature and T_{∞} is the free-stream

Temperature. We now introduce the following relations for u, v and θ as

$$u = \frac{\partial \psi}{\partial y}, \qquad v = -\frac{\partial \psi}{\partial x}, \tag{10}$$

$$\theta = \frac{T - T_{\infty}}{T_w - T_{\infty}}, \tag{11}$$

Where ψ is the stream function. The temperature dependent –viscosity is given by (Bird et al 1960)

$$\mu = \mu^* \, e^{a(T_w - T)}, \tag{12}$$

Where μ^* is the reference viscosity and a is a constant.

Using the relations (10), (11) and (12) in the equations (2) to (7), we obtain the following

$$\frac{\partial \psi}{\partial y}\frac{\partial^2 \psi}{\partial x \partial y} - \frac{\partial \psi}{\partial x}\frac{\partial^2 \psi}{\partial y^2} = -A v^* e^{A(T_w - T)} \frac{\partial \theta}{\partial y}\frac{\partial^2 \psi}{\partial y^2} + v^* e^{A(1-\theta)} \frac{\partial^3 \psi}{\partial y^3} - M\frac{\partial \psi}{\partial y} \tag{13}$$

$$\frac{\partial \psi}{\partial y}\frac{\partial \theta}{\partial x} - \frac{\partial \psi}{\partial x}\frac{\partial \theta}{\partial y} = \frac{k}{\rho C_p}\frac{\partial^2 \theta}{\partial y^2} + \frac{16\sigma T_{\infty}^3}{3\rho c_p a_R}\frac{\partial^2 \theta}{\partial y^2} \tag{14}$$

Where $\alpha = \dfrac{k}{\rho C_p}, v^* = \dfrac{\mu^*}{\rho}, M = \dfrac{\sigma B_0^2}{\rho c}$ and c has a dimension (1/ time).

Using the following similarity transformations

$$\psi = (c v^*)^{\frac{1}{2}} x f(\eta), \ \eta = \left(\frac{c}{v^*}\right)^{\frac{1}{2}} y, \tag{15}$$

in to the equations (13) and (14), we get

$$\left(\frac{df}{d\eta}\right)^2 - f\frac{d^2 f}{d\eta^2} + A e^{A(1-\theta)}\frac{d\theta}{d\eta}\frac{d^2 f}{d\eta^2} = e^{A(1-\theta)}\frac{d^3 f}{d\eta^3} - M\frac{df}{d\eta} \tag{16}$$

$$\frac{d^2\theta}{d\eta^2} + \left(1 + \frac{4}{3R}\right)^{-1} \Pr f \frac{d\theta}{d\eta} = 0 \tag{17}$$

where $\Pr = \frac{\mu c_p}{k}$, $R = \frac{k a_R}{4\sigma T_\infty^3}$

The corresponding boundary conditions reduced to

$$f(0)=0, \quad f'(0)=1, \quad \theta(0)=1 \tag{18}$$

$$f'(\infty)=0, \quad \theta(\infty)=0$$

IV. SOLUTION OF THE PROBLEM

Empirical method of equations (16) and (17) cannot be found together with boundary conditions (18). So these coupled non linear ordinary differential equations are solved numerically using MATLAB inbuilt solver bvp4c.

Skin friction coefficient and Nusselt number

As the local skin-friction coefficient is an important characteristic of the boundary layer flow, as defined by

$$C_f = \frac{-2f''(0)}{(cx^2 v^*)^{\frac{1}{2}}}$$

We calculate $-f''(0)$ and $-\theta'(0)$ taking various values of heat source parameter S

V. RESULTS AND DISCUSSION

In order to get the physical understanding of the problem and to investigate the significance of the various physical parameters, a parametric study is conducted. To be realistic, the values of Prandtl number (Pr) are chosen to be Pr = 0.71 and Pr = 7.0, which represent air and water at temperature 20◦C and one atmosphere pressure, respectively... Numerical results for local skin friction and local rate of heat transfer are also presented in table for various values of radiation parameter. The governing Partial differential equations are reduced to nonlinear ordinary differential equations by applying the similarity transformation. Calculations have been carried out by the MATLAB inbuilt solver for changed values of the non-dimensional parameters Figures (1) show the effect of radiation parameter R on velocity and temperature field respectively. It is obvious from figures that, the thermal radiation leads to reduce the temperature of the fluid. This due to the fact that an increase in the thermal radiation leads to reduce in the rate of radiative heat, transferred to the fluid. So it causes a reduce in kinetic energy of the fluid particles. This result leads to decrease in the temperature of the fluid. Fig (2) displays the effect of magnetic parameter M on velocity field u in the presence of radiation Magnetic parameter M describes the ratio of electromagnetic force to the viscous force. It is observed from the figure that the velocity decreases an increase in Magnetic parameter. This is due to fact that the interaction of the magnetic field with an electrically conducting fluid produces a body force known as Lorentz force, which plays the role of a resistive type force on the velocity and this force acts against to the fluid flow when the magnetic field is applied perpendicular to it. Fig (3) shows the effect of Pr in the presence of radiation on temperature profile. It is observed that the presence of heavier Prandtl number fluid is found to decelerate temperature profile. This is owing to the reality that a fluid with high Prandtl number has a relatively low thermal conductivity which results in the reduction of the thermal boundary layer. Further, it is interesting to note that temperature of the fluid decreases in the presence of radiation.

The impact of the physical parameter i.e. radiation parameter R on local skin friction and local rate of heat transfer is shown in table 1 for the case of a fixed

Prandtl number Pr=0.71 and temperature dependent viscosity $A=1$. The local rate of heat transfer and the local skin friction coefficient i.e. $-\theta'(0)$ and $-f''(0)$ enhance as the value of R increases. This implies $\theta'(0)$ and $f''(0)$ reduces for the growing values of radiation parameter R.

VI.CONCLUSION

- The temperature of the fluid decrease in the presence of thermal radiation i
- The local rate of heat transfer and the local skin friction coefficient i.e. $-\theta'(0)$ **and** $-f''(0)$ increase with the increasing values of radiation parameter R.

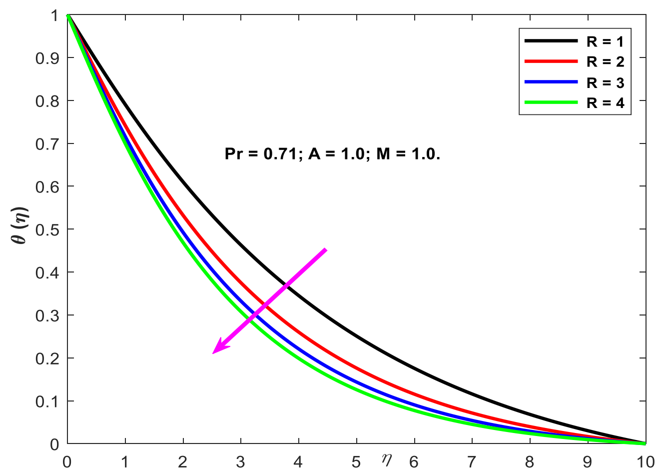

Fig 1: Effect of Radiation on temperature field **θ**

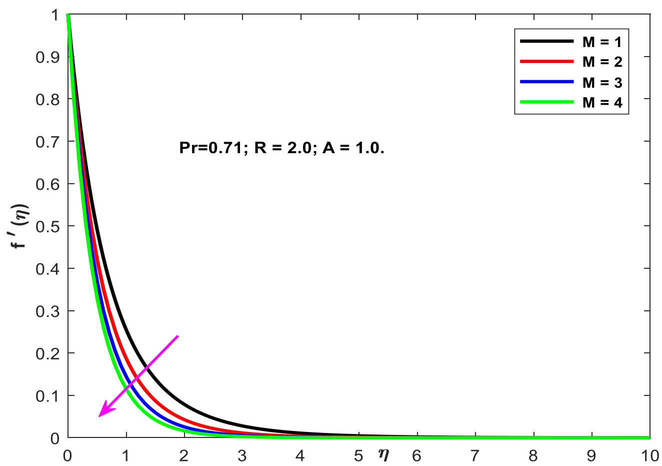

Fig 2: Effect of Magnetic parameter M on velocity field

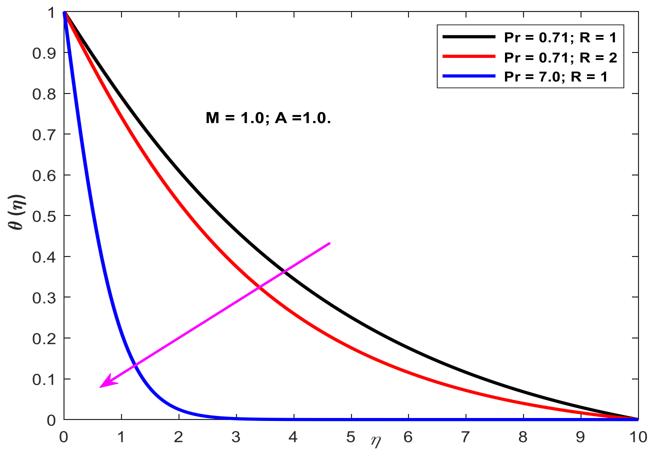

Fig 3: Effect of Prandtl number in the presence of radiation on temperature field (A=1.0, M=1.0 and Pr =0.71)

Table 1 : Skin-friction and Nusselt umber

	Bvp4c	
R	$-f''(0)$	$-\theta'(0)$
1.0	1.464848	0.217797
2.0	1.477736	0.272407
3.0	1.484768	0.302098

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