

Thermodynamic and Dynamic Analysis of Four-Cylinder Crankshaft

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ABSTRACT

In this paper, dynamic analysis of four-cylinder crankshaft has been studied using theoretical and numerical methods. Crankshaft is large volume production component with a complex geometry in the Internal Combustion (I.C) Engine. This converts the reciprocating displacement of the piston into a rotary motion of the crank. The objective of this paper is to analyze the kinematics and dynamics of the crankshaft and compare the theoretical results with Solidworks Motion results. Kinematics is the study of motion without regard for the forces that cause the motion. The kinematic parameter of the mechanism is: linear displacements, linear velocity and linear acceleration of the piston. A kinematic analysis is conducted before dynamic behavior of the mechanism can be simulated properly. Dynamic analysis is the study of motion in response to externally applied loads. The dynamic behavior of a mechanism is governed by Newton's second laws of motion. The main target of dynamics analysis is to compute the values of inertia forces. The calculated values and the simulation results of inertia forces are nearly the same. The maximum percentage error is within the acceptable limit. From the result of Comsol simulation, the output torque of engine at 2000 rpm is approximately 180 N-m whereas the manufacturer specification data for output torque at 2000 rpm is 192 N-m. So, the percentage error between simulation result and manufacturer specification data is 6.25 %.

Keywords: Acceleration, Crankshaft, Displacements, Dynamic Analysis, Velocity.

I. INTRODUCTION

Crankshaft is one of the most important moving parts in internal combustion engine. Crankshaft is a large component with a complex geometry in the engine, which converts the reciprocating displacement of the piston to a rotary motion with a four link mechanism. Crankshaft must be strong enough to take the downward force of the power stroke without excessive bending. So the reliability and life of the internal combustion engine depend on the strength of the crankshaft largely. In addition, the linear displacement of an engine is not smooth; as the displacement is caused by the combustion chamber therefore the displacement has sudden shocks. The concept of using crankshaft is to change these sudden displacements to as smooth rotary output, which

is the input to many devices such as generators, pumps and compressors. It should also be stated that the use of a flywheel helps in smoothing the shocks. Crankshaft experiences large forces from gas combustion. This force is applied to the top of the piston and since the connecting rod connects the piston to the crankshaft, the force will be transmitted to the crankshaft.

II. SPECIFICATION OF ENGINE

The crankshaft numerically analyzed in this paper is modeled on the basic of the following specification. Technical Specifications of Wai Mandalay light truck diesel engines are described as follows.

The rated power, $BHP = 91 \text{ hp}$

The crankshaft speed,	n	= 4000 rpm
The compression ratio,	r	= 22:1
Number of cylinders,	i	= 4
Calorific heating value,	CV	= 42.5 MJ/kg
The temperature of residual gases, T_{res}		= 700 K
Displacement volume		= 2184 cm ³
Torque		= 192N-m
		at 2000 rpm
Bore		= 86 mm
Stroke		= 94 m

TABLE. I THE MAIN DIMENSION OF CRANKSHAFT [6]

Physical Parameters	Values	Units
Diameter of Crankpin	50.5	mm
Length of Crankpin	28	mm
Diameter of Main Journal	57	mm
Length of Main Journal	28	mm
Width of Crank Web	110	mm
Thickness of Crank Web	15	mm

The engine assembly consists of a crankshaft, a flywheel, and three identical sets of cylinder, piston and connecting rod. The flywheel is mounted on the crankshaft and this crankshaft-flywheel assembly is supported by journal bearings at both ends. All the cylinders are fixed, while the other components are free to move in space. The main dimensions of crankshaft are shown in Table. I. Fig.1 shows the geometry model of four-cylinder crankshaft.

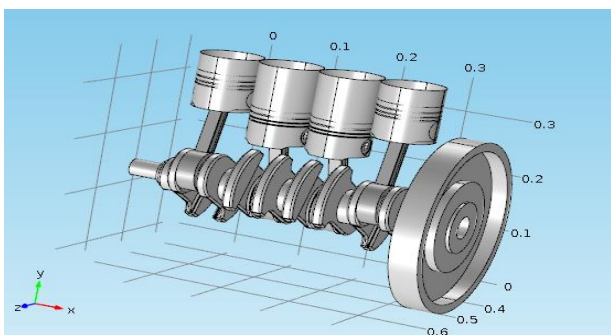


Figure 1: Geometry Model of Four-cylinder Crankshaft

III. THERMAL CALCULATION

The pressure and temperature at the end of admission may be calculated by using the following equations (1) and (2). [7]

$$P_a = P_0 - (\beta^2 + \xi_{is}) \frac{V_{is}^2}{2} \times \rho_0 \quad (1)$$

$$T_a = \frac{(T_0 + \Delta T) + \gamma_{res} T_{res}}{(1 + \gamma_{res})} \quad (2)$$

The pressure and temperature at the end of compression may be calculated by using equations (3) and (4). [7]

$$P_c = P_a \times r^{n_1} \quad (3)$$

$$T_c = T_a \times r^{n_1 - 1} \quad (4)$$

The pressure and temperature at the end of combustion may be calculated by using the following equations (5) and (6). [7]

$$\frac{\xi_z \times CV}{M_1(1 + \gamma_{res})} + \frac{U_c + \gamma_{res} U_c''}{1 + \gamma_{res}} + 8.314 \epsilon T_{comp} = \mu_{act} (U_z'' + 8.314 T_z) \quad (5)$$

$$P_z = \epsilon P_c \quad (6)$$

The pressure and temperature at the end of admission may be calculated by using the following equations (7) and (8). [7]

$$T_e = \frac{T_z}{\delta^{n_2 - 1}} \quad (7)$$

$$P_e = \frac{P_z}{\delta^{n_2}} \quad (8)$$

Parameters obtained from thermal calculation of engine are described in Table.II.

TABLE. II
CALCULATED RESULTS

Parameters	Pressure (MN/m ²)	Temperature(K)
Residual gas	0.1216	700
Admission	0.0915	319.4175
Compression	5.4128	858.8829
Combustion	9.743	2145
Expansion	0.2996	1001.4747

IV. CONSTRUCTION OF INDICATOR DIAGRAM

$$20 \quad P(\text{MN/m}^2)$$

$$19$$

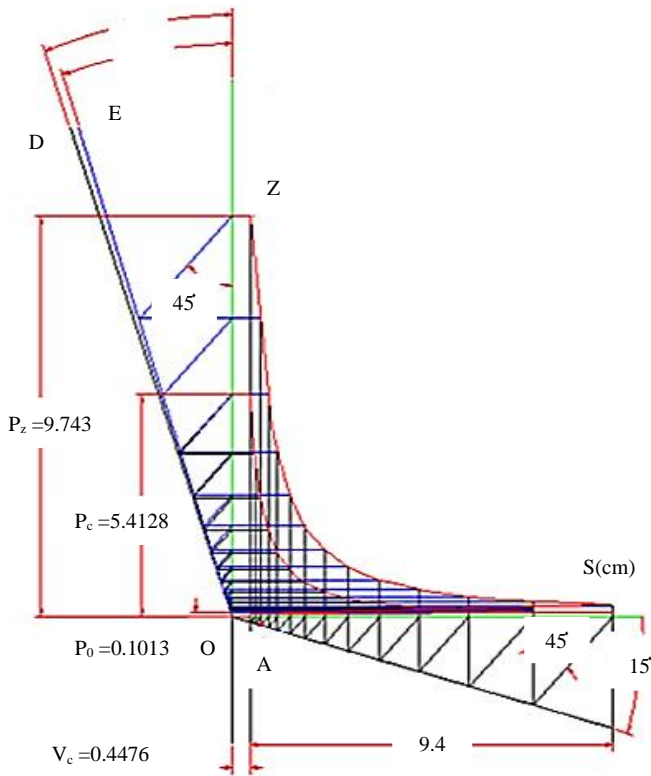


Figure 2: Indicator Diagram Drawn in Auto CAD

Indicator diagram may be drawn by using the parameters obtained from thermal calculation. Pressure can be located on vertical axis, and the stroke of piston can also be located on the horizontal axis. Clearance volume line can be drawn by the formula;

$$V_c = OA = \left(\frac{AB}{r-1} \right) \quad (9)$$

where, AB = stroke length,
r = compression ratio

V. DYNAMICS ANALYSIS OF CRANKSHAFT

The balance of this part is devoted to an analysis of the dynamics of crankshaft. To simplify this work, it is assumed that the engine is running at constant crankshaft speed and gravitational force and friction forces can be neglected in comparison to dynamic force effects. The forces on a crank gear are divided into: the forces of gas pressure in the cylinder, the forces of inertia of the moving parts in the mechanism, and the inertia and centrifugal forces of the rotating parts. The gas pressure can be read from the indicator diagram. The following notation is used in Fig.3 [8].

ϕ = crank angle,

β = angle between the connecting rod and the cylinder axis,

ω = angular velocity of crankshaft,

S = piston stroke,

L = connecting rod length,

R = crank radius,

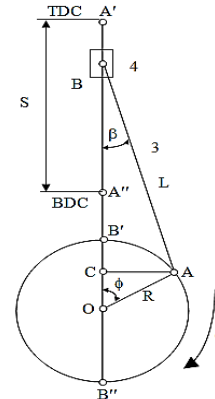


Figure 3: Central Crank Gear

The displacements, velocity and acceleration of the piston are obtained through equations (10) to (13).

$$x = L - \frac{R^2}{4L} + R(\cos\phi + \frac{R}{4L} \cos 2\phi) \quad (10)$$

$$\lambda = \frac{R}{L} \quad (11)$$

$$v = -R\omega \left[\sin\phi + \frac{\lambda}{2} \sin 2\phi \right] \quad (12)$$

$$a = -R\omega^2 [\cos\phi + \lambda \cos 2\phi] \quad (13)$$

$$\text{The angular velocity, } \omega = \frac{2 \times \pi \times N}{60} \quad (14)$$

$$\text{Connecting rod angle, } \beta = \sin^{-1} (R \sin \phi / L) \quad (15)$$

The maximum and minimum velocities of piston from the calculated results are 19.91163 m/s at 0.0125 s and -19.6873 m/s at 0.00375 s and they are shown in Fig. 4.

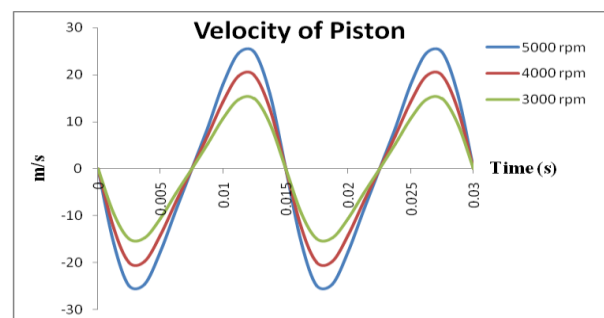


Figure 4 : Velocity of Piston at Various Speed

Fig.5 shows the acceleration of piston at 4000 rpm. In this figure, the maximum and minimum acceleration of piston from the calculated results are 5757.516 m/s^2 at 0.00875 s and $-11015.1041 \text{ m/s}^2$ at 0.015 s .

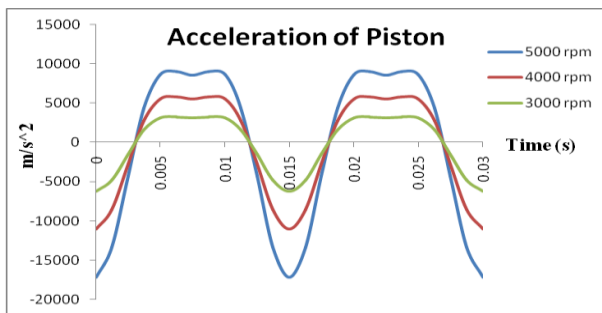


Figure 5: Acceleration of Piston at Various Speed

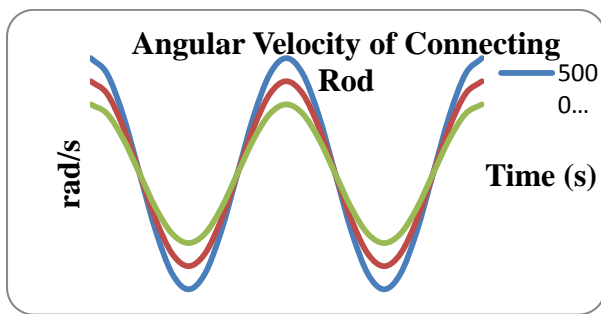


Figure 6: Angular Velocity of Connecting Rod at Various Speed

The angular velocity of connecting rod can be calculated by using equation (16). Fig.6 shows the angular velocity of connecting rod. In this figure, the maximum and minimum angular velocities of connecting rod from the calculated results are $8057.1441 \text{ deg/sec}$ at 0.015 s and $-8057.1441 \text{ deg/sec}$ at 0.0075 s .

$$\omega_{\text{rod}} = \omega \frac{R}{L} \cos \phi \quad (16)$$

The angular acceleration of connecting rod can be calculated by using equation (17). Fig.7 shows the angular acceleration of connecting rod. In this figure, the maximum and minimum angular acceleration of connecting rod from the calculated results are $3374968.48 \text{ deg/sec}^2$ at 0.00375 s and $-3374968.48 \text{ deg/sec}^2$ at 0.01125 s .

$$\alpha_{\text{rod}} = -\omega^2 \frac{R}{L} \sin \phi \quad (17)$$

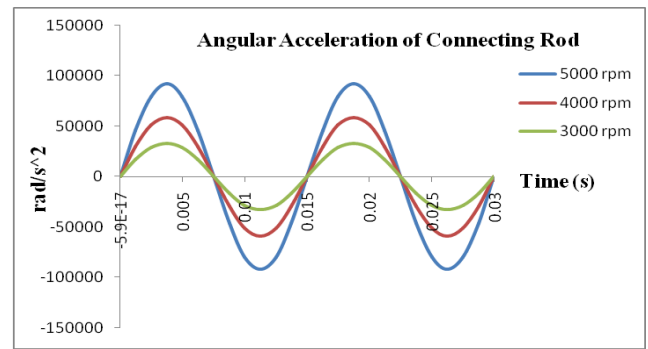


Figure 7: Angular Acceleration of Connecting Rod at Various Speed

In analyzing the inertia forces due to the connecting rod of an engine, it is often convenient to concentrate a portion of the mass at the crankpin A and the remaining portion at the piston pin B as in Fig.8.

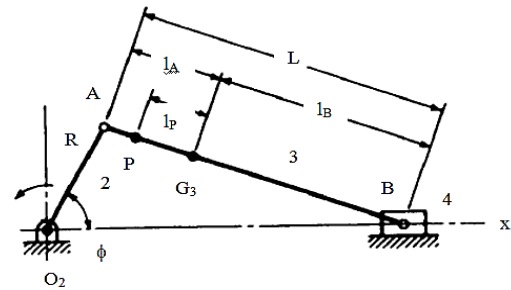


Figure 8: Equivalent Masses for Connecting Rod

Mass of connecting rod at crankpin A,

$$m_{3A} = \frac{m_3 l_B}{L} \quad (18)$$

Mass of connecting rod at piston pin B,

$$m_{3B} = \frac{m_3 l_A}{L} \quad (19)$$

Equivalent mass of crank at the crankpin (point A),

$$m_{2A} = m_2 \frac{r_G}{R} \quad (20)$$

The rotating mass at crankpin (point A),

$$m_A = m_{2A} + m_{3A} \quad (21)$$

The reciprocating mass at piston pin (point B),

$$m_B = m_{3B} + m_4 \quad (22)$$

Fig.9 shows the equivalent masses for crank. In the inertia force analysis, simplification is obtained by locating m_{2A} at the crankpin.

$$m_2 r_G = m_{2A} R \quad (23)$$

$$m_{2A} = m_2 \frac{r_G}{R} \quad (24)$$

where,

m_2 = mass of crank,

m_4 = mass of piston,

m_{2A} = equivalent mass of crank at the crankpin,

r_G = distance from the axis of rotation.

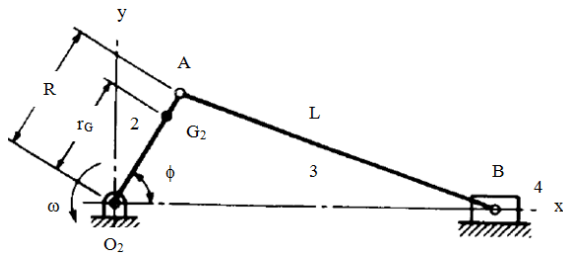


Figure 9: Equivalent Masses for Crank

The inertia force of rotating parts and reciprocating parts are shown in Fig.10. The inertia force of rotating parts can be calculated by using equation (25).

$$-m_A a_A = m_A (R \alpha \sin \phi + R \omega^2 \cos \phi) i + m_A (-R \alpha \cos \phi + R \omega^2 \sin \phi) j \quad (25)$$

Since the analysis is usually made at constant angular velocity ($\alpha = 0$),

$$-m_A a_A = m_A R \omega^2 \cos \phi i + m_A R \omega^2 \sin \phi j \quad (26)$$

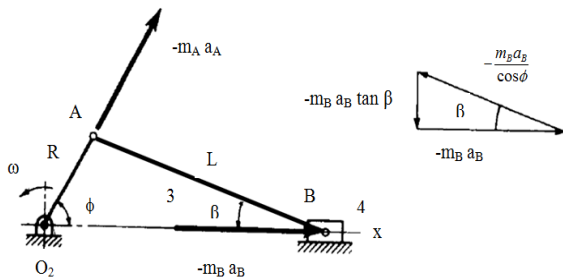


Figure 10: Inertia Forces

Fig.11 shows the inertia forces of reciprocating parts at 4000 rpm. The inertia force of reciprocating parts can be calculated by using equation (27). The maximum inertia force is 10505.1 N and the minimum inertia force is -5367.88 N.

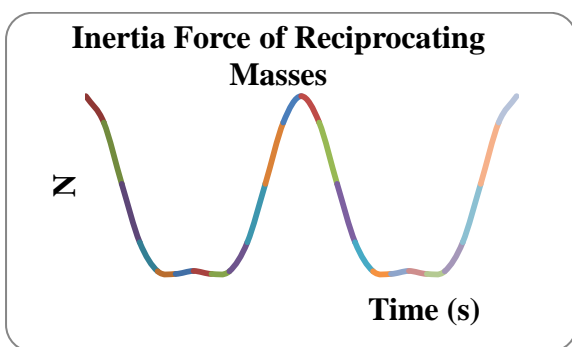


Figure 11: Inertia Forces of Reciprocating Masses at 4000 rpm

$$-m_B a_B = m_B R \omega^2 \left[\cos \phi + \frac{R}{L} \cos 2\phi \right] i \quad (27)$$

The torque delivered by the crankshaft to the load is called the crankshaft torque and can be calculated by the following equation. Fig. 12 shows the torque delivered by the crankshaft. From this figure, the maximum torque is 350 N-m and minimum torque is -250 N-m in two revolution.

$$T_{21} = [(m_{3B} + m_4) a_B + F] \tan \beta \times x k \quad (28)$$

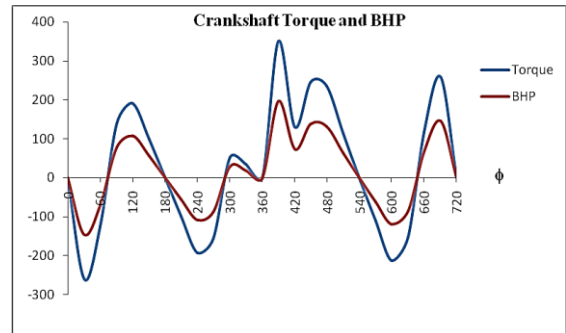


Figure 12: Crankshaft Torque at 4000 rpm

VI. SIMULATION FOR CRANKSHAFT

A. Combustion Pressure Analysis

In the combustion pressure analysis, the pressure variation in the combustion chamber due to the compression and combustion of air-fuel mixture is determined. This analysis is performed using the heat transfer and coefficient form partial differential equations (PDEs) physics interface. In this analysis, the pressure variation in one of the cylinders is determined during one full revolution of a crankshaft and the geometry modeled for combustion pressure analysis is shown in Fig.13.

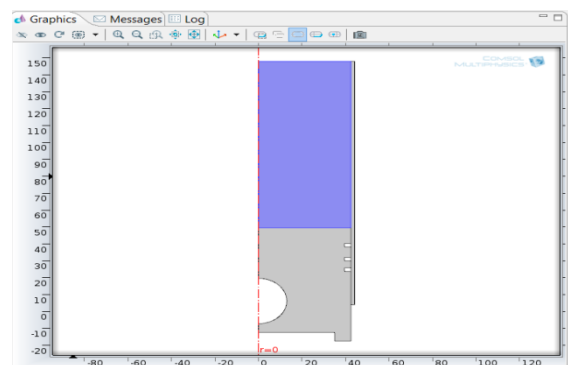


Figure 13: Axisymmetric View of Cylinder Assembly

The P-V diagram for one of the cylinders of the engine, as computed from the thermodynamic analysis, is shown in Fig.14. The pressure at the end of compression is 5.3 MN/m². From theoretical calculation, the pressure at the

end of compression is 5.4128 MN/m^2 . The pressure at the end of combustion is 9.6 MN/m^2 by simulation whereas the pressure at the end of combustion is 9.743 MN/m^2 by theoretical calculation.

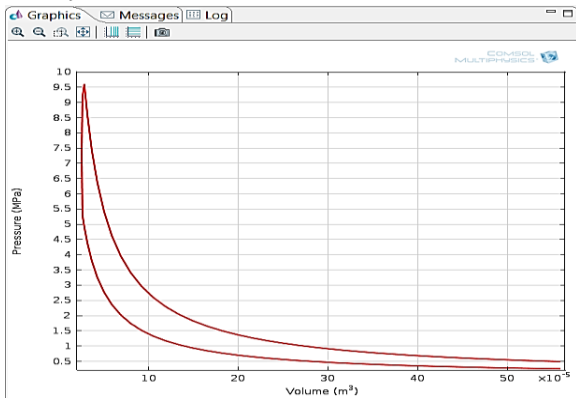


Figure 14: P-V Diagram for One Cylinder

Fig.15 shows the variation of cylinder pressure with the crankshaft rotation. The pressure at the end of combustion is 9.6 MN/m^2 . The curve is exported and used to prescribe the pressure on the top surfaces of the piston in the multibody dynamics analysis.

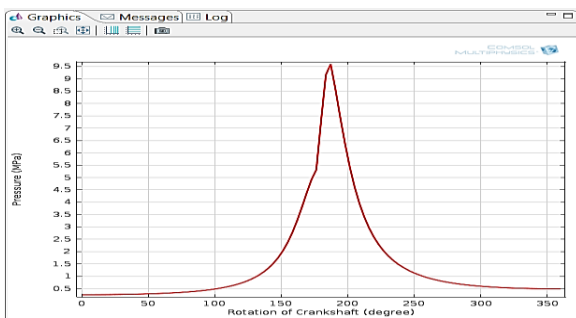


Figure 15: Cylinder Pressure with Crank Rotation

B. Multibody Dynamics Analysis

The pressure variation obtained from the thermodynamic analysis, with appropriate phase difference, is applied on the top surfaces of each piston.

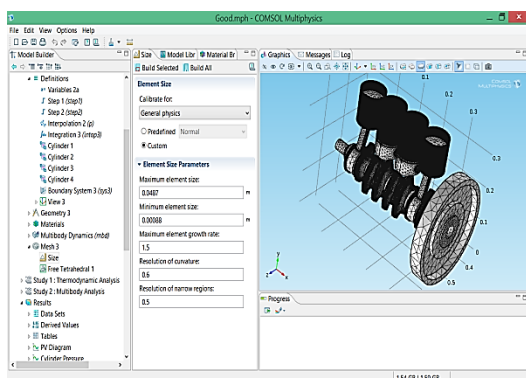


Figure 16: Geometry of Crankshaft with Meshing

Cast Alloy steel, Structural steel and Aluminum 6063 T-83 are chosen for the material selection as shown in Figure 6.6. The first material is used for the crankshaft, flywheel and cylinder walls. The second material is used for connecting rods and the third material is used for pistons. To get the accurate simulation result, the mesh elements have to be fine enough. Fig.16 shows the geometry of four-cylinder crankshaft with the meshing. From the mesh study statement, there are 33 domain elements, 1028 boundary elements, 2156 edge elements and 1250 vertices.

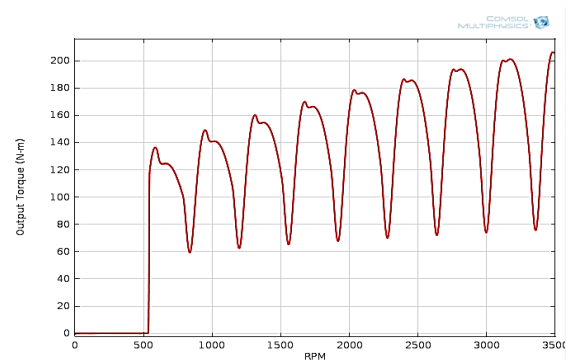


Figure 17: Output Torque of Engine

Fig.17 shows output torque of engine. From this figure, the output torque of engine at 2000 rpm is nearly 180 N-m. The manufacturer specification data for output torque at 2000 rpm is 192 N-m. So, the deviation between simulation result and manufacturer specification data is 6.25 %.

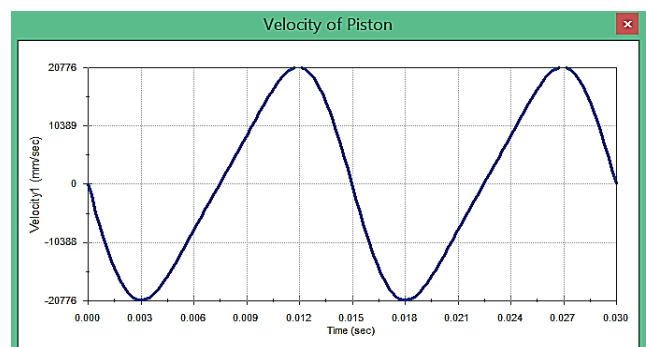


Figure 18: Velocity of Piston

Fig.18 shows the velocity of piston at 4000 rpm. The maximum velocity of piston is 20.7786 m/s at 0.0119 s. The minimum velocity of piston is -20.7695 m/s at 0.0031 s. The maximum and minimum velocities of piston from the calculated results are 19.91163 m/s at 0.0125 s and -19.6873 m/s at 0.00375 s and they are shown in Fig.4. So, the calculated result and the

simulation result are nearly the same in magnitude and direction.

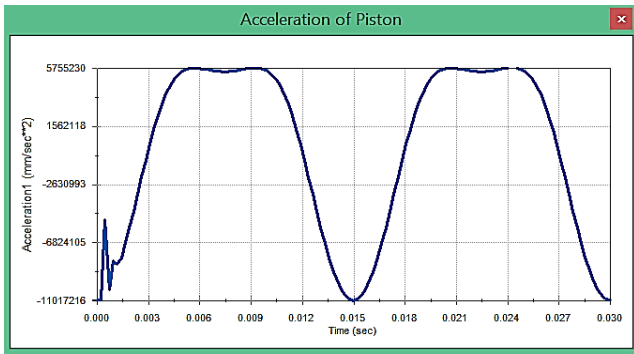


Figure 19: Acceleration of Piston

Fig.19 shows the acceleration of piston at 4000 rpm. In this figure, the maximum acceleration of piston is 5755.2297 m/s^2 at 0.0093 s. The minimum acceleration of piston is $-11017.2164 \text{ m/s}^2$ at 0.015 s. The maximum and minimum acceleration of piston from the calculated results are 5757.516 m/s^2 at 0.00875 s and $-11015.1041 \text{ m/s}^2$ at 0.015 s and shown in Fig.5. So, the calculated result and the simulation result are nearly the same in magnitude and direction.

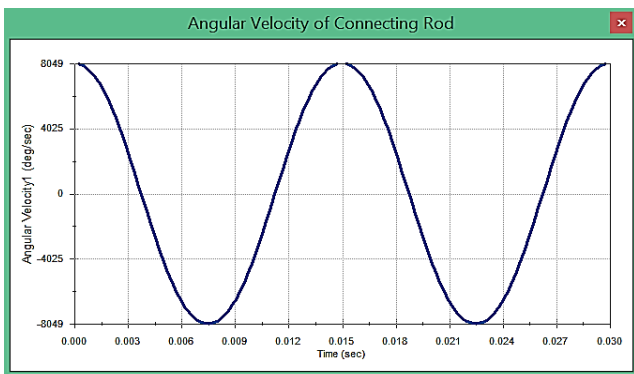


Figure 20: Angular Velocity of Connecting Rod

Fig.20 shows the angular velocity of connecting rod. In this figure, the maximum angular velocity of connecting rod is $8057.1445 \text{ deg/sec}$ at 0.015 s. The minimum angular velocity of connecting rod is $-8048.2645 \text{ deg/sec}$ at 0.0074 s. The maximum and minimum angular velocities of connecting rod from the calculated results are $8057.1441 \text{ deg/sec}$ at 0.015 s and $-8057.1441 \text{ deg/sec}$ at 0.0075 s. So, the calculated and simulation results are nearly the same in magnitude and direction.

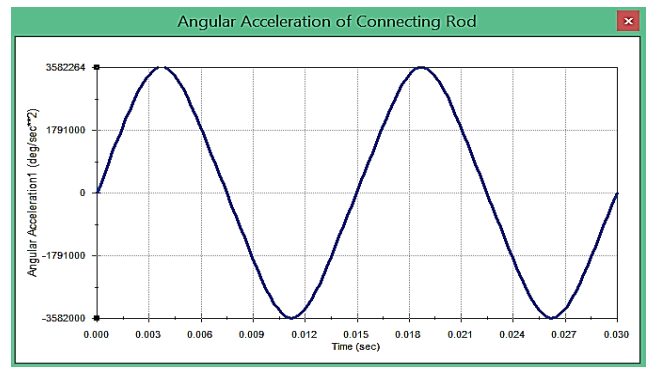


Figure 21: Angular Acceleration of Connecting Rod

Fig.21 shows the angular acceleration of connecting rod. In this figure, the maximum angular acceleration of connecting rod is $3582264.3432 \text{ deg/sec}^2$ at 0.0038 s. The minimum angular acceleration of connecting rod is $-3582711.6177 \text{ deg/sec}^2$ at 0.0112 s.

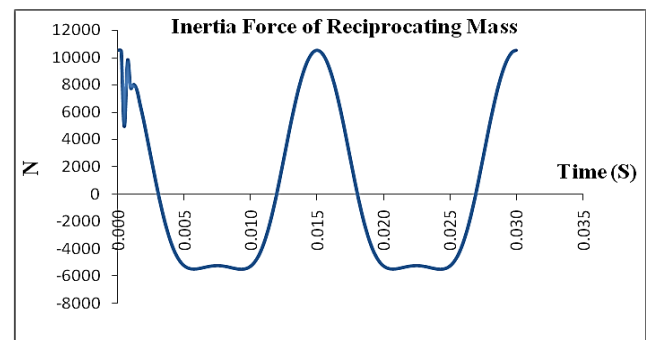


Figure 22: Inertia Force of Reciprocating Masses at 4000 rpm

The maximum and minimum angular acceleration of connecting rod from the calculated results are $3374968.48 \text{ deg/sec}^2$ at 0.00375 s and $-3374968.48 \text{ deg/sec}^2$ at 0.01125 s. So, the calculated result and the simulation result are nearly the same in magnitude and direction. The main target of dynamics analysis is to compute the values of inertia forces. The comparison of calculated value and simulation result of inertia forces are shown in Table. III. From this table, the calculated values of Fig.11 and the simulation results of Fig.22 are nearly the same in magnitude and direction. Table.III shows the comparison of calculated values and simulation results of inertia force. The maximum percentage error between theoretical and simulation result is 12 %.

TABLE.III
COMPARISON OF THEORETICAL AND SIMULATION
RESULTS OF INERTIA FORCE

Time (s)	Inertia Force (N)		% Error
	Theoretical Results	Simulation Results	
0	10505.1	10505.11	0
0.001667	6483.261	6534.376	0.7822
0.003333	-1115.38	-1265.19	11.8409
0.005	-5252.55	-5241.55	0.2094
0.006667	-5367.88	-5318.48	0.7957
0.008333	-5367.88	-5325.17	0.7957
0.01	-5252.55	-5256.83	0.0814
0.011667	-1115.38	-1247.11	10.5628
0.013333	6483.261	6585.414	1.5512
0.015	10505.1	10507.12	0.0192
0.016667	6483.261	6561.79	1.1968
0.018333	-1115.38	-1265.19	10.9736
0.02	-5252.55	-5241.55	0.2094
0.021667	-5367.88	-5318.48	0.7957
0.023333	-5367.88	-5325.17	0.7957
0.025	-5252.55	-5256.83	0.0814
0.026667	-1115.38	-1247.11	10.5628

VII. CONCLUSION AND DISCUSSION

In the thermal calculation, the range of the sum of the drop in the charge velocity, β and resistance coefficient of the intake system, ζ is 2.5 to 4. The deviation of combustion pressure between numerical and theoretical results will be reduced by increasing the sum value of β and ζ . Because of the power output of the engine depends upon the combustion pressure, inertia forces and the masses of rotating and reciprocating parts, it is need to analyze dynamic loads acting on the crankshaft due to their effects. Theoretical investigations are carried out based on a four-cylinder light truck diesel engine and then they are compared with simulation results by using Comsol and SolidWorks softwares. In this paper, the temperatures and pressures for various engine strokes, inertia forces and output torque for 4000 rpm are calculated. The maximum percentage error of inertia force between theoretical and simulation results is 12 % and so it is within the limitation range and can be accepted. According to simulation result of Fig.17, the output torque of engine at 2000 rpm is approximately 180 N-m although specification for output torque of engine is 192 N-m. So, the deviation between simulation

result and manufacturer specification data is 6.25 %. On the results of Fig. 17, the fluctuations in the engine output torque are caused by the different strokes in a cycle. From this research, crankshaft torque would be increased by increasing combustion pressure and inertia forces of reciprocating parts. For further research, experimental study should be conducted to validate the simulation results for the mentioned parameters.

VIII. REFERENCES

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