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Simulation and Analysis of Fractional Order PID Controller

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ABSTRACT

This work presents the development of a new tuning method and performance of the fractional order PID controller includes the integer order PID controller parameter. The tuning of the PID controller is mostly done using Zeigler and Nichols tuning method. All the parameters of the controller, namely K_p (Proportional gain), K_i (integral gain), K_d (derivative gain) can be determined by using Zeigler and Nichols method. Fractional order PID (FOPID) is a special kind of PID controller whose derivative and integral order are fractional rather than integer. To design FOPID controller is to determine the two important parameters λ (integrator order) and μ (derivative order). In this paper it is shown that the response and performance of FOPID controller is much better than integer order PID controller for the same system.
Keywords: Fractional order PID controller, Zeigler-Nichols method, Astrom-Hagglund method, PID Controller

I. INTRODUCTION

PID controller is a well-known controller which is used in the most application. PID controller becomes a most popular industrial controller due to its simplicity and the ability to tune a few parameters automatically. According to the Japan electric measuring instrument manufacture's association in 1989, PID controller is used in more than 90% of the control loop. As an example for the application of PID controller in industry, slow industrial process can be pointed, low percentage overshoot and small settling time can be obtained by using this controller.

This controller provides feedback, it has ability to eliminate steady state offsets through derivative action. The derivative action in the control loop will improve the damping and therefore by accelerating the transient response, a lighter proportional gain can be obtained during the past half century, many theoretical and industrial studies have been done in PID controller setting rules Zeigler and Nichol's in 1942 proposed a method to set the PID controller parameter Hagglund and Astrom in 1955 and chengching in 1999, introduced other technique. By generalizing the derivative and integer orders, from the integer field to non-integer numbers, the fractional order PID control is obtained.

The performance of the PID controller can be improved by making the use of fractional order derivatives and integrals. This flexibility helps the design more robust system. Before using the fractional order controller in design an introduction to the fractional calculus is required. The first time, calculus generation to fractional, was proposed Leibniz and Hopital for the first time after words, the systematic studies in this field by many researchers such as Liouville (1832), Holmgren (1864) and Riemann (1953) were performed.

I. CONVENTIONAL PID CONTROLLER

PID- most widely-used type of controller for industrial applications. And exhibit robust performance over a wide range of operating conditions. The three main parameters involved are Proportional (P), Integral (I) and Derivative (D).

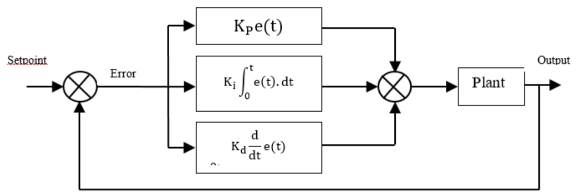

Figure 1 Block-diagram of PID

The proportional part is responsible for following the desired set-point, while the integral and derivative part account for the accumulation of past errors and the rate of change of error in the process respectively. PID controller's algorithms are mostly used in feedback loops. PID controllers can be implemented in many forms. It can be implemented as a stand-alone controller or as part of Direct Digital Control (DDC) package or even Distributed Control System (DCS). A PID controller has three tuning parameters. If these are adjusted in an ad hoc fashion, it may take a while for satisfactory performance to be obtained.

Table 1 PID tuning method

Zeigler –Nichols Tuning method			
Controller Type	K_c	T_i	T_d
P	$\tau/k_P t_d$	-	-
PI	$0.9 * \tau/k_P t_d$	$3.33\ t_d$	-
PID	$1.2 * \tau/k_P t_d$	$2t_d$	$0.5t_d$

Also, each tuning technician will end up with a different set of tuning parameters. There is plenty of motivation then to develop an algorithmic approach to controller tuning. The Ziegler-Nichols closed-loop tuning technique was perhaps the first rigorous method to tune PID controllers. The technique is not widely used today because the closed-loop behaviour tends to be oscillatory and sensitive to uncertainty.

I. FRACTIONAL ORDER $PI^{\lambda}D^{\mu}$ CONTROLLER

One of the possibilities for improvements in the quality and robustness of PID controllers is to use fractional order controllers with noninteger derivation and integration parts. The $PI^{\lambda}D^{\mu}$ controller involving an integrator of order λ and a differentiator of order μ.

The differential equation of the $PI^{\lambda}D^{\mu}$ controller is given as follows:

$$u(t) = K_P e(t) + K_I D^{-\lambda} e(t) + K_D D^{\mu} e(t) \quad (1)$$

The continuous transfer function of the FOPID controller is obtained by means of the Laplace transformation, as given by:

$$G_c(s) = \frac{U(s)}{E(s)} = K_P + K_I s^{-\lambda} + K_D s^{\mu}, \ (\lambda, \mu > 0) \quad (2)$$

For designing a FOPID controller, 3 parameters (Kp, Ki, Kd) and 2 orders (λ, μ) with nonintegers should optimally determined for a given system.

II. MATHEMATICAL MODELING AND ANALYSIS OF FOPID

To obtain the K_P (proportional gain), a constant of integral term (K_i), the constant of derivative term K_d, the fractional order of differentiator μ and the fractional order of integrator λ. The method uses classical Zeigler – Nichols tuning rule to obtain K_P and K_i. To obtain initial value of K_d, then some fine tuning has been done by using Astrom Hagglund method described earlier. The fractional order λ and μ are obtained to achieve specified phase margin.

Let Φ_{pm} be the required phase margin and $j\omega_{cp}$ be the frequency of the critical point on the Nyquist curve of G(s) at which arg $(G(j\omega_{cp})) = -180)$, then the gain margin defined as

$$g_m = \frac{1}{|G(j\omega_{CP})|} = K_c \qquad (3)$$

In order to make the phase margin of the system equal to Φ_{pm} and $\left|C(j\omega_{cp})G(j\omega_{cp}) = 1\right|$, the following equation must be satisfied.

$$C(j\omega_{cp}) = \frac{1}{|G(j\omega_{CP})|} e^{j\Phi_{pm}} = K_c \cos\Phi_{pm} + jK_c \sin\Phi_{pm} \qquad (4)$$

Then we write $C(j\omega_{cp})$ using equation

$$C(j\omega_{cp}) = K_p + K_i\omega_{cp}^{-\lambda}\cos\left(\frac{\pi}{2}\lambda\right) + K_d\omega_{cp}^{\mu}\cos\left(\frac{\pi}{2}\mu\right) + \left[-K_i\omega_{cp}^{-\lambda}\sin\left(\frac{\pi}{2}\lambda\right) + K_d\omega_{cp}^{\mu}\sin\left(\frac{\pi}{2}\mu\right)\right] \qquad (5)$$

Considering equation (7) and (8) we can write

$$f_1(\lambda,\mu) = K_p + K_i\omega_{cp}^{-\lambda}\cos\left(\frac{\pi}{2}\lambda\right) + K_d\omega_{cp}^{\mu}\cos\left(\frac{\pi}{2}\mu\right) - K_c\left(\cos\Phi_{pm}\right) = 0 \qquad (6)$$

$$f_2(\lambda,\mu) = -K_i\omega_{cp}^{-\lambda}\sin\left(\frac{\pi}{2}\lambda\right) + K_d\omega_{cp}^{\mu}\sin\left(\frac{\pi}{2}\mu\right) - K_c\left(\sin\Phi_{pm}\right) = 0 \qquad (7)$$

The numerical solution for λ and μ can be obtained by the equations (6) and (7)

Algorithm for tuning of $PI^{\lambda}D^{\mu}$ controller

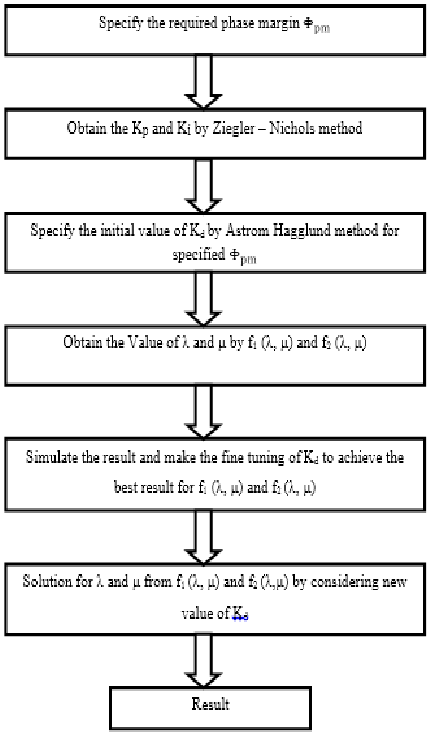

Figure 2 Flowchart for FOPID Algorithm

III. PROBLEM FORMULATION

The transfer function consider for the implementation of PID and FOPID controller is given as,

$$G(s) = \frac{1}{s(s^2 + 3s + 2)} \qquad (8)$$

To tune the PID controller, Zeigler Nichols closed loop tuning is used

K_p , K_i and K_d of the controller has been obtained are 3.6, 1.63 and 1.98. The PID controller obtain can be given as $C_1(s)$

$$C_1(s) = 3.6 + \frac{1.63}{s} + 1.98s \qquad (9)$$

By using classical Astrom- Hagglund method, the value of the PID controller parameters have been calculated for the specified phase margin (Φpm) at 40˚.

$$C_2(s) = 4.59 + \frac{1.51}{s} + 3.48s \qquad (10)$$

The fractional order PID controller takes the value of K_p and K_i from Zeigler and Nichols method. The value of K_d have been obtained by using Astrom - Hagglund

Phase Margin	Proposed Method (FOPID)				
Φ_{pm}	K_P	K_i	K_d	λ	μ
40°	3.6	1.63	3.75	1.39	0.79

method for specified Phase Margin. By using fine tuning for K_d to achieve the best solution by the following equation with the specified phase margin.

$$f_1(\lambda,\mu) = K_p + K_i\omega_{cp}^{-\lambda}\cos\left(\frac{\pi}{2}\lambda\right) + K_d\omega_{cp}^{\mu}\cos\left(\frac{\pi}{2}\mu\right) - K_c\left(\cos\Phi_{pm}\right) = 0 \quad (11)$$

$$f_2(\lambda,\mu) = -K_i\omega_{cp}^{-\lambda}\sin\left(\frac{\pi}{2}\lambda\right) + K_d\omega_{cp}^{\mu}\sin\left(\frac{\pi}{2}\mu\right) - K_c\left(\sin\Phi_{pm}\right) = 0 \quad (12)$$

Using the above two equations can be solved by using "fsolve" toolbox of the MATLAB to obtain the value of λ and μ for the new value of K_d for the phase margin (Φpm) at 40˚. The proposed method values have been compared with Zeigler and Nichols method and Astrom - Hagglund method.

Table 2 Phase Margin with proposed FOPID Controller

The $PI^{\lambda}D^{\mu}$ controller obtained can be written as

$$C_3(s) = 3.6 + \frac{1.63}{s^{1.39}} + 3.75s^{0.79}$$

IV. RESULTS AND DISCUSSION

MATLAB / SIMULINK is one of the most successful software packages currently available. It is a powerful, comprehensive and user friendly software for simulation studies. Especially, functions are then interconnected to form a SIMULINK block diagram that defines the system structure. The Step responses of the system using Zigler-Nichols, Astrom – Hagglund and Fractional Order PID controllers are given below.

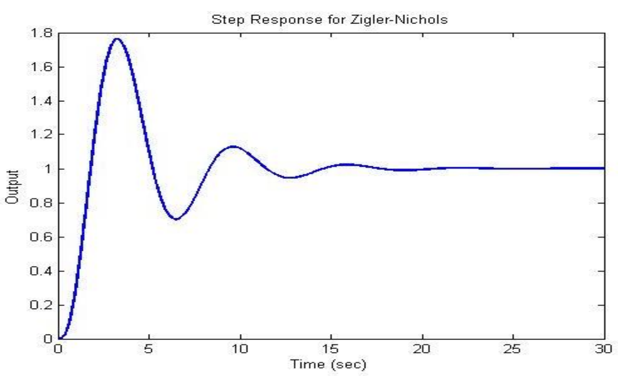

Figure 3 Tuning of the System using Zigler - Nichols PID Controller

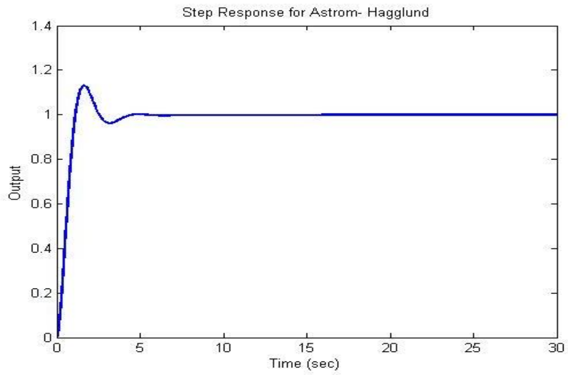

Figure 4 Tuning of the System using Astrom-Hugglund PID Controller

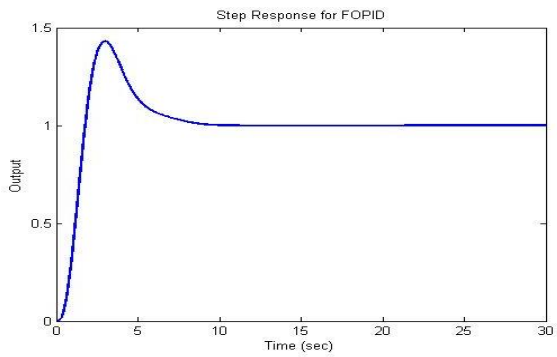

Figure 5 Tuning of the System using Fractional Order PID Controller

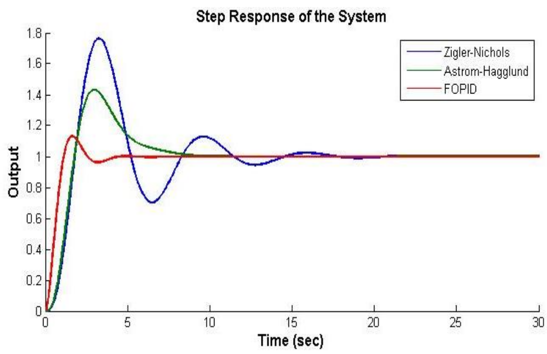

Figure 6 Tuning of the System using Z-N, A-H and FOPID Controller

Step response of the system gives valuable information such as Maximum overshoot (Mp %), rise time (Tr), peak time (Tp) and settling time (Ts) as shown the table:

Table 3 Comparative analysis of FOPID and Existing Controllers

Step response Specification	Maximum overshoot M_p %	Rise time T_r(sec)	Peak time T_p(sec)	Settling time T_s(sec)
Z-N PID	1.76	3.3	1.0615	20.5
A-H PID	1.43	3	1.0524	10.2
FO PID	1.13	1.6	0.7674	4.8

From the Table 3 the proposed method gives much better performance with respect to Z-N method and Astrom-Hagglund method especially for Maximum overshoot (Mp %),), rise time(T_r), peak time(T_p) and settling time(T_s).

V. CONCLUSION

A method for tuning of PID and fractional order PID controller has been proposed. The presented method is based on idea of using Zeigler-Nichols for K_p and K_i while Astrom Hagglund method is used for determining K_d for the conventional PID. Similarly K_p and K_i parameter for fractional order PID controller have been computed from Zeigler and Nichols method and the remaining parameter K_d, λ and μ have been found from Astrom – Hagglund method. The simulation result demonstrated that the fractional order PID controller has better response than the conventional PID controller. The comparison study of the proposed method for tuning of fractional order PID controller certainly will be very important. FOPID controller provides stability region even when an Integer order PID controller cannot provide any stability region.

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