# Finding out square root of an integer number using Single Electron Transistor 

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#### Abstract

The single-electron transistor (SET) attracts the researchers, scientists or technologists to design and construct large scale circuits for the sake of the consumption of ultra-low power and its small size. All the incidences in a SET-based circuit happen when only a single electron tunnels through the transistors under the proper applied bias voltage and a small gate voltage or multiple gate voltages. The oscillatory conduction as the function of the variable-multiple /single gate voltage is exhibited by SET. This uncommon characteristic provides the ability of executing the functions of AND, OR, XOR, Inverter and some combinational circuits like multiplexer, subtractor etc. For implementing a square root circuit, SET would be a best candidate to fulfil the requirements. The processing speed of SET based devices will be nearly close to electronic speed. Noise during processing gets ultra-low when the circuits is built with SETs. The square root circuit is presented here for sixteen bit input numbers. The input bit numbers can be increased with the increasing of the depth of the pattern very easily. And this will provide us the greater accuracy about the squared root value. Power consumption in the single electron circuit is low irrespective of bipolar junction transistor (BJT) or Complementary Metal Oxide Semiconductor (CMOS) circuits. Reducing the numbers of nodes, the power consumption is reduced. Keywords : electron-tunneling, Coulomb blockade, single-electron transistor (SET), logic gates, square root


## I. INTRODUCTION

## Coulomb blockade in a tunnel junction

Between two conducting materials, a thin insulator is sliced to make a tunnel junction. These conducting materials considered as two electrodes
required for connecting the bias voltage for tunneling phenomenon. If anybody selects the electrodes as superconducting substances, the current carrying through the junction will be by virtue of the two types of elementary charges, electron and proton, defined as Cooper pairs.

Instead of superconducting electrodes, one's intention to use normal conducting things whose characteristics differ from semiconductor and superconductor is to provide the current by only a single elementary charge-electron.
For the case of Classical electrodynamics, through the insulating wafer no current can pass. On the other hand, for the case of Quantum mechanics, there is a nonfading probability of an electrontransport from one electrode to another through the insulator. For the Quantum mechanics case, if we apply bias voltage to the tunnel electrodes then the current following through it will be in proportionate to the voltage provided, we are neglecting all other additional effects like shunt effect. So the tunnel junction acts as a resistor whose resistance varies exponentially with the thickness (nanoscale) of the wafer-barrier.
The tunnel junction discussed above has a resistance along with a capacitance. In this case the capacitance is made up of two conducting plates and the dielectric insulator between them. The value of the capacitance is finite (auto scale).
When the current passing through the tunnel junction, it is the fact that there is a sequence of events of tunneling of exactly one electron at a time i.e., not co-tunneling in which a pair of electrons passes (tunnels) concurrently.
For the discreteness of electrical charge, the current passing through a tunnel junction is a series of events and only a single electron tunnels through the tunnel-barrier at a time when tunneling. The capacitor of the tunnel junction is charged with one elementary charge (i.e. $1.602 \times 10^{19}$ coulomb) by the tunneling electron, causing a voltage buildup $\mathrm{V}=\frac{e}{C}$, where $e$ is the elementary charge and $C$ the capacitance of the junction. If the value of $C$ is very small, the voltage built up can be large enough to prevent another electron from tunneling We are able to suppress the electric current with a low bias voltage, as a result the resistance of the
device will remain no longer constant. The increment of the differential resistance around zero bias is defined as the Coulomb blockade.

## II. Inverter

The inverter [7, 9, 10, 12, 20] depicted in Fig. 1 is constructed with two Single electron transistors (SETs) in series connection. Input voltage is directly coupled to both of the islands of the SET1 and SET2 through the capacitances $C_{1}$ and $C_{2}$ respectively. The island of each SET is a very small size of gold having total capacitance of $C_{t o t}=1 \mathrm{pF}$. The output terminal $V_{0}$ is connected to the common channel of the two SETs and to the ground via a capacitor $C_{L}$ to put down charging effects. The two extra gates $V_{g 1}$ and $V_{g 2}$ through two capacitors $C_{g 1}$ and $C_{g 2}$ respectively are intentionally connected to the islands of SET1 and SET2 to tune the induced charges on the islands of SET1 and SET2.


Fig. 1 SET based inverter
For the case of inverter, the parameters values chosen are: $V_{g 2}=0.1 \times \frac{q_{e}}{C}, C_{L}=9 C, t_{4}=\frac{1}{10} C, t_{3}=$ $\frac{1}{2} C, t_{2}=\frac{1}{2} C, t_{1}=\frac{1}{10} C, C_{1}=\frac{1}{2} C, C_{2}=\frac{1}{2} C, C_{g 1}=$ $\frac{17}{4} C$ and $C_{g 2}=\frac{17}{4} C$. For the simulation cases the value of $C$, we consider, is=1aF.

The operation of the inverter will be like this: - the output $V_{0}$ value will be high when the input voltage $V_{i n}$ is low and $V_{0}$ value will be low when the input voltage is high. To obtain this target, we put the voltage $V_{g 1}=V_{g 2}$ which will be equal to $\frac{e}{C_{t o t}}=16 \mathrm{mV}$ along with the tuning gate voltages (at
present $V_{i n}$ ) of SET1 and SET2, which are adjusted so that as soon as the $V_{\text {in }}$ is low the SET1 is in conduction mode and the SET2 is in Coulomb blockade.

This effectively results the output to bias voltage $V_{b}$ and causes the output voltage to high, i.e, this effectively connects the output to the supply voltage and makes the output high.

Coulomb blockade interferes the steady flow of current as when the high voltage is applied to the input it causes to shift the induced charge on each of the islands of the SETs by a part of an electron charge and keeps the SET1 in the state of Coulomb blockade and the SET2 in the conducting mode. As a consequence, the output result shifts from high to low.
In this work I assume the Boolean logic inputs corresponding to the voltages like: logic " 0 " $=0$ Volts and logic " 1 " $=0.1 \times \frac{q_{e}}{c}$
For our simulation and other purposes, we must assume $\mathrm{C}=1 \mathrm{aF}$ then Logic " 1 " $=0.1 \times$ $\frac{1.602 \times 10^{-19}}{1 \times 10^{-18}}=0.1 \times 1.602 \times 10^{-2}=16.02 \times$ $10^{-3}=16.02 \cong 16 \mathrm{mV}$

## III. Single Electron NAND and AND gates

By using four SETs, a two input NAND gate [1, 18] can be constructed as depicted in Fig-2(a). The Single Electron Transistor circuits are configured as CMOS circuits, the pull up transistor called pchannel (P-SET) is tied to $\mathrm{V}_{\mathrm{d}}$ and the pull down transistor called n-channel transistor (N-SET) tied to ground. The p-channel bears doubles the area factor of the n -channel SET which is the equivalent of setting twice the gate width. For the case of Single Electron NAND, the upper two SETs called P-channel Single Electron Transistors (P-SETs) are connected in parallel and the lower two SETs called N -channel Single Electron Transistors ( N -SETs) are connected in series. When each of the input
voltages $V_{\text {in } 1}$ and $V_{\text {in } 2}$ are at high levels, the lower two SETs should be in tunneling mode and a single electron must pass through them and virtually the output $V_{\text {out }}$ connected to ground which leads $V_{\text {out }}$ to low. As opposed to, if any one or both of the inputs $V_{\text {in1 }}$ and $V_{\text {in2 }}$ are low, the output is pulled to $V_{d}$ by the P-SETs which (either one or both) are in tunneling mode and the resultant output becomes high as the N -SETs are in Coulomb blockade.

When the output of the NAND gate is connected to the input of inverter shown in Fig-1, we get the Single Electron AND gate that is drawn in Fig-2(b) For the case of AND and NAND Gates, the parameters values chosen are: $C_{L}=9 a F$, Drain $=$ $0.5 a F$, Source $=0.1 a F$, and $C_{1}=C_{2}=0.5 a F$. The same parameters are chosen for the case of OR, NOR, XOR, 2:1 Multiplexer, and subtraction circuit.


Fig. 2 (a) NAND Gate


Fig. 2 (b) AND Gate


Fig.2(c) simulation result of AND and NAND Gate

## IV. Single Electron NOR and OR gates

For the case of NOR gate [1, 13, 18] four SETs are taken of which upper two P-SETs are connected in series and the lower two N-SETs in parallel shown in Fig-3(a), When both input voltages $V_{i n 1}$ and $V_{\text {in2 }}$ are low, the upper two SETs are in tunneling mode towards $V_{d}$, the lower two N -SETs are in Coulomb blockade and the output $V_{\text {out }}$ connected to bias voltage $V_{d}$, so the output $V_{\text {out }}$ becomes high. When any one or both of the inputs $V_{i n 1}$ and $V_{i n 2}$ are low, the upper P-SETs are non-tunneling and the lower N -SETs (any or both) are in tunneling and the output is virtually grounded. This indicates the output voltage $V_{\text {out }}$ low. If we connect the output of NOR gate to the input of the inverter, the combined circuit is treated as Single Electron OR gate shown in Fig-3(b).


Fig. 3 (a) NOR gate


Fig. 3 (b) OR Gate


Fig. 3(c) simulated result of OR and NOR gate

## V. Single Electron XOR gate

Single Electron transistor exhibits an oscillatory conductance [Fig. 4] when we apply gate voltage as input voltage. This enables the SET the capability of functioning as an exclusive-OR gate.


Fig. 4 XOR Gate
There are two gates capacitors $C_{g 1}$ and $C_{g 2}$ having almost the same value for their symmetric constructions in fabrication. A discrete load capacitor $C_{L}$ is connected to the drain and the output voltage is taken as the voltage $V_{o}$ of the load capacitor in response to the combined input gate voltages $V_{g 1}$ and $V_{g 2}$ in the range 0 to 16 mV .

When both the input gate voltages $V_{g 1}$ and $V_{g 2}$ are 0 V or 16 mV the output shows low and when one of them is high i.e., 16 mV , the output is high. These phenomena indicate the operation of an XOR gate $[2,3,4,6,13,20]$.

When anybody wishes to use $V_{g 1}$ as electrode and $V_{g 2}$ as control gate, then the device can transmit current signal for a particular voltage at $V_{g 1}$.
By using conventional logic circuits we need 16 transistors to make an XOR gate $[1,5,16,17,19]$ on the other hand the function of it can be implemented with a single SET. The two gate capacitances Cg 1 and Cg 2 are almost equal on account of the symmetric structure of the gate. The range of $V_{g 1}$ and $V_{g 2}$ are kept between 0 and 16 mV .


Fig.4(a)


Fig. 4(b)

## VI. Multiplexer

A Multiplexer is combinational circuit that connects to one input out of many inputs and send that to the output. It is a combinational circuit whose present output don't rely on the previous output. The circuit needs $n$ control lines for $2^{n}$ inputs. A circuit for 2:1 Multiplexer [21, 22] drawn in Fig. 5(a) containing two inputs $V_{i n 1}$ and $V_{i n 2}$, output $V_{o}$ and a single control line input 'Sel' that have two operational stages. The outputs of the first two stages (upper and lower in Fig. 5(a)) which are really two AND gates give the outputs Vin1. $\overline{\operatorname{Sel}}$ and Vin2.Sel respectively. The last stage is a two input OR gate which gives the output (Vin1. $\overline{\text { Sel }}$ + Vin2.Sel). If Sel $=0$, the output is $V_{0}=V_{\text {in } 1}$ and if Sel $=1$, the output is $V_{0}=V_{\text {in } 2}$. So the circuit given in Fig.5(a) acts as a 2:1 Multiplexer.
When we trigger the selection line (Sel) from low to high or from high to low, the delay time becomes approximately 1 picosecond. Same time delay is for the case of inputs. The time it takes for stable in high stage is approximately 15-20 picoseconds.


Fig. 5(a) 2:1 Multiplexer


Fig, 5(b) simulated result of 2:1 Multiplexer

## VII. Subtraction Circuit

For the implementation of a square root circuit we need to construct a subtraction circuit. The inputoutput relationship of this circuit and the related
equations that can be exposed by using Karnaugh map is shown in the Table-1. Based on the single electron transistor gates like AND gate, XOR gate, OR-gate and inverter gate a subtractor (SUB=DIFF) with its borrow ( $B_{\text {out }}$ ) has been drawn in Fig.6. We can write the equation related to the subtractor as:

Diff $=X^{\prime} Y^{\prime} Z+X Y^{\prime} Z^{\prime}+X^{\prime} Y Z '$ XYZ $=X \oplus Y \oplus Z=$
$\mathrm{X} \oplus \mathrm{Y} \oplus \mathrm{B}_{\text {in }}$
$B_{\text {out }}=X^{\prime} Y^{\prime} Z+X^{\prime} Y Z '+X^{\prime} Y Z+$
XYZ

| Table-1 |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| X | Y | $\mathrm{B}_{\text {in }}$ | DIFF | $\mathrm{B}_{\text {out }}$ | Diff $=X^{\prime} \mathrm{Y}^{\prime} \mathrm{Z}+\mathrm{XY} \mathrm{Z}^{\prime}+\mathrm{X}^{\prime} \mathrm{YZ}{ }^{\prime}+\mathrm{XYZ}=\mathrm{X} \oplus \mathrm{Y} \oplus \mathrm{Z}=\mathrm{X} \oplus \mathrm{Y} \oplus \mathrm{B}_{\text {in }}$ |
| 0 | 0 | 0 | 0 | 0 |  |
| 0 | 0 | 1 | 1 | 1 | $B_{\text {out }}=X^{\prime} Y^{\prime} \mathrm{Z}+\mathrm{X}^{\prime} \mathrm{YZ}{ }^{\prime}+\mathrm{X}^{\prime} \mathrm{YZ}+\mathrm{XYZ}$ |
| 0 | 1 | 0 | 1 | 1 | $=X^{\prime} Z\left(Y^{\prime}+Y\right)+Y\left(X^{\prime} Z^{\prime}+X Z\right)$ |
| 0 | 1 | 1 | 0 | 1 | = $\mathrm{X}^{\prime} \mathrm{Z}+\mathrm{Y}(\mathrm{X} \oplus \mathrm{Z})^{\prime}$ |
| 1 | 0 | 0 | 1 | 0 | X $\mathrm{Z}+\mathrm{Y}(\mathrm{X} \oplus \mathrm{Z})^{\prime}$ |
| 1 | 0 | 1 | 0 | 0 | $=\mathrm{X}^{\prime} \mathrm{Y}+\mathrm{Z}(\mathrm{X} \oplus \mathrm{Y})^{\prime}$ |
| 1 | 1 | 0 | 0 | 0 |  |
| 1 | 1 | 1 | 1 | 1 |  |



Fig. 6 A Subtraction Circuit based on SET

## VIII. Restoring Algorithm for calculating the square-root

## Algorithm for Square-root

(i) Start
(ii) Initialize n -bit radicand which is to be squared root
(iii) Segment the radicand into groups of two bits starting from least significant bit
(iv) Starting from left, if the radicand size is odd, the first group is considered as one bit otherwise the first group is of two bits.
(v) Take ' 1 ' squared and subtract it from first group. if the subtracted value/remainder is positive the first square root is ' 1 ', if negative square root is ' 0 '.
(vi) Shift $2^{\text {nd }}$ group of two bits from the remaining of the radicand to the right of the subtracted value, subtract it by the guess square with append ' 01 '. If the subtracted value is positive the $2^{\text {nd }}$
square root is ' 1 ', if negative $2^{\text {nd }}$ square root is ' 0 '.
(vii) When negative, restore

Its meaning is: add this negative value with previous "the guess squared with append 01 " to get the remainder.
And follow the steps until last group of two bits is shifted from.

Following restoring algorithm, we have tried to find out the square root of the number $1111110110_{2}=1014_{10}$. After manipulation of 16 steps, the result given at the topmost row in the Table-2 is $11111.110101111_{2}$. The decimal value of this binary number is being $=31.841796875_{10}$.

| Table-2 |  |  |
| :---: | :---: | :---: |
| 1 |  1 1 1 1 1 $\cdot$ 1 1 0 1 0 1 1 1 1   <br> $\sqrt[2]{1}$ 1 1 1 1 1 0 1 1 0.0 0 0 0 0 0 0 0 0 <br> 0 0 0 0 0 0 0 0 0 0         | 01 |
| 2 | 1011 +ve [when $1^{\text {st }}+$ ve or - ve then sqrt bit starts] $\rightarrow$ 101 | $\begin{array}{ll} 1 & \\ 101 \end{array}$ |
| 3 | $\begin{array}{llllll} \hline 1 & 1 & 0 & 1 & 1 \\ & 1 & 1 & 0 & 1 \end{array}$ | $\begin{array}{ll} \hline 1 & \\ 1 & \\ 1 & 101 \end{array}$ |
| 4 | $\begin{array}{lllllll} 1 & 1 & 1 & 0 & 0 & 1 \\ & 1 & 1 & 1 & 0 & 1 \end{array}$ | $\begin{array}{lllll} 1 & & & \\ 1 & 1 & 1 & 0 & 1 \end{array}$ |
| 5 | $\begin{array}{lllllllllll} \hline 1 & 1 & 1 & 0 & 0 & 1 & 0 & & & & +\mathrm{ve} \\ & 1 & 1 & 1 & 1 & 0 & 1 & & 1 & & \end{array}$ | $\begin{aligned} & \hline 1 \\ & 111101 \end{aligned}$ |
| 6 |  | $\begin{aligned} & 15^{\text {th }} \text { from right } \\ & 1111101 \end{aligned}$ |
| 7 | $\begin{array}{rlllllllllll} \hline 1 & 0 & 1 & 0 & 1 & 1 & 1 & 0 & 0 & & & \text { +ve } \\ 1 & 1 & 1 & 1 & 1 & 1 & 0 & 1 & & \\ \hline \end{array}$ | $\begin{aligned} & \hline .1 \\ & 11111101 \end{aligned}$ |
| 8 | $\begin{array}{rllllllllllllllllll} 0 & 1 & 0 & 1 & 1 & 1 & 1 & 1 & 0 & 0 & +v e \\ & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 1 & & \end{array}$ | $\begin{aligned} & 1 \\ & 111111101 \end{aligned}$ |
| 9 | $101111100 \quad$-ve <br> 111111101 restore | 0 |
| 10 | $\begin{array}{lllllllllll} 1 & 0 & 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 0 & 1 \end{array}$ | $1111111001$ |
| 11 | $\begin{array}{ccccccccccccc} 1 & 1 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 0 & 0 & +v e ~ \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 1 & 0 & 1 & & \end{array}$ | $\begin{aligned} & 1 \\ & 11111110101 \end{aligned}$ |
| 12 | 11111110101 restore | 0 |
| 13 | $\begin{array}{llllllllllllll} 1 & 1 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 1 & 0 & 0 & 1 \end{array}$ | $111111101001$ |
| 14 | $\begin{array}{llllllllllllll} 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 \\ & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 1 & 0 & 1 & 0 & 1 \end{array}$ | $\begin{aligned} & \hline 1 \quad \text { +ve } \\ & 1111111010101 \end{aligned}$ |
| 15 | $\begin{array}{lllllllllllllll} 1 & 1 & 1 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 \\ & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 1 & 0 & 1 & 1 & 0 & 1 \end{array}$ | $\begin{aligned} & 1 \text { +ve } \\ & 11111110101101 \end{aligned}$ |
| 16 | $\begin{array}{lllllllllllllll} \hline 1 & 1 & 1 & 0 & 0 & 1 & 0 & 1 & 1 & 0 & 1 & 1 & 1 & 1 & 0 \\ & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 1 & 0 & 1 & 1 & 1 & 0 \end{array}$ | $\begin{aligned} & \hline 1 \text { +ve } \\ & 111111101011101 \end{aligned}$ |
|  | 110011001011111 | $1+\mathrm{ve}$ |

The original number of $\sqrt[2]{1014}=$ 31.84336665618132

From the Table-2, we observe the different square root values from the top-most row as
$11111.110101_{2}=31.828125_{10}$ [from top most row of Table 2] six digits after dot
$11111.1101011_{2}=31.8359375{ }_{10}$ [from Table 2] seven digits after dot

$$
11111.11010111_{2}=31.8398437510
$$

[from Table 2] eight digits after dot

$$
11111.110101111_{2}=31.841796875_{10}[\text { from }
$$

Table 2] nine digits after dot
The error for the last step is calculated with respect to 31.84336665618132 is

$$
\begin{aligned}
& \frac{\sqrt[2]{1014}-31.841796875}{\sqrt[2]{1014}} \\
& =\frac{0.0015697811813153}{31.84336665618132} \\
& =4.929696028263887 \times 10^{-5}
\end{aligned}
$$

$$
=0.0049296960282639 \% \approx 0.0049 \% \text { (error) }
$$

So the error is $0.0049 \%$ which is acceptable. The more the steps we proceed, the less the error we obtain.

## IX. Implementation of Square root circuit



Fig. 7 Building block (cell) of the Square root circuit

(a)

(b)

Fig. 8(a) "Symbol sqrt ckt" (a) with inverted borrow $\overline{\mathrm{Bo}}$ " and 8(b) "with borrow Bo"
create complement $\bar{B}$ o instead of $B o$. The leftmost column cells of Fig. 9 are of 8(a) and the remaining cells are of $8(b)$. The square root circuit drawn in Fig 9 has eight steps to calculate eight bits square root of 16 bits radicand $R(0), R(1), R(2), R(3)$, $R(4), \ldots \ldots, R(15)$. The square root bits are $S R(7)$, $\operatorname{SR}(6), \operatorname{SR}(5), \operatorname{SR}(4), \ldots . \operatorname{SR}(0)$. When we increase
the steps along with the increase of two cells each time, we get the more accurate result(s).

If our total pipeline stages are 16, we require 32 bits input (radicand). Some input-output instances are exposed in the Table-3 for the diagram given in Fig. 9. Here are 16 radicand bits and the output are of 8 bits


Fig. 9 SET-based Square root calculation circuit
Now we shall fix the dot (point) in the mid-point
of input then the dot is also to be fixed in the
middle of the output bits for getting the fractional
output squared values. Noted that 16 stages
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of input then the dot is also to be fixed in the
middle of the output bits for getting the fractional
output squared values. Noted that 16 stages

For the Fig. 9 we have taken only integer input. This is why there are some error in the calculated root values as the output bit length equals to 8 . If the pipeline stages are increased to 16,32 bits inputs will be required.
pipelines are not shown in the figure for the cause of limited space. But results of 16 bit for 32 bit inputs are delivered in Table-4.

| Table 3 |  |  |
| :---: | :---: | :---: |
| Sl．No． | Result from C－Program | Input and calculated root （Pipeline stage＝8） |
| 1 | ```Radicand r1,r2,r3,r4,r5,r6,r7,r8,r9,r10,r11,r12,r13,r14,r15,r16 is =1010101010111111 INPUT=43711 Square root of r1,\ldots,r16 is =11010001 Calculated root = 209``` | $\begin{aligned} & \text { Input } \\ & =1010101010111111 \\ & \text { INPUT=43711 } \\ & \text { Square root is }=11010001 \\ & \text { Calculated root }=209 \end{aligned}$ |
| 2 | ```Radicand r1,r2,r3,r4,r5,r6,r7,r8,r9,r10,r11,r12,r13,r14,r15,r16 is =0 11111 1000 111100 1 1 0 INPUT=30950 Square root of r1,\ldots,r16 is =1 0 1 0 1 1 1 1 Calculated root = 175``` | $\begin{aligned} & \text { Radicand } \\ & =0111100011100110 \\ & \text { INPUT }=30950 \\ & \text { Square root }=10101111 \\ & \text { Calculated root }=175 \end{aligned}$ |
| 3 | ```Radicand r1, r2, r3, r4, r5,r6,r7, r8, r9, r10, r11, r12,r13,r14, r15,r16 is =0 0 0 0 0 0 0 0 11111 0 1 0 1 INPUT=245 Square root of r1,\ldots,r16 is =0000 1 1 1 1 Calculated root = 15``` | $\begin{aligned} & \text { Radicand } \\ & =0000000011110101 \\ & \text { INPUT }=245 \\ & \text { Square root }=00001111 \\ & \text { Calculated root }=15 \end{aligned}$ |
| 4 | ```Radicand r1, r2, r3, r4, r5, r6,r7,r8,r9, r10, r11, r12, r13, r14, r15, r16 is =11111001100110011 INPUT=62259 Square root of r1,\ldots,r16 is =11111001 Calculated root = 249``` | $\begin{aligned} & \text { Radicand } \\ & =1111001100110011 \\ & \text { INPUT=62259 } \\ & \text { Square root is }=11111001 \\ & \text { Calculated root }=249 \end{aligned}$ |


| Table 4 |  |  |
| :---: | :---: | :---: |
| S1． No | Result from C－Program | Input and calculated root （Pipeline stage＝16） |
| 1 | ```Radicand=10101010101111111.0000000000000000 INPUT=43711.000000 Square root of r1,\ldots.,r32 is -1 1010001.0001001 0 Calculated root = 209.070313``` | ```Radicand \(=10101010101111\) 11.0000000000000000 INPUT=43711.000000 Square root is \(=11010001.000\) 10010 Calculated root \(=209.070313\)``` |
| 2 |  | $\begin{aligned} & \text { Radicand }=011110001111001 \\ & 10.0000000000000000 \\ & \text { INPUT }=30950.000000 \\ & \text { Square root is }=10101111.111 \\ & 01101 \\ & \text { Calculated root }=175.925781 \end{aligned}$ |
| 3 |  | ```Radicand \(=00000000111101\) 01.0000000000000000 INPUT \(=245.000000\) Square root is \(=00001111.101\) 00111 Calculated root \(=15.652344\)``` |
| 4 | ```Radicand \(11111001100110011.000000000000000 \theta\) INPUT=62259.e日e日e日 Square root of \(\mathrm{r} 1, \ldots, \mathrm{r} 32\) is -11111001.10000100 Calculated root \(=249.515625\)``` | Radicand $=11110011001100$ 11.000000000000000 INPUT $=62259.000000$ Square root is $=11111001.100$ 00100 Calculated root $=249.515625$ |

calculator，there will be an error and this error
When we are interested in comparing the
shown in Table－5 is in tolerable level．

In Table-6, any binary value of word-length 32 is taken keeping the dot (point) in the middle
position (between $16^{\text {th }}$ and $17^{\text {th }}$ bit) and result is shown.

| Table 5 |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $\begin{aligned} & \hline \text { Sl. } \\ & \text { No } \end{aligned}$ | Inpu <br> t <br> valu <br> e | Square root taken from calculator(X) | Calculate d sqr root (Y) from ckt | Difference=X-Y | $\begin{aligned} & \text { Error } \\ & \frac{X-Y}{X} \\ & \times 100 \% \end{aligned}$ |
| 1 | $\begin{aligned} & 4371 \\ & 1 \end{aligned}$ | $\begin{aligned} & 209.0717580162370950162870796 \\ & 3854 \end{aligned}$ | $\begin{aligned} & 209.0703 \\ & 13 \end{aligned}$ | 0.0014450162370950162870796385 4225 | $\begin{aligned} & \hline 0.00069 \\ & \% \end{aligned}$ |
| 2 | $\begin{aligned} & 3095 \\ & 0 \end{aligned}$ | $\begin{aligned} & 175.9261208575918097452961727 \\ & 8488 \end{aligned}$ | $\begin{aligned} & 175.9257 \\ & 81 \end{aligned}$ | $\begin{aligned} & 3.3985759180974529617278488189 \\ & 341 \mathrm{e}-4 \end{aligned}$ | $\begin{aligned} & 0.00019 \\ & \% \end{aligned}$ |
| 3 | 245 | $\begin{aligned} & 15.65247584249852787486421568 \\ & 1119 \end{aligned}$ | $\begin{aligned} & 15.65234 \\ & 4 \end{aligned}$ | 1.3184249852787486421568111893 365e-4 | $\begin{aligned} & 0.00084 \\ & \% \end{aligned}$ |
| 4 | $\begin{aligned} & 6225 \\ & 9 \end{aligned}$ | $\begin{aligned} & 249.5175344539938339441299518 \\ & 8689 \end{aligned}$ | $\begin{aligned} & 249.5156 \\ & 25 \end{aligned}$ | 0.0019094539938339441299518868 8775 | $\begin{aligned} & 0.00076 \\ & \% \end{aligned}$ |


| Sl.No. | Table 6 |
| :---: | :---: |
| 1 | ```Radicand =1 0 1 0 1 1 1 1 1 0 0 0 1 1 1 0..000111100011 0 0 1 1 0 1 1 INPUT=44942.225021 Square root of r1,\ldots,r32 is =1 1 0 1 00011. 1 1 1 1 1 1 1 0 Calculated root = 211.992188``` |
| 2 | ```Radicand =0 1 0 0 0 1 1 1 0 1 1 1 0 1 1 1. . 1 1 0 0 1 1 0 0 1 1 0 0 1 1 0 0 INPUT=18295.799988 Square root of r1,\ldots,r32 is =1 000001111.0 1 0 0 0 0 1 1 Calculated root = 135.261719``` |
| 3 | ```Radicand =0 0 0 0 0 0 0 0 0 0 0 0 1 0 1 0. 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 1 INPUT=10.000076 Square root of r1,\ldots,r32 is =0 0 0 0 0 0 1 1. . 0 0 1 0 1 0 0 1 Calculated root = 3.160156``` |
| 4 | ```Radicand =0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0. 1 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 INPUT=0.750000 Square root of r1,\ldots,,r32 is =0 000000000.1 1 0 1 1 1 0 1 Calculated root = 0.863281``` |

## X. Time delay

For a CMOS/TTL logic gate the time delay for a gate is 12 ns [ 7,15 ], on the contrary the time required for tunneling through a SET is approximately 4ns [2] [3]. The XOR gate using conventional logic circuits needs

16 transistors, whereas this function can be implemented with just one SET [1,2,3,8].
All the gates and devices drawn in Fig. 1 through 7 are combinational circuits. Considering that they are constructed as different pipeline stages, we are easily
able to calculate the time required to find out the output of a circuit. In table-7, the delays involved up to the Fig. 7 are set in.

| Table-7 |  |  |
| :--- | :---: | :---: |
| Sl. <br> No. | Circuit name | SET <br> Circuit Delay(ns) |
| 1 | Inverter | $4 \times 2=8$ |
| 2 | NAND/AND | $4 \times 3=12$ |
| 3 | NOR/OR | $4 \times 3=12$ |
| 4 | XOR | $4 \times 1=4$ |
| 5 | SUTRACTOR | $4 \times 10=32$ |
| 6 | MUX | $4 \times 10=40$ |
| 7 | Symbol8(a)/ <br> Symbol8(b | $4 \times 12=48$ |
| 8 | Squared <br> Root Circuit | (Stage number $\times 48)$ <br> For 8 stage pipelines <br> $8 \times 48=384 n s$ <br> For 16 stage pipelines <br> $16 \times 48=768 \mathrm{~ns}$ |

From the Table-7, we can find the Square root without fractional parts of a 16 bit input number after 265 ns . Hence the frequency of the output is $\frac{1}{384 n s}=$ $\frac{10^{9}}{384}=2604167 \mathrm{~Hz}$
For more accurate result we need 16 pipeline stages and 32 bit input number. In this case, the time delay is 512 ns and the frequency of the output is $\frac{1}{768 n \mathrm{~s}}=\frac{10^{9}}{512}$ $=1302083 \mathrm{~Hz}$

## XI. Switching Power

For switching energies required for the AND, OR, NOR, NAND and INVERTER are $10.8 \mathrm{meV}, 10.8 \mathrm{meV}$, $10.7 \mathrm{meV}, 10.7 \mathrm{meV}$ and 10.4 meV respectively [8]. Depending on these, the switching energies necessary for the different gates, combinational circuits and square root circuits in this work are given in the Table-8. It is really clear that the energy required will be of order $\mathrm{O}\left(10^{-17}\right)$. So very negligible energy is
required in comparison with our conventional TTL-, JFET- or CMOS-based gates $\mathrm{O}\left(10^{-6}\right)$ and combinational circuits.

| Table-8 |  |  |
| :--- | :--- | :--- |
| Gate <br> Name | Required elements | Energy <br> (Joule) |
| AND | fourteen (13) | $1.73016 e^{-18}$ |
| OR | fourteen (13) | $1.73016 e^{-18}$ |
| NOR | fourteen (13+9) | $2.92796 e^{-18}$ |
| NAND | fourteen (13+9) | $2.92796 e^{-18}$ |
| Inverter | Nine(9) | $1.19750 e^{-18}$ |
| XOR | Four(4) | $0.53235 e^{-18}$ |
| Subtractio <br> n | Inverter(5)+OR(1)+AN <br> D(2) | $14.50672 e^{-18}$ |
| MUX 2:1 | Sixty six (66) | $8.78389 e^{-18}$ |
| Building <br> block(cell <br> ) | MUX(1)+Inv(6)+NAN <br> D(2)+NOR(1) | $21.69354 e^{-18}$ |
| Building <br> block(cell <br> )with an <br> inverter | Cell(1)+Inverter(1) | $22.49203 e^{-18}$ |
| Square <br> root <br> circuit | Cell(43)+Cell with <br> Inv(8) | $7.849977 e^{-17}$ |

## XII. CONCLUSION

In this work, we have elaborately discussed concerning a square root circuit, different radicands as inputs and their relevant outputs as square root values which are reliable. The more the stages the less the error. We have studied the time delays both for CMOS/TTL logic and SET based logic and found that SET related circuit at least 3 times faster than first one. Observed that when the number of stages are increased for the square root circuit, more accurate results are produced and which satisfy our expectations. Switching power in the case of SET based devices are the order of $\mathrm{O}\left(10^{-17}\right)$ which is very low. Single electron-based devices have turned out to
be valuable tools in the research of science and engineering fields. It has drawn one's attention as the SET-devices consume very low power and require small number of nodes so it provides higher order integrity of population (i.e. large-scale integration) and gives the output rate more than $\mathrm{O}\left(10^{6}\right)$.

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