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$\bar{R}$-Projective Motion in a Finsler Space $F_n{}^*$ with a Non-symmetric Connection

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ABSTRACT

We have studied the $\bar{R}$-Projective motion in a Finsler space $F_n{}^*$ equipped with a non-symmetric connection. $\bar{R}$ - Curvature collineation and Ricci- Collineation have also been studied in the above context and new results have been obtained.

Keywords: $\bar{R}$ -Projective Motion, $\bar{R}$ -Curvature Collineation, Ricci-Collineation, Non-Symmetric Connection.

I. INTRODUCTION

Davies [1] has studied the generalization of the Lie-derivatives to the Finsler space $F_n{}^*$, and its application to the theory of subspaces. By considering the infinitesimal point transformation $\bar{x}^i = x^i + v^i\,(x)dt$, Rund, [2] and Yano, [3] have defined the Lie-derivatives of an arbitrary vector $X^i(x,\dot{x})$ and the symmetric connection parameter $\Gamma^{*i}_{jk}(x,\dot{x})$. Katzin, Levine and Davis [4] have defined the curvature collineation in a Riemannian space and have studied its properties. They developed that a Riemannian space V_n , admits a curvature collineation provided that there exists a vector $v^i(x)$ such that $\pounds_v R^i_{jkh} = 0$ where R^i_{jkh} is the Riemannian curvature tensor. The properties of curvature collineation in a conformally flat Riemannian space have also been studied by them. This theory of curvature collineation has been extended in a Finsler space by Pande and Kumar [5], Singh and Prasad [6] and many others. The relations which exist in a Finsler space admitted by curvature collineation and other symmetries with special reference to Berwald's and Cartan's curvature tensor fields have also been studied by these researchers.

Vranceanu [7] has introduced a non-symmetric connection Γ^i_{jk} $(\neq \Gamma^i_{kj})$ in an n-dimensional space A_n, we expand this concept to the theory of n-dimensional Finsler spaces. Suppose an n-dimensional Finsler space $F_n{}^*$ with non-symmetric connection Γ^i_{jk} $(\neq \Gamma^i_{kj})$ which is based on a non-symmetric fundamental tensor $g_{ij}(x,\dot{x})$ $(\neq g_{ji}\,(x,\dot{x}))$.

Let us write

$$\Gamma^i_{jk} = M^i_{jk} + \frac{1}{2} N^i_{jk}\,, \tag{1.1}$$

where M^i_{jk} and $\frac{1}{2} N^i_{jk}$ are respectively the symmetric and skew-symmetric parts of Γ^i_{jk}.

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We introduce another connection coefficient $\tilde{\Gamma}^{\,i}_{jk}(x,\dot{x})$ defined as

$$\tilde{\Gamma}^{\,i}_{jk} = M^i_{jk} - \tfrac{1}{2} N^i_{jk}. \qquad (1.2)$$

Using (1.1) and (1.2), we get

$$\tilde{\Gamma}^{\,i}_{jk}\,(x,\dot{x}) = \Gamma^i_{kj}(x,\dot{x}). \qquad (1.3)$$

Let a vertical stroke (|) by an index denotes covariant derivative with respect to x, we defined the covariant derivative of any contravariant vector field $X^i(x,\dot{x})$ is two distinct ways [8], as follows:

$$X^{i\,+}|_{\,j} = \partial j X^i - (\dot{\partial}_m X^i)\;\Gamma^m_{kj}\,\dot{x}^k + X^k\,\Gamma^i_{kj} \qquad (1.4)$$

and $$X^{i\,-}|_{\,j} = \partial j X^i - (\dot{\partial}_m X^i)\;\tilde{\Gamma}^{\,m}_{kj}\,\dot{x}^k + X^k\,\tilde{\Gamma}^{\,i}_{kj}\,. \qquad (1.5)$$

Using (1.3), the equation (1.5) can be written as:

$$X^{i\,-}|_{\,j} = \partial j X^i - (\dot{\partial}_m X^i)\;\Gamma^m_{jk}\,\dot{x}^k + X^k\,\Gamma^i_{jk}\,, \qquad (1.6)$$

where, a positive sign below vertical stroke (|) by an index indicates that the covariant derivatives with respect to the connection Γ^i_{jk} concerning that index.

Similarly, a negative sign below an index vertical stroke (|) by an index indicates that the covariant derivative with respect to the connection $\tilde{\Gamma}^i_{kj}$ concerning that index.

The covariant derivative defined in (1.4) and (1.5) will be called $\oplus$- covariant differentiation of $X^i(x,\dot{x})$ with respect to $\dot{x}^j$ and $\ominus$- covariant differentiation of $X^i(x,\dot{x})$ with respect to $\dot{x}^j$ respectively throughout the thesis. Allowing $\oplus$- covariant differentiation in (1.4) with respect to x^k and then using the part of the skew-symmetric result so obtained with respect to j and k, we get

$$X^{i\,+}|_{\,jk} - X^{i\,+}|_{\,kj} = -\,(\dot{\partial}_m X^i)\;R^m_{pjk}\,\dot{x}^p + X^m\,R^i_{mjk} + X^{i\,+}|_{\,m}\,N^m_{kj}, \qquad (1.7)$$

where

$$R^h_{ijk} \overset{\text{def}}{=} \partial_k\Gamma^h_{ij} - \partial_j\Gamma^h_{ik} + \dot{\partial}_m\Gamma^h_{ik}\,\Gamma^m_{sj}\,\dot{x}^s - \dot{\partial}_m\Gamma^h_{ij}\,\Gamma^m_{sk}\,\dot{x}^s + \Gamma^p_{ij}\,\Gamma^h_{pk} - \Gamma^p_{ik}\,\Gamma^h_{pj}\,. \qquad (1.8)$$

Similarly, $\ominus$-covariante differentiation with respect to x^k and proceeding as above, we have

$$X^{i\,-}|_{\,jk} - X^{i\,-}|_{\,kj} = -\,(\dot{\partial}_m X^i)\;\tilde{R}^m_{pjk}\,\dot{x}^p + X^m\,\tilde{R}^i_{mjk} + X^{i\,-}|_{\,m}\,N^m_{kj}\;, \qquad (1.9)$$

where, $$\tilde{R}^h_{ijk} = \partial_k\tilde{\Gamma}^h_{ij} - \partial_j\tilde{\Gamma}^h_{ik} + \dot{\partial}_m\tilde{\Gamma}^h_{ik}\,\Gamma^m_{sj}\,\dot{x}^s - \dot{\partial}_m\tilde{\Gamma}^h_{ij}\,\Gamma^m_{sk}\,\dot{x}^s + \tilde{\Gamma}^p_{ij}\,\tilde{\Gamma}^h_{pk} - \tilde{\Gamma}^p_{ik}\,\tilde{\Gamma}^h_{pj}\,. \qquad (1.10)$$

In view of (1.10) the above result can be rewritten as

$$\tilde{R}^h_{ijk} = \partial_k\Gamma^h_{ji} - \partial_j\Gamma^h_{ki} + \dot{\partial}_m\Gamma^h_{ki}\,\Gamma^m_{js}\,\dot{x}^s - \dot{\partial}_m\Gamma^h_{ji}\Gamma^m_{ks}\,\dot{x}^s + \Gamma^p_{ji}\Gamma^h_{kp} - \Gamma^p_{ki}\,\Gamma^h_{jp}\,. \qquad (1.11)$$

Where the entities R^h_{ijk} and $\tilde{R}^h_{ijk}$ respectively defined by (1.8) and (1.11) are said to be curvature tensors, which becomes the duality in the nature of covariant derivatives defined by (1.7) and (1.9).

Now, we shall use the following identities [2]:

(a) $x^{i+}|_{\,k} = \dot{x}^{i\,-}|_{\,k} = 0$, (b) $R^i_{jk} \overset{\text{def}}{=} R^i_{hjk}\,\dot{x}^h$, (c) $R^i_j \overset{\text{def}}{=} R^i_{hj}\,\dot{x}^h$, (1.12)

(d) $R^i_{hjk} = -\,R^i_{hkj}$, (e) $R^i_i = (n-1)\,R$,

(f) $N^i_{jk} = -\,N^i_{kj} = \Gamma^i_{jk} - \Gamma^i_{kj}$, (g) $\Gamma^i_{hjk} \overset{\text{def}}{=} \dot{\partial}_h\Gamma^i_{jk}$.

Differentiating (1.4) partially with respect to $\dot{x}^k$ and using the result thus obtained, we have the following commutation formula,

$$\dot{\partial}_k\big(T^{i+}_j|_h\big) - (\dot{\partial}_k T^i_j)^+|_h = T^m_j\;\Gamma^i_{kmh} - T^i_m\;\Gamma^m_{kjh} - (\dot{\partial}_m T^i_j)\,\Gamma^m_{kph}\dot{x}^p, \qquad (1.13)$$

Differentiating (1.5) partially with respect to $\dot{x}^k$ and using the result thus the set of equation (1.13), becomes

$$\dot{\partial}_k\left(T_j^i{}^-|_h\right) - (\dot{\partial}_k T_j^i)^-|_h = T_j^m \ \tilde{\Gamma}_{kmh}^i - T_m^i \ \tilde{\Gamma}_{kjh}^m - (\dot{\partial}_m T_j^i) \ \tilde{\Gamma}_{kph}^m \dot{x}^p, \qquad (1.14)$$

Let $v^i(x)$ is a vector field of class C^2 defined over a region R of $F_n{}^*$ with this field we can associate an infinitesimal transformation of the form

$$\bar{x}^i = x^i + v^i(\mathrm{x})\ \mathrm{d}t, \qquad (1.15)$$

where $\mathrm{d}t$ is to be stated as an infinitesimal constant. We can interpret (1.15) by associating to every point x^i of $F_n{}^*$ a shift or displacement

$$dx^i = v^i(\mathrm{x})\ \mathrm{d}t \qquad (1.16)$$

where, it is natural to stipulate that the corresponding variation of the component $\dot{x}^i$ of the element of support is represented by

$$\bar{\dot{x}}^i = \dot{x}^i + (\partial_h v^i)\ \dot{x}^h\ \mathrm{d}t. \qquad (1.17)$$

Let $X^i(x,\dot{x})$ be a vector field defined over region R of $F_n{}^*$. We suppose that $X^i(x,\dot{x})$ is homogeneous of degree zero with respect to $\dot{x}^k$. This field shall be affected by variation (1.15) and (1.17).

Let $d^v X^i$ denote the following from (1.15) and (1.17) and therefore we obtain

$$d^v X^i = (\partial_k X^i)\ v^k\ \mathrm{d}t + (\dot{\partial}_h X^i)(\partial_k v^h)\ \dot{x}^k\ \mathrm{d}t. \qquad (1.18)$$

Cartan [8] has defined the covariant derivative of $T_{ij}(x,\dot{x})$ with respect to x^k as follows:

$$T_{ij}{}^-|_h = \partial_h T_{ij} - \dot{\partial}_k T_{ij} \dot{\partial}_h G^k - T_{kj}\ \Gamma_{ih}^{*k} - T_{ik} \Gamma_{jh}^{*k}\ , \qquad (1.19)$$

where, (a) $G_h^k \overset{\text{def}}{=} \dot{\partial}_h G^k$, (1.20)

(b) $2G^k \overset{\text{def}}{=} \gamma_{ij}^k\ (x,\dot{x})\ \dot{x}^i \dot{x}^j = \Gamma_{ij}^{*k}(x,\dot{x})\ \dot{x}^i \dot{x}^j$,

In view of (1.19) and (1.20), the set of equation (1.18) can be rewritten as

$$d^v X^i = (\partial_k X^i - \dot{\partial}_h X^i \partial_k G^l)\ v^k\ \mathrm{d}t + \dot{\partial}_h X^i\ (v^h{}^-|_k\ \dot{x}^k)\ \mathrm{d}t. \qquad (1.21)$$

However, if we interpret (1.15) not as a general shift but only as an infinitesimal coordinate transformation with which (1.16) will be consistent and if we denote by $\bar{X}^i$ the component of the field X^i in the new coordinate system then we will have

$$\bar{X}^i = (\dot{\partial}_j \bar{x}^i) X^j = (\delta_j^i + \partial_j v^i\ \mathrm{d}t) X^j. \qquad (1.22)$$

We can say that, $\bar{X}^i$ is the vector X^i displaced from $(x,\dot{x})$ to $(\bar{x},\bar{\dot{x}})$. Let us define

$$d^m X^i = \bar{X}^i - X^i = (\partial_j v^i) X^j\ \mathrm{d}t. \qquad (1.23)$$

The Lie derivative of the vector field X^i in the Finsler space $F_n{}^*$. can be defined by

$$\pounds_v X^i \overset{\text{def}}{=} d^v X^i - d^m X^i / \mathrm{dt} \quad \text{(Rund [2])}. \qquad (1.24)$$

With the help of (1.21) and (1.23), $\pounds_v X^i$ can be expressed as under:

$$\pounds_v X^i = (X^i{}^+|_k) v^k - (v^i{}^-|_k) X^k + \left(\dot{\partial}_h X^i\right)\left(v^h{}^-|_k\right) \dot{x}^k. \qquad (1.25)$$

In view of (1.12) and (1.25), it can be easily verified that the Lie-derivative of directional argument $\dot{x}^i$ vanishes identically, 1.e.,

$$\pounds_v \dot{x}^i = 0. \tag{1.26}$$

Also $\pounds_v X_i$ can be expressed as under:

$$\pounds_v X_i = (X_i{}^+|_k) v^k + (v^k{}^-|_i) X_k + (\dot{\partial}_h X_i)(v^h{}^-|_k)\dot{x}^k. \tag{1.27}$$

In view of (1.25) and (1.27), the Lie derivative for the mixed tensor field $T_j^i(x, \dot{x})$ can be written as

$$\pounds_v T_j^i = (T_j^i{}^+|_k) v^k - (v^i{}^-|_k) T_j^k + (v^k{}^-|_j) T_k^i - (\dot{\partial}_h T_j^i)(v^h{}^-|_k)\dot{x}^k. \tag{1.28}$$

In a similar manner, we find the Lie-derivative of an arbitrary tensor $T^{i_1 i_2 \dots i_r}_{j_1 j_2 \dots j_s}$ as given by

$$\pounds_v T^{i_1 i_2 \dots i_r}_{j_1 j_2 \dots j_s} = v^k \, T^{i_1 i_2 \dots\dots i_r}_{j_1 j_2 \dots j_s}{}^+|_k + \dot{\partial}_h T^{i_1 i_2 \dots i_r}_{j_1 j_2 \dots j_s}(v^h{}^-|_k \dot{x}^k) +$$
$$+ \sum_\mu T^{i_1 i_2 \dots\dots\dots i_r}_{j_1 j_2 \dots\dots j_{\mu-1}\, m\, j_{\mu+1} \dots j_s}\left(v^m{}^-|_{j_\mu}\right) - \sum_\nu T^{i_1 i_2 \dots i_{\nu-1}\, m\, i_{\nu+1} \dots i_r}_{j_1 j_2 \dots\dots\dots j_s}\left(v^{i_\nu}{}^-|_m\right). \tag{1.29}$$

Now, differentiating (1.28) partially with respect to $\dot{x}^k$ and subtracting the expression thus obtained from (1.28) we get the following commutation formula

$$\pounds_v(\dot{\partial}_k T_j^i) - \dot{\partial}_k(\pounds_v T_j^i) = 0, \tag{1.30}$$

The relation (1.30) shows that the operations of Lie-differentiation and partial differentiation with respect to directional arguments commute with each other in the Finsler space $F_n{}^*$ with non-symmetric connection with special reference to $\ominus$ − covariant derivative.

Using (1.5) and (1.28), we get

$$\pounds_v(T_j^i{}^-|_k) - (\pounds_v T_j^i)^-|_k = (T_j^i{}^-|_{kh} - T_j^i{}^-|_{hk}) v^h + T_j^i{}^-|_h \, v^h{}^-|_k + \{\dot{\partial}_h(T_j^i{}^-|_k) -$$
$$-(\dot{\partial}_h T_j^i)^-|_k\} v^h{}^-|_s \dot{x}^s - T_h^i\,(v^h{}^-|_j)^-|_k + T_j^h\,(v^i{}^-|_h)^-|_k - (\dot{\partial}_h T_j^i)(v^h{}^-|_s)^-|_k \dot{x}^s. \tag{1.31}$$

Also the Lie derivative of connection parameter with non-symmetric connection with reference to $\ominus$ − covariant derivative (Rund [2]), we can write

$$\pounds_v \tilde{\Gamma}^i_{jk} = (v^i{}^-|_j)^-|_k + (\dot{\partial}_r \tilde{\Gamma}^i_{kj})(v^r{}^-|_h)\dot{x}^h + v^h \tilde{\mathrm{R}}^i_{jkh}. \tag{1.32}$$

In view of (1.14), (1.32) and (1.31) reduces to its simplest form as

$$\pounds_v(T_j^i{}^-|_k) - (\pounds_v T_j^i)^-|_k = T_j^h \pounds_v \Gamma^i_{kh} - T_h^i \pounds_v \Gamma^h_{kj} - (\dot{\partial}_h T_j^i)(\pounds_v \Gamma^h_{ks})\dot{x}^s. \tag{1.33}$$

II. $\bar{R}$-CURVATURE COLLINEATION AND $\bar{R}$-PROJECTIVE MOTION IN A FINSLER SPACE $F_n{}^*$:

We will use following definitions in our discussions:

DEFINITION(2.1):

A Finsler space $F_n{}^*$ equipped with non-symmetric connection is said to be $\bar{R}$-symmetric or special symmetric if the curvature tensor $\tilde{R}^i_{jkh}$ with respect to $\ominus$ − covariant derivative satisfying

$$\tilde{R}^i_{hjk}{}^-|_m = 0. \tag{2.1}$$

On contracting (2.1) with respect to the indices i and h, we get

$$\tilde{R}_{jk} \; {}^{-}|_{m} = 0 \quad \text{where} \quad \tilde{R}^{i}_{ijk} = \tilde{R}_{jk} \tag{2.2}$$

DEFINITION(2.2):

A Finsler space $F_n{}^*$ is said to be an $\bar{R}$- affinely connected motion if

$$\dot{\partial}_l \tilde{\Gamma}^{i}_{jk} = 0 \; . \tag{2.3}$$

DEFINITION(2.3):

The infinitesimal point transformation $\bar{x}^i = x^i + v^i\,(x)dt$ is said to define a special $\bar{R}$ -curvature collineation in a Finsler space $F_n{}^*$ provided there exists a field $v^i(x)$ satisfying

$$\pounds_v \tilde{R}^{i}_{jkh} = 0. \tag{2.4}$$

DEFINITION(2.4):

A Finsler space $F_n{}^*$ is said to admit $\bar{R}$- Ricci collineation provided there exists a field $v^i(x)$ satisfying

$$\pounds_v \tilde{R}_{kh} = 0. \tag{2.5}$$

DEFINITION(2.5):

The infinitesimal point transformation $\bar{x}^i = x^i + v^i\,(x)dt$ defines an infinitesimal $\bar{R}$- projective motion in an $F_n{}^*$ if

$$\pounds_v \, \tilde{\Gamma}^{i}_{jk} = \delta^i_j \lambda_k + \delta^i_k \lambda_j + \lambda_{jk} \dot{x}^i \; , \tag{2.6}$$

Where λ is an arbitrarily chosen positively homogeneous scalar function of degree 1 in $\dot{x}\,'s$ and satisfying the following relations:

$$\text{(a) } \dot{\partial}_j \lambda = \lambda_j \;, \quad \text{(b) } \dot{\partial}_k \lambda_j = \lambda_{jk} \;, \quad \text{(c) } \lambda_{jk}\, \dot{x}^k = \lambda_j \;, \quad \text{(d) } \lambda_j \dot{x}^j = \lambda \; . \tag{2.7}$$

Using (2.6) and (1.15) in (1.33), we get

$$\pounds_v \tilde{R}^{i}_{jkh} = \delta^i_h (\lambda_k{}^{-}|_j - \lambda_j{}^{-}|_k) + \delta^i_k \lambda_h{}^{-}|_j - \delta^i_j \lambda_h{}^{-}|_k + 2\dot{x}^i \lambda_{[kj}{}^{-}|_{h]}, \tag{2.8}$$

where we have taken into account the fact that the $\ominus -$ covariant derivative of $\dot{x}^i$ and δ^i_j vanishes identically.

Let us now assume that the Finsler space F^*_n admits a $\bar{R}$ -projective motion as well as $\bar{R}$ -curvature collineation then from (2.8), we get

$$\delta^i_h (\lambda_k{}^{-}|_j - \lambda_j{}^{-}|_k) + \delta^i_{[k} \lambda_h{}^{-}|_{j]} + 2\dot{x}^i \lambda_{\,[kj}{}^{-}|_{h\,]} = 0, \tag{2.9}$$

Contracting (2.9) with respect to the indices i and j and noting (2.7) therefore, we get

$$\lambda_k{}^{-}|_h - n\lambda_h{}^{-}|_k + \lambda\,{}^{-}|_k = 0\, . \tag{2.10}$$

We now make the equation obtained after commutating the indices h and k in (2.8), we get

$$(1-n)\; \lambda_{\,[h}{}^{-}|_{k]} = 0. \tag{2.11}$$

So, we now state our first result as below:

Theorem (2.1):

In an affinely connected $\bar{R}$ –projective F_n^* , $\bar{R}$ -projective motion must be $\bar{R}$ -curvature collineation (2.11).

Contracting equation (2.8) with respect to the indices i and j and get

$$\pounds_v \tilde{R}_{kh} = \lambda_k{}^-|_h - n\lambda_h{}^-|_k + 2\dot{x}^i \lambda_k{}^-|_h = 0. \qquad (2.12)$$

Let us now make the assumption that the space F_n^* under consideration admits a $\bar{R} - Ricci$ collineation then from (2.12), we get

$$\lambda_k{}^-|_h - n\lambda_h{}^-|_k + 2\dot{x}^i \lambda_k{}^-|_h = 0. \qquad (2.13)$$

We now state our next results as:

Theorem (2.2):

In an affinely connected $\bar{R}$ –projective F_n^*, the necessary and sufficient condition that $\bar{R}$ -projective motion be $\bar{R}$ -Ricci collineation is that form of function $\lambda(x, \dot{x})$ is given by (2.13).

Applying the commutation formula (1.14) to the curvature tensor field $\tilde{R}^i_{jkh}$ and thereafter use (2.1), (2.6) and (2.7) and get

$$(\pounds_v \tilde{R}^i_{jkh})^-|_h = (\dot{\partial}_q \tilde{R}^i_{jkh})\lambda_l \dot{x}^q + (\dot{\partial}_l \tilde{R}^i_{jkh})^-|_p + (\dot{\partial}_s \tilde{R}^i_{jkh}) + \lambda_{hl}\tilde{R}^i_{jkq}\dot{x}^q + \lambda_h \tilde{R}^i_{jkl} + \lambda_{kl}\tilde{R}^i_{jh} + \lambda_k \tilde{R}^i_{jlh} + \lambda_{jl}\tilde{R}^i_{kh} + \lambda_j \tilde{R}^i_{lkh} + 2\lambda_l \tilde{R}^i_{jkh} - \lambda_{ql}\tilde{R}^q_{jkh}\dot{x}^i - \delta^i_l \lambda_q \tilde{R}^q_{jkh}\,. \qquad (2.14)$$

Transvecting (2.14) successively by $\dot{x}^k$ and $\dot{x}^h$ and thereafter using the equations given by (2.7), we get

$$(\pounds_v \tilde{R}^i_{jkh})^-|_h \dot{x}^k \dot{x}^h = \delta^i_l \lambda_q \tilde{R}^q_{jkh}\dot{x}^h \dot{x}^k + \lambda_{ql}\tilde{R}^q_j \dot{x}^i - 3\tilde{R}^i_j - \tilde{R}^i_l \lambda_j - \lambda\, \tilde{R}^i_{jlk}\dot{x}^h + \lambda\, \tilde{R}^i_{jh} - (\dot{\partial}_s \tilde{R}^i_{jkh})\lambda_l \dot{x}^h \dot{x}^k - \tilde{R}^i_{jkh}\lambda_{sl}\,. \qquad (2.15)$$

Now suppose the Finsler space F_n^* under consideration admits a special $\bar{R}$-curvature collineation characterized by (2.4) then from (2.15) we get

$$\delta^i_l \lambda_q \tilde{R}^q_{jkh}\dot{x}^h \dot{x}^k + \lambda_{ql}\tilde{R}^q_j \dot{x}^i - 3\tilde{R}^i_j - \tilde{R}^i_l \lambda_j - \lambda\, \tilde{R}^i_{jlk}\dot{x}^h + \lambda\, \tilde{R}^i_{jh} - (\dot{\partial}_s \tilde{R}^i_{jkh})\lambda_l \dot{x}^h \dot{x}^k - -\tilde{R}^i_{jkh}\lambda_{sl} = 0\,. \qquad (2.16)$$

Contracting (2.16) with respect to indices i and j and thereafter allowing a suitable interchange of the dummy indices, we get

$$\tilde{R}^q_{jkh}\lambda_q \dot{x}^h \dot{x}^k + 2\lambda_{ql}\tilde{R}^q_i \dot{x}^i - 4\tilde{R}^i_j \lambda_l - \lambda\, \tilde{R}_{lk}\dot{x}^h + \lambda\, \tilde{R}^i_{jh} - (\dot{\partial}_s \tilde{R}^i_{kh})\lambda_l \dot{x}^h \dot{x}^k - (\dot{\partial}_l \tilde{R}_{kh})\lambda_s \dot{x}^h \dot{x}^k - -\tilde{R}_{kh}\lambda_s = 0\,. \qquad (2.17)$$

Now our last results is stated as below:

Theorem (2.3):

In a special $\bar{R}$ –symmetric F_n^*, $\bar{R}$-projective motion must be special $\bar{R}$-curvature collineation given by equation (2.17) .

III. CONCLUSION

In this paper we have studied $\bar{R}$-Projective motion in a Finsler space ${F_n}^*$ equipped with non-symmetric connection with special reference to $\ominus -$ covariant derivative. We have studied $\bar{R}$ - Curvature collineation and Ricci- Collineation also in the above context . It has been found that in an affinely connected $\bar{R}$ –projective F_n^* , the $\bar{R}$ -projective motion consist of $\bar{R}$ -curvature collineation and $\bar{R}$ -Ricci collineation. We have also found that the $\bar{R}$-projective motion will be a special $\bar{R}$-curvature collineation in a special $\bar{R} -$ symmetric F_n^*.

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