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Study on Weyl – Sasakian Concircular Recurrent and Conharmonic Recurrent Spaces of Order Two

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ABSTRACT
In this paper we have studied Weyl- Sasakian Concircular, Weyl- Sasakian Conharmonic Recurrent Spaces and Ricci-recurrent space of order two and obtained some new results. In continuation we have also studied Weyl–Sasakian Concircular symmetric space, Weyl–Sasakian Conharmonic Symmetric Spaces and Weyl–Sasakian Ricci symmetric space of order two and obtained some new results.
Keywords: Weyl – Sasakian Concircular, Weyl - Sasakian Conharmonic, Ricci-recurrent space, symmetric space.

INTRODUCTION
Matsumoto [1] have studied and defined Kaehlerian space with parallel or Vanishing Bochner Curvature tensor. Negi and Rawat [2] have studied and defined Kaehlerian space with recurrent and symmetric Bochner Curvature tensor. Otsuki [3] and Walker [8] have defined Ruse's spaces of recurrent curvature and curves in Kaehlerian space. Rawat and Silswal [4] studied and defined Kaehlerian bi-recurrent and bi-symmetric spaces. Further, Rawat and Kumar [5] studied and defined Weyl–Sasakian projective and conformal bi-recurrent and bi-symmetric spaces. Also, Singh [6] have studied and defined Kaehlerian recurrent and Ricci-recurrent space of order two.

An n – dimensional Kaehlerian space K_n be a Riemannian space, which defined a tensor field F_i^h and satisfying by (Yano [9])

(1.1) $F_j^h F_h^i = -\delta_j^i$

(1.2) $F_{ij} = -F_{ji}$,

(1.3) $F_{ij} = F_i^a g_{aj}$

and (1.4) $F_{i,j}^h = 0$,

where the comma (,) denotes the covariant differentiation with respect to the metric tensor g_{ij} of the Riemannian space. The Riemannian Curvature tensor field is denoted by R_{ijk}^h , such that

(1.5) $R_{ijk}^h = \partial_j \Gamma_{ik}^h - \partial_k \Gamma_{ij}^h + \Gamma_{ik}^l \Gamma_{mj}^h - \Gamma_{ij}^l \Gamma_{lk}^h$.

The Ricci tensor and the Scalar curvature in S_n are respectively given by

(1.6) $R_{ij} = R_{ijh}^h$ and $R = g^{ij} R_{ij}$,

where scalar R is called the scalar curvature. If we define a tensor S_{ij} by

(1.7) $S_{ij} = F_i^h R_{hj}$,

Then, we have

(1.8) $S_{ij} = -S_{ji}$,

(1.9) $F_i^h S_{hj} = -S_{ih} F_j^h$,

and (1.10) $F_i^h S_{jk,h} = R_{ji,k} - R_{ki,j}$.

Yano [9] is defined the metric tensor g_{ij} and the Ricci-tensor R_{ij} are mixed in *i* and *j*. Therefore, we get

(1.11) $g_{ij} = g_{pq} F_i^p F_j^q$,

and (1.12) $R_{ij} = R_{pq} F_i^p F_j^q$.

Weyl-Concircular curvature and Weyl- Conharmonic curvature tensor in S_n respectively give by the following form

(1.13) $M_{ijk}^h = R_{ijk}^h - \frac{R}{n(n-1)}(g_{ij}\delta_k^h - g_{ik}\delta_j^h)$

and (1.14) $L_{ijk}^h = R_{ijk}^h - \frac{1}{(n-2)}(g_{ij}R_k^h - g_{ik}R_j^h + R_{ij}\delta_k^h - R_{ik}\delta_j^h)$

From equations (1.13) and (1.14), we can have

(1.15) $L_{ijk}^h = M_{ijk}^h + \frac{R}{n(n-1)}(g_{ij}\delta_k^h - g_{ik}\delta_j^h) - \frac{1}{(n-2)}(g_{ij}R_k^h - g_{ik}R_j^h + R_{ij}\delta_k^h - R_{ik}\delta_j^h)$

Weyl- Sasakian Concircular Recurrent Spaces and Conharmonic Recurrent Spaces of order two:

We have taken following definitions:

Definition (2.1): (Singh [7]) A Sasakian space satisfying the condition

(2.1) $R_{ijk,pq}^h - \lambda_{pq} R_{ijk}^h = 0$,

where non-zero tensor λ_{pq}, is said to be Sasakian recurrent space of order two and denoted by S_n^{**} and is said to be Sasakian Ricci-recurrent space of order two, if it satisfies by

(2.2) $R_{ij,pq} - \lambda_{pq} R_{ij} = 0$,

Multiplying by g^{ij} in equation (2.2), we have

(2.3) $R_{,pq} - \lambda_{pq} R = 0$,

Definition (2.2): A Sasakian space S_n satisfying the Condition

(2.4) $M_{ijk,pq}^h - \lambda_{pq} M_{ijk}^h = 0$,

where non-zero tensor λ_{pq}, said to be Weyl – Sasakian Concircular recurrent space of order two and is denoted by M^{**}- space.

Definition (2.3): A Sasakian space S_n satisfying the Condition

(2.5) $L_{ijk,pq}^h - \lambda_{pq} L_{ijk}^h = 0$,

where non-zero tensor λ_{pq}, is said to be Weyl–Sasakian Conharmonic recurrent space of order two and is denoted by L^{**}- space. Now, we have taken the following Theorems in the form of results:

Theorem (2.1):

In a Sasakian space S_n, A Sasakian recurrent space of order two is Weyl – Sasakian Concircular recurrent space of order two.

Proof : The set of equation (1.13) covariantly differentiating with respect to x^p, and after differentiation obtained result again covariantly differentiate with respect to x^q, we have

(2.6) $M_{ijk,pq}^h = R_{ijk,pq}^h - \frac{R_{,pq}}{n(n-1)}(g_{ij}\delta_k^h - g_{ik}\delta_j^h)$.

Transvecting the set of equation (1.13) by λ_{pq}, and subtracting the set of equation from (2.6) we have

(2.7) $M_{ijk,pq}^h - \lambda_{pq} M_{ijk}^h = R_{ijk,pq}^h - \lambda_{pq} R_{ijk}^h - \frac{(R_{,pq} - \lambda_{pq} R)}{n(n-1)}(g_{ij}\delta_k^h - g_{ik}\delta_j^h)$.

If the space is $S_n{}^{**}$- space, then the set of equations (2.1) and (2.2) are satisfied and using the set of equations (2.1) and (2.2) in (2.7), we have

$M_{ijk,pq}^h - \lambda_{pq} M_{ijk}^h = 0$, this equation shows that (2.4) i.e., the space is Weyl – Sasakian Concircular recurrent space of order two or M^{**} - space. Hence completes the proof.

Theorem (2.2):

In a Sasakian space S_n, A Sasakian recurrent space of order two is Weyl–Sasakian Conharmonic recurrent space of order two.

Proof : Theset of equation (1.14) covariantly differentiating w.r. to x^p, after differentiation obtained result again differentiate covariantly with respect to x^q, we have

(2.8) $L^h_{ijk,pq} = R^h_{ijk,pq} - \frac{1}{(n-2)}(g_{ij}R^h_{k,pq} - g_{ik}R^h_{j,pq} + R_{ij,pq}\delta^h_k - R_{ik,pq}\delta^h_j)$.

Transvecting (1.14) by λ_{pq}, then subtracting the set equation from (2.8), we have

(2.9) $L^h_{ijk,pq} - \lambda_{pq}L^h_{ijk} = R^h_{ijk,pq} - \lambda_{pq}R^h_{ijk} + \frac{1}{(n-2)}\ [(R^h_{k,pq} - \lambda_{pq}R^h_k)g_{ij} - (R^h_{j,pq} - \lambda_{pq}R^h_j)g_{ik} -$

$-(R_{ij,pq} - \lambda_{pq}R_{ij})\delta^h_k - (R_{ik,pq} - \lambda_{pq}R_{ik})\delta^h_j$.

If the space is $S_n{}^{**}$- space, then the set of equations (2.1), (2.2) and (2.3) are satisfied and using the set of equations (2.1), (2.2) and (2.3) in equation (2.9), we have

$L^h_{ijk,pq} - \lambda_{pq}L^h_{ijk} = 0$,

which implies that the space isWeyl–Sasakian Conharmonic recurrent space of order two. Hence completes the proof.

Theorem (2.3):

In a Sasakian space S_n, if the space is Ricci-recurrent of order two and the space is Weyl – Sasakian Concircular recurrent space of order twothen the space is Weyl–Sasakian Conharmonic recurrent space of order two.

Proof: The set of equation(1.15) covariantly differentiating with respect to x^p, and after differentiation obtained result again differentiate covariantly w.r.to x^q, we have

(2.10) $L^h_{ijk,pq} = M^h_{ijk,pq} + \frac{R_{,pq}}{n(n-1)}(g_{ij}\delta^h_k - g_{ik}\delta^h_j)$

$-\frac{1}{(n-2)}(g_{ij}R^h_{k,pq} - g_{ik}R^h_{j,pq} + R_{ij,pq}\delta^h_k - R_{ik,pq}\delta^h_j)$.

Transvecting equation (1.15) byλ_{pq}, then subtracting the set of equation from (2.10), we have

(2.11) $L^h_{ijk,pq} - \lambda_{pq}L^h_{ijk} = M^h_{ijk,pq} - \lambda_{pq}M^h_{ijk} + \frac{(R_{,pq} - \lambda_{pq}R)}{n(n-1)}(g_{ij}\delta^h_k - g_{ik}\delta^h_j)$ -

$-\frac{1}{(n-2)}[\ (R^h_{k,pq} - \lambda_{pq}R^h_k)g_{ij} - (R^h_{j,pq} - \lambda_{pq}R^h_j)g_{ik} +$

$+(R_{ij,pq} - \lambda_{pq}R_{ij})\delta^h_k - (R_{ik,pq} - \lambda_{pq}R_{ik})\delta^h_j)]$.

Using the set of equation (2.1), (2.2), (2.3), (2.4), (2.5) in (2.11), we have

$L^h_{ijk,pq} - \lambda_{pq}L^h_{ijk} = 0$,

which implies that the space is Weyl–Sasakian Conharmonic recurrent space of order two. Hence competes the proof.

Theorem (2.4):

In the scalar recurrent of order two, the space is Weyl–Sasakian Conharmonic recurrent space of order two iff the space is Sasakian recurrent space of order two.

Proof: Let M^{**}- space be $S_n{}^{**}$- space, then the set of equation (2.1) and (2.4) are satisfied and using (2.3) in (2.7), we have

(2.12) $(R_{,pq} - \lambda_{pq}R\ \ )(g_{ij}\delta^h_k - g_{ik}\delta^h_j) = 0$.

From (2.12) we have obtain the result which shows that the space is scalar recurrent of order two.

Conversely, let the M^{**}- space be scalar recurrent of order two, then the set of equation (2.3) is satisfied, then using the set of equations (2.3) and (2.4) in (2.7), we have

$(R^h_{ijk,pq} - \lambda_{pq} R^h_{ijk}) = 0,$

which implies that the space is Sasakian recurrent space of order two. Hence completes the proof.

Weyl–Sasakian Concircular Symmetric Spaces and Conharmonic Symmetric Spaces of order two:

Definition (3.1) : A Sasakian space S_n satisfying the Condition

(3.1) (a) $R^h_{ijk,pq}$=0, or (b) $R_{ijkl,pq} = 0$,

is called Sasakian symmetric space of order two and is said to be Sasakian Ricci symmetric space of order two, if it satisfies by

(3.2) $R_{ij,pq}$= 0

Multiplying equation (3.2) by g^{ij}, we have

(3.3) $R_{,pq}$=0.

From the set of equation (3.1), we have obtained the result in the form of theorem:

Theorem (3.1):

A Sasakian symmetric space of order two is Sasakian Ricci–symmetric space of order two.

Definition (3.2): A Sasakian space S_n satisfying the Condition

(3.4) (a) $M^h_{ijk,pq}$ or (b) $M_{ijkl,pq} = 0$,

is said to be Weyl – Sasakian Concircular Symmetric space of order two.

Definition (3.3) : A Sasakian space S_n satisfying the Condition

(3.5) (a) $L^h_{ijk,pq}$=0, or (b) $L_{ijkl,pq} = 0$,

is said to be Weyl–Sasakian Conharmonic symmetric space of order two.

Theorem (3.2):

A Sasakian symmetric space of order two is Weyl–Sasakianconcirular symmetric space of order two.

Proof: Let the space is Sasakian symmetric space of order two, then equations (3.1) and (3.2) are satisfied and using the set of equations (3.1) and (3.2) in (2.6), we have

$M^h_{ijk,pq}$=0 ,

which implies that the space is Weyl – SasakianConcirular symmetric space of order two.

Theorem (3.3):

A Sasakian symmetric space of order two is Weyl–Sasakian Conharmonic symmetric space of order two.

Proof : Let the space is Sasakian symmetric space of order two, then the set of equations (3.1) and (3.2) are satisfied and using the set of equations (3.1) and (3.2) in (2.8), we have

$L^h_{ijk,pq}$=0,

which implies that the space is Weyl–Sasakian Conharmonic symmetric space of order two.

Theorem (3.4):

In a Sasakian space S_n, if the Space is Ricci–symmetric of order two and the Space is Weyl–Sasakian Concircular symmetric space of order two then the Space is Weyl –Sasakian Conharmonic symmetric space of order two.

Proof : Let the space is Sasakian Ricci - symmetric space of second order and the Space is Weyl–Sasakian Concircular symmetric space of order two then the set of equation (3.4) and (3.5) are satisfies and using the set equations (3.2), (3.4), (3.5) in (2.10), we have

$L^h_{ijk,pq} = 0,$

which implies that the space is Weyl –Sasakian Conharmonic symmetric space of order two.

CONCLUSION

In this paper we have studied a Sasakian recurrent space of order two is Weyl- Sasakian Concircular recurrent space of order two, Weyl–Sasakian Conharmonic recurrent space of order two also we have obtained if the space is Ricci-recurrent of order two and the space is Weyl – Sasakian Concircular recurrent space of order two then the space is Weyl–Sasakian Conharmonic recurrent space of order two. In the scalar recurrent of order two, the space is Weyl–Sasakian Conharmonic recurrent space of order two if the space is Sasakian recurrent space of order two and conversely also true. In this continuation we have also studied a Sasakian symmetric space of order two is Sasakian Ricci–symmetric space of order two, Weyl–Sasakianconcirular symmetric space of order two, Weyl–Sasakian Conharmonic symmetric space of order two. Also if the Space is Ricci–symmetric of order two and the Space is Weyl–Sasakian Concircular symmetric space of order two then the Space is Weyl –Sasakian Conharmonic symmetric space of order two.

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