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Upgrade of CCW Prediction for Cylindrical Weak Shock Waves in Strong Magnetic Field with Self- Gravitating Gas

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ABSTRACT

In this paper CCW Method used for study the motion of cylindrical weak shock waves with strong magnetic field in a self-gravitating gas in the presence of constant axial and azimuthal components of magnetic field. Consider density ($\rho_0 = \rho' r^{-\omega}$) decrease in atmospheric strong magnetic field and derive the analytical expression for flow variables of weak shock with strong magnetic field. In the end of e.o.d behind the flow variables have been included and modified forms of analytical expression for flow variables so obtained have been numerically computed only at psfl.

Keywords :

e.o.d $\rightarrow$ Effect of overtaking disturbances

$H_{\theta_0}, H_{z_0} \rightarrow$ Axial magnetic and azimuthal magnetic field.

$H_{\theta_0} = H_{z_0}$ = constant.

FD$\rightarrow$ Free propagation

Psfl$\rightarrow$ Permissible shock front location

$\rho_0 = \rho' r^{-\omega} \rightarrow$ Density distribution

1.1 INTRODUCTION :

The study focus on method of characteristics and similarity have been used for the motion of shock/hydro magnetic weak shock wave through uniform and non-uniform media. similarity method work only for strong shocks but characteristics method works for both weak and strong shocks.

In the paper the e.o.d. behind the flow on the propagation of diverging cylindrical weak shocks waves through an ideal and electrically perfectly conducting self-gravitating gas

in the presence of a strong magnetic field having constant axial (H_{θ_0}) and azimuthal (H_{z_0}) components. The density in the unperturbed state has been consider as $\rho_0 = \rho' r^{-\omega}$ where ρ' density at the plane / exes of symmetry, ω non-dimensional constant.

The propagation of diverging cylindrical shock wave has been studied by CCW method for **weak shock with strong magnetic field only**.

The analytical relation for shock strength, shock velocity and pressure have been detect for strong magnetic field.

1.2 PRIMARY EQUATION :

The equation governing the cylindrical flow at the gas and constant axial and azimuthal components are written as.

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial r}+\frac{1}{\rho}\frac{\partial p}{\partial r}+\frac{\mu}{2\rho}\frac{\partial}{\partial r}(H_{\theta_0}^2+H_{z_0}^2)+\frac{\mu}{\rho}\frac{H_0^2}{r}+\frac{Gm}{r^2}-0,$$
$$\frac{\partial \rho}{\partial t}+u\frac{\partial \rho}{\partial r}+\rho\left(\frac{\partial u}{\partial r}+\frac{\alpha u}{r}\right)=0,$$
$$\frac{\partial p}{\partial t}+u\frac{\partial p}{\partial r}+\gamma p\left(\frac{\partial u}{\partial r}+\frac{\alpha u}{r}\right)=0, \qquad (1)$$
$$\frac{\partial H_\theta}{\partial t}+u\frac{\partial H_\theta}{\partial r}+H_\theta\frac{\partial u}{\partial r}=0,$$
$$\frac{\partial H_z}{\partial t}+u\frac{\partial H_z}{\partial r}+H_z\frac{\partial u}{\partial r}=0,$$
$$\frac{\partial m}{\partial r}-2\pi\rho r=0$$

Where $r\rightarrow$ radial co-ordinate $u,\rho,\rho H_Z,H_\theta,\mu$ and m are. respectively, the velocity of particle, thy density, the pressure, azimuthal and axial components at magnetic field, permeability of gas and mass inside a cylinder of unit cross-section.

1.3 BOUNDARY LIMITATION :

The Boundary limitation for diverging cylindrical weak shock waves written in term of single parameter.

$$\xi=\frac{\rho}{\rho_0} \quad as \quad \rho=\rho_0\xi, \quad H=H_0\xi \ , \quad u=\frac{\xi-1}{\xi}U \ , \qquad (2)$$

$$U^2=\frac{2\xi}{(\gamma+1)-(\gamma-1)\xi}\left[a_0^2+\frac{b_0^2}{2}\left\{(2-\gamma)\xi+\gamma\right\}\right]$$

$$P=P_0+\frac{2\rho_0(\xi-1)}{(\gamma+1)-(\gamma-1)\xi}\left[a_0^2+\frac{(\gamma-1)}{4}b_0^2\left(\xi+1\right)^2\right]$$

where 'o' stande for the state immediately ahead of the shock front, U is the shock strength, a_0 is the sound speed $\left(\sqrt{\frac{\gamma p_0}{\rho_0}}\right)$ and b_0 is the alfven speed $\left(\frac{\mu H_0^2}{\rho_0}\right)^{\frac{1}{2}}$.

1.4 CONDITION FOR WEAK SHOCK:-For every weak shock

$$\rho/\rho_0=1+\varepsilon(r) \qquad (3)$$

But $\varepsilon(r)<<1$. now we consider when magnetic field is strong

then $b_0^2>>a_0^2, i.e., \quad \mu H_0^2>>\gamma p_0$ by using equation (3)

The Boundary condition written as

$$\rho = \rho_0 (1+\varepsilon), \;\; H_\theta = H_{\theta_0} (1+\varepsilon), \;\; H_z = H_{z_0} (1+\varepsilon), \tag{4}$$

$$U = \left(1+\frac{3}{4}\varepsilon\right) b_0, \;\; P = P_0 (1+\gamma\varepsilon) \;\; and \;\; u = \varepsilon b_0$$

1.5 CHARACTERISTIC EQUATION : For diverging cylindrical weak shock characteristic form of the system of equation (1) is easily obtained by forming a linear combination of first and third equation of (1) in only one direction in (r, t,) plane can be written as.

$$dp + \mu H_\theta dH_\theta + \mu H_z dH_z + \rho c du + \mu H_\theta^2 \frac{dr}{r} + \rho c^2 \frac{u}{u+c}\frac{dr}{r} + \rho c \frac{Gm}{u+c}\frac{dr}{r^2} = 0 \quad (5)$$

The equation (5) represent the characteristic form of the system of equation (1) for diverging shock. In order to estimate the strength of overtaking disturbances and independent C_+ characteristic is considered.

The differential relation valid across C_+ disturbances obtained by replacing c by -c in equation (5) and written as.

$$dp + \mu H_\theta dH_\theta + \mu H_z dH_z + \rho c du + \mu H_\theta^2 \frac{dr}{r} + \rho c^2 \frac{u}{u-c}\frac{dr}{r} + \rho c \frac{Gm}{u-c}\frac{dr}{r^2} = 0 \tag{6}$$

equation (6) represent characteristic form of equation (1) for Conversing shock.

1.6 ANALYTICAL RELATIONS FOR FLOW VARIABLES :

To substitute the shock condition (4) in to (5) and (6) and a first order differential equation in $\varepsilon(r)$ or U is obtained which is the determine the shock the equilibrium state of the gas as.

$$\frac{\partial}{\partial t} = 0 = u, \; H_{z_0} = \text{constant} = H_{\theta_0} \tag{7}$$

$$\rho_0 = \rho^1 r^{-w} \tag{8}$$

with the help of equation (7) and first of (1) pressure as.

$$\frac{p_0}{G\rho^{'2}} = K - \beta_2^2 D \log r - K_1 r^{1-2w} \tag{9}$$

where K is constant and

$$\frac{a_0}{a^1} = \left(\frac{\gamma r^w}{D}\left(K - \beta_2^2 D \log r - K_1 r^{1-2w}\right)\right)^{\frac{1}{2}} \tag{10}$$

where $K_1 = \dfrac{2\pi}{(1-2\omega)(2-\omega)}$

$D = \dfrac{a'^2}{G\rho'^2}$ and $\beta_2^2 = \dfrac{\mu H_{\theta_0}^2}{\gamma p'}$

ρ' it the density at the plane of symmetry in unperturbed state and G is universal Gravitational constant.

1.7 CONDITION FOR WEAK SHOCK WITH STRONG MAGNETIC FIELD (WSSMF) :

By using equation (4), (5) we get

$$\frac{d\varepsilon}{\varepsilon} + \frac{1}{2}\left(1 - \frac{1}{2}\frac{\gamma p_0}{\mu H_0^2}\right)\left(\frac{\gamma p_0}{\mu H_0^2} + \frac{dp_0}{p_0} + \frac{db_0}{b_0} + \frac{2\mu H_{\theta_0}^2}{\mu H_0^2}\frac{dr}{r} + \frac{dr}{r}\right) = 0 \tag{11}$$

with help of (11) we get

$$\boxed{\varepsilon_+(r) = K_2^1 r^{A_1} \exp.(\Delta_1)} \qquad \textbf{(For Free propagation)} \qquad (12)$$

where K_2^1 is a constant of integration

$$\Delta_1 = A_2^1 \frac{r^{1-2\omega}}{1-2\omega} + A_3^1 \frac{r^{2-4\omega}}{2-4\omega} + A_4^1 \frac{r^{2-6\omega}}{2-6\omega} + A_5^1\left(\log r - \frac{1}{1-2\omega}\right)\frac{r^{1-2\omega}}{1-2\omega},$$

$$A_1^1 = \frac{\beta_2^2 D}{2}\left(1-\frac{\gamma K}{\beta^2}\right) + \frac{\gamma K}{2D\beta^2}(\omega+5) - \frac{\omega}{4} - \frac{3}{2} + \left(1-\frac{\gamma K}{2D\beta^2}\right),$$

$$A_2^1 = \frac{\gamma K_1}{8D\beta^2}(2-K\omega) - \frac{\beta_2^2}{4K\beta^2} + \frac{K_1}{2K}\left(1-2\omega+\frac{\beta_2^2 D}{K}\right)\left(1-\frac{\gamma K}{2\beta^2 D}\right),$$

$$A_3^1 = \frac{K_1^2}{K}(1-2\omega)\left(1-+\frac{\gamma K}{4\beta^2 D}\right) - \frac{K_1}{K}\frac{\gamma}{4\beta^2 D}\left(1-2\omega+\frac{\beta_2^2 D}{K}\right),$$

$$A_4^1 = (2\omega-1)\frac{K_1^2}{K}\frac{\gamma}{4\beta^2 D},$$

$$A_5^1 = \frac{K_1}{K.}\left(1-2\omega+\frac{\beta_2^2 D}{K}\right)\frac{\gamma\beta_2^2}{4\beta^2} + (1-2\omega)\left(\frac{\gamma K}{8D\beta^2}\right)\beta_2^2 D\frac{K_1^1}{K}$$

equation (12) describes for Free Propagation similarly the propagation parameter ε(r) which include the e.o.d., behind the flow on the motion of weak shock in strong magnetic field written as.

$$\boxed{\varepsilon_-(r) = K_1^* r^{K_2} \exp.(\Delta_2)} \qquad \textbf{(For e.o.d)} \qquad (13)$$

where K_1^* is a constant of integration

$$\Delta_2 = K_3^/ \frac{r^{1-2\omega}}{1-2\omega} + K_4^/ \frac{r^{2-4\omega}}{2-4\omega} + K_5^/ \frac{r^{3-6\omega}}{3-6\omega} - K_6^/ \frac{r^{-(1-3\omega)}}{(1+3\omega)},$$

$$K_2 = \frac{\beta_2^2 D}{K} + \frac{\beta_2^2}{\beta^2}\left(\frac{\gamma}{2}-1\right) + \omega\left(\frac{1}{2}+\frac{\gamma K}{4\beta^2 D}+\frac{\beta^2 D}{\gamma K}\right) - \frac{1}{2} + \frac{\beta^2 D}{\gamma K} - 2\frac{\beta_2^2 D}{\gamma K}$$

$$K_3^/ = \frac{1}{K}\left(1+\frac{\gamma K}{2\beta^2 D}\right)\left\{\frac{K_1}{K}\beta_2^2 D + K_1(1-2\omega)\right\} - \frac{\gamma K_1}{2\beta^2}$$

$$K_4^/ = \frac{(1-2\omega)}{K}\left(1+\frac{\gamma K}{2\beta^2 D}\right) - \frac{\gamma K_1}{2\beta^2 D}\left\{\frac{K_1}{K}\beta_2^2 D + K_1(1-2\omega)\right\}$$

$$K_5^/ = -\frac{\gamma K_1^2}{2\beta^2 D}(1-2\omega) \quad , \quad K_6^/ = -\frac{4\pi}{\gamma K(2-\omega)},$$

1.8 ANALYTICAL EXPRESSIONS FOR FLOW VARIABLES FOR SS :

The analytical expression for flow variable of weak cylindrical shock in strong magnetic field may obtain with the help of equation (12), (13) and (14) we get for both **F.P.** and **e.o.d.**

Flow variable for F.P

$$\frac{U}{a_0}=\left[1+\frac{3}{4}\left\{K_2^{/}r^{A_1^{/}}\exp.(\Delta_1)\right\}\right]\left(\frac{\beta^2 D}{\gamma\left(K-\beta_2^2 D\log r-K_1 r^{1-2\omega}\right)}\right)^{\frac{1}{2}}$$

$$\frac{U}{\sqrt{G\rho^{/}}}\left[1+\frac{3}{4}\left\{K_2^{/}r^{A_1^{/}}\exp.(\Delta_1)\right\}\right]\sqrt{\beta^2 D\gamma r^{\omega}}$$

$$\frac{P}{G\rho^{/2}}=\left(K-\beta_2^2 D\log r-K_1 r^{1-2\omega}\right)\left[1+\gamma\left(K_2^{/}r^{A_1^{/}}\exp.(\Delta_1)\right)\right]$$

$$\frac{u}{\sqrt{G\rho^{/}}}=\left[\gamma r^{\omega}\beta^2\left(K-\beta_2^2 D\log r-K_1 r^{1-2\omega}\right)\right]^{\frac{1}{2}}\left[K_2^{/}r^{A_1^{/}}\exp.(\Delta_1)\right]$$

$$\frac{\rho}{\rho^{/}}=r^{-\omega}\left[1+\left\{K_2^{/}r^{A_1^{/}}\exp.(\Delta_1)\right\}\right]$$

Flow variable for e.o.d.

$$\frac{U}{a_0}=\left[1+\frac{3}{4}\left\{K_2^{*}r^{K_2}\exp.(\Delta_2)\right\}\right]\left(\frac{\beta^2 D}{\gamma\left(K-\beta_2^2 D\log r-K_1 r^{1-2\omega}\right)}\right)^{\frac{1}{2}}$$

$$\frac{U}{\sqrt{G\rho^{/}}}=\left[1+\frac{3}{4}\left\{K_1^{*}r^{K_2}\exp.(\Delta_2)\right\}\right]\sqrt{\beta^2 D\gamma r^{\omega}}$$

$$\frac{P}{G\rho^{/2}}=\left(K-\beta_2^2 D\log r-K_1 r^{1-2\omega}\right)\left[1+\gamma\left(K_1^{*}r^{K_2}\exp.(\Delta_2)\right)\right]$$

$$\frac{u}{\sqrt{G\rho^{/}}}=\left[\gamma r^{\omega}\beta^2\left(K-\beta_2^2 D\log r-K_1 r^{1-2\omega}\right)\right]^{\frac{1}{2}}\left[K_1^{*}r^{K_2}\exp.(\Delta_2)\sqrt{\beta^2 D\gamma r^{\omega}}\right]$$

$$\frac{\rho}{\rho^{/}}=r^{-\omega}\left[1+\left\{K_1^{*}r^{K_2}\exp.(\Delta_2)\right\}\right]$$

1.9 TABLE AND GRAPHS FOR FLOW VARIABLES OF WSSMF :

S.N.	Variation of flow variable	F.P.	EOD	C.P. %
1	$\left(\frac{U}{a_0}\right)$ Shock strength versus r	Increase	Increase	**0.2492% to 0.2998%**
	β_2^2	Increase	Decrease	
	β^2	Increase	Increase	
	ω	Decrease	Decrease	
2	$\left(\frac{U}{\sqrt{G\rho'}}\right)$ Shock velocity			**0.2492% to 0.2998%**
	r	Increase	Increase	
	β_2^2	Increase	Decrease	
	β^2	Increase	Increase	
	ω	Decrease	Decrease	
3	$\left(\frac{u}{\sqrt{G\rho'}}\right)$ Particle velocity			
	r	Decrease	Increase	[illegible]

	β_2^2	Increase	Decrease	
	β^2	Increase	Increase	
	ω	Decrease	Decrease	

Flow variables also increase ***with*** ξ**.**

GRAPHS FOR FLOW VAFIABLES:

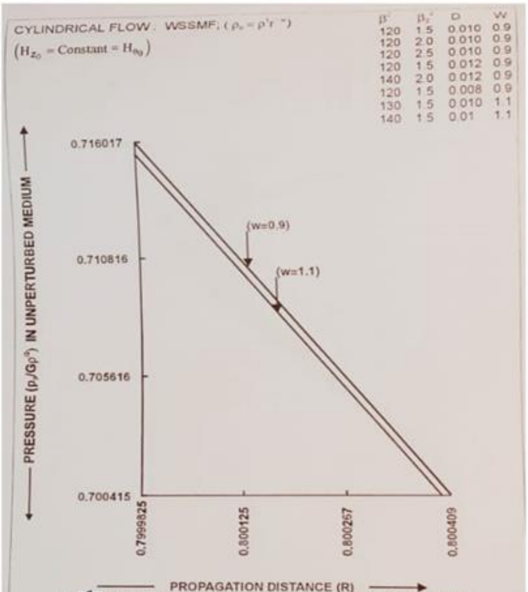

FIG 1 VARIATION OF THE PRESSURE IN UNPERTURBED MEDIUM WITH PROPAGATION DISTANCE (r) FOR CYLINDRICAL WSSMF IN A SGG.

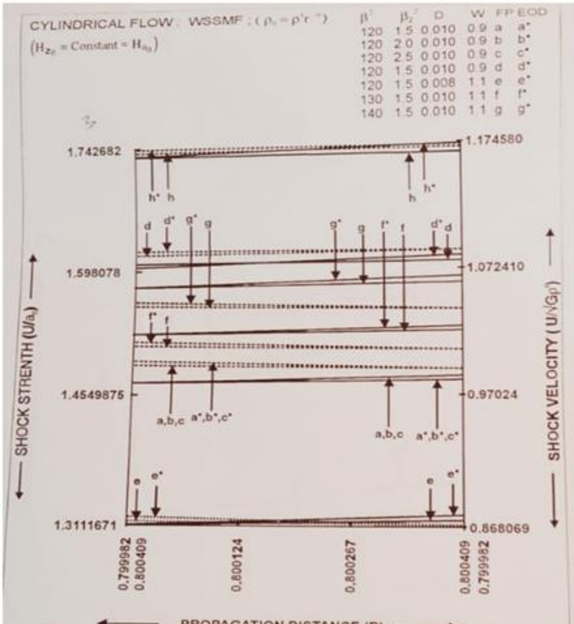

FIG 2 VARIATION OF SHOCK STRENGTH (—) AND SHOCK VELOCITY (---) WITH PROPAGATION DISTANCE (r) FOR CYLINDRICAL WSSMF IN A SGG

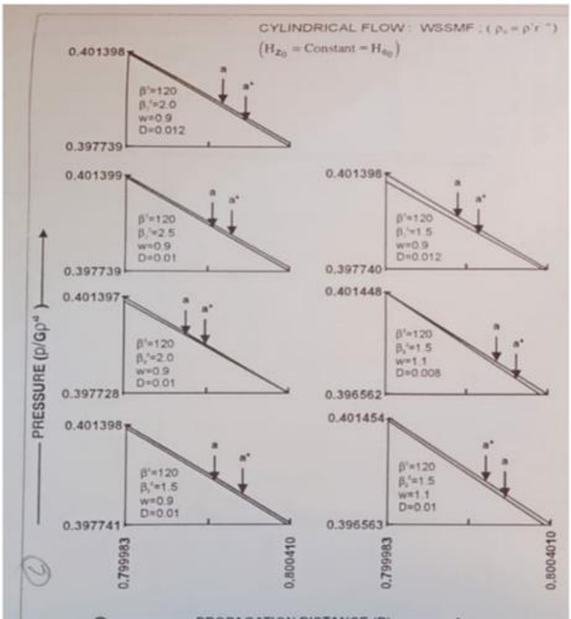

FIG 3 VARIATION OF PRESSURE WITH PROPAGATION DISTANCE (r) FOR CYLINDRICAL WSSMF IN A SGG.

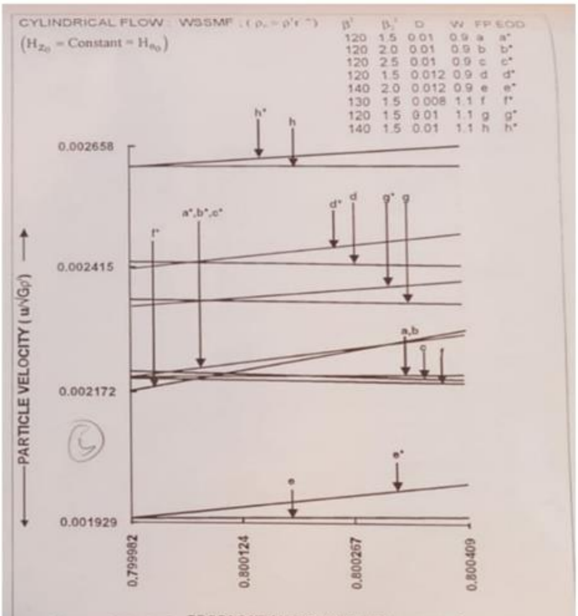

FIG 4 VARIATION OF PARTICLE VELOCITY WITH PROPAGATION DISTANCE (r) FOR CYLINDRICAL WSSMF IN A SGG.

1.9 RESULT AND DISCUSSION:

The modified analytical expression include the e.o.d. behind the flow on the motion of the diverging cylindrical shock waves in a self-gravitating gas in presence of strong magnetic field (β^2).

Overall the strength of shock, the velocity of shock and the velocity of particle increase whereas as the pressure and the density decrease with propagation distance r. Increasing in β_2^2 from 1.5 to 2.0 lead the decrease the strength of shock the velocity of GRAshock, the particle velocity.

all flow variable increase with increase in β^2, ω.

for above result taking -

$\varepsilon(r) = 0.00223\ \ at\ \ r = 0.8,\ \ \gamma = 1.4,\ \ P_0 = 0.4G\rho'^2, \beta_2^2 = 1.5, 2.0,\ \ \omega = 0.9, 1.1,\ \ \beta^2 = 120,130,140$

(strong magnetic field) and including the e.o.d. results varying from -3.912% to 4.4427%.

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