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Special $\bar{R}$-Projective Motion in a Finsler Space

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ABSTRACT
We have studied the Special $\bar{R}$-Projective motion in a Finsler space $F_n{}^*$ equipped with a non-symmetric connection. New results, in the view of Special $\bar{R}$ - Curvature collineation and Ricci- Collineation, has been obtained and presented.

Keywords: Non-Symmetric Connection, $\bar{R}$-Projective Motion, $\bar{R}$-Curvature Collineation, Ricci- Collineation.

I. INTRODUCTION

Use of infinitesimal transformations, which in terms defines motion, has been discussed in details by Misra [1]. Davies [2] has also studied similar problem where he has generalized the Lie-derivatives to the Finsler space and its application to the theory of subspaces. By considering the infinitesimal point transformation in Lie-derivatives of an arbitrary vector $X^i(x,\dot{x})$ and the symmetric connection parameter $\Gamma_{jk}^{*i}(x,\dot{x})$, has been discussed in details by Rund, [3] and Yano, [4]. The concept of curvature collineation in a Riemannian space and its properties is discussed in detail by Katzin, Levine and Davis [5]. They showed that a Riemannian space V_n, admits a curvature collineation provided that there exists a vector & $v^i(x)$ such that $\pounds_v R^i_{jkh} = 0$ where R^i_{jkh} is the Riemannian curvature tensor. Recently Pandey [6] has studied Projective motion in a Finsler space equipped with a non-symmetric connection in some special cases.

A non-symmetric connection $\Gamma^i_{jk}\ (\neq \Gamma^i_{kj})$ in an n-dimensional space $A_n$was introduced by Vranceanu [7]. In the present work we expand this concept to the theory of n-dimensional Finsler spaces.

Consider an n-dimensional Finsler space with non-symmetric connection $\Gamma^i_{jk}\ (\neq \Gamma^i_{kj})$ which is based on a non-symmetric fundamental tensor $g_{ij}(x,\dot{x})\ (\neq g_{ji}\ (x,\dot{x}))$.

Let us write

$$\Gamma^i_{jk} = M^i_{jk} + \frac{1}{2} N^i_{jk}\ , \tag{1.1}$$

where M^i_{jk} and $\frac{1}{2} N^i_{jk}$ are the symmetric and skew-symmetric parts of Γ^i_{jk} respectively.

Now introducing another connection coefficient $\tilde{\Gamma}{}^i_{jk}(x,\dot{x})$ defined as

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$$\tilde{\Gamma}^{\,i}_{jk} = M^i_{jk} - \frac{1}{2} N^i_{jk}. \tag{1.2}$$

Equations (1.1) and (1.2) together yields

$$\tilde{\Gamma}^{\,i}_{jk}(x, \dot{x}) = \Gamma^i_{kj}(x, \dot{x}). \tag{1.3}$$

Now suppose a vertical stroke (|) denotes covariant derivative with respect to x. We define the covariant derivative of any contravariant vector field $X^i(x, \dot{x})$ is two distinct ways (see [8] for details), as follows:

$$X^{i}\,{}^{+}|_{\,j} = \partial j X^i - (\dot{\partial}_m X^i)\ \Gamma^m_{kj}\, \dot{x}^k + X^k\, \Gamma^i_{kj}, \tag{1.4}$$

and $$X^{i}\,{}^{-}|_{\,j} = \partial j X^i - (\dot{\partial}_m X^i)\ \tilde{\Gamma}^{\,m}_{kj}\, \dot{x}^k + X^k\, \tilde{\Gamma}^{\,i}_{kj}\,. \tag{1.5}$$

Using equation (1.3) we can write (1.5) as

$$X^{i}\,{}^{-}|_{\,j} = \partial j X^i - (\dot{\partial}_m X^i)\ \Gamma^m_{jk}\, \dot{x}^k + X^k\, \Gamma^i_{jk}\,, \tag{1.6}$$

where, a positive sign below vertical stroke (|) by an index indicates that the covariant derivatives with respect to the connection Γ^i_{jk} concerning that index and a negative sign below an index vertical stroke (|) by an index indicates that the covariant derivative with respect to the connection $\tilde{\Gamma}^i_{kj}$ concerning that index. The covariant derivative defined in (1.4) and (1.5) will be known as ⊕- covariant differentiation of $X^i(x, \dot{x})$ with respect to $\dot{x}^j$ and ⊖- covariant differentiation of $X^i(x, \dot{x})$ with respect to $\dot{x}^j$ respectively throughout the thesis. Allowing ⊕- covariant differentiation in (1.4) with respect to x^k and then using the part of the skew-symmetric result so obtained with respect to j and k, we get

$$X^{i}\,{}^{+}|_{\,jk} - X^{i}\,{}^{+}|_{\,kj} = -(\dot{\partial}_m X^i)\ R^m_{pjk}\, \dot{x}^p + X^m\, R^i_{mjk} + X^{i}\,{}^{+}|_{\,m}\, N^m_{kj}, \tag{1.7}$$

where

$$R^h_{ijk} \overset{\text{def}}{=} \partial_k \Gamma^h_{ij} - \partial_j \Gamma^h_{ik} + \dot{\partial}_m \Gamma^h_{ik}\, \Gamma^m_{sj}\, \dot{x}^s - \dot{\partial}_m \Gamma^h_{ij}\, \Gamma^m_{sk}\, \dot{x}^s + \Gamma^p_{ij}\, \Gamma^h_{pk} - \Gamma^p_{ik}\, \Gamma^h_{pj}\,. \tag{1.8}$$

and is known as curvature tensor.

Similarly, ⊖-covariant differentiation with respect to x^k and proceeding as above, we have

$$X^{i}\,{}^{-}|_{\,jk} - X^{i}\,{}^{-}|_{\,kj} = -(\dot{\partial}_m X^i)\ \tilde{R}^m_{pjk}\, \dot{x}^p + X^m\, \tilde{R}^i_{mjk} + X^{i}\,{}^{-}|_{\,m}\, N^m_{kj}\,, \tag{1.9}$$

where, $$\tilde{R}^h_{ijk} = \partial_k \tilde{\Gamma}^h_{ij} - \partial_j \tilde{\Gamma}^h_{ik} + \dot{\partial}_m \tilde{\Gamma}^h_{ik}\, \Gamma^m_{sj}\, \dot{x}^s - \dot{\partial}_m \tilde{\Gamma}^h_{ij}\, \Gamma^m_{sk}\, \dot{x}^s + \tilde{\Gamma}^p_{ij}\, \tilde{\Gamma}^h_{pk} - \tilde{\Gamma}^p_{ik}\, \tilde{\Gamma}^h_{pj}\,. \tag{1.10}$$

note that $\tilde{R}^h_{ijk}$ is also known as curvature tensor.

With the help of the equation (1.3) we can write equation (1.10) as

$$\tilde{R}^h_{ijk} = \partial_k \Gamma^h_{ji} - \partial_j \Gamma^h_{ki} + \dot{\partial}_m \Gamma^h_{ki}\, \Gamma^m_{js}\, \dot{x}^s - \dot{\partial}_m \Gamma^h_{ji} \Gamma^m_{ks}\, \dot{x}^s + \Gamma^p_{ji} \Gamma^h_{kp} - \Gamma^p_{ki}\, \Gamma^h_{jp}\,. \tag{1.11}$$

Now, we shall use the following identities (for details see [3])

(a) $x^{i+}|_{\,k} = \dot{x}^{i\,-}|_{\,k} = 0$, (b) $R^i_{jk} \overset{\text{def}}{=} R^i_{hjk}\, \dot{x}^h$, (c) $R^i_j \overset{\text{def}}{=} R^i_{hj}\, \dot{x}^h$, (1.12)

(d) $R^i_{hjk} = -R^i_{hkj}$, (e) R^i_i = (n-1) R,

(f) $N^i_{jk} = -N^i_{kj} = \Gamma^i_{jk} - \Gamma^i_{kj}$, (g) $\Gamma^i_{hjk} \overset{\text{def}}{=} \dot{\partial}_h \Gamma^i_{jk}$.

Let $v^i(x)$ is a vector field of class C^2 defined over a region R of $F_n{}^*$ with this field we can associate an infinitesimal transformation of the form

$$\bar{x}^i = x^i + v^i(\mathrm{x})\, \mathrm{d}t, \tag{1.13}$$

where dt is to be stated as an infinitesimal constant.

The Lie derivative for the mixed tensor field $T^i_j(x, \dot{x})$ can be written as [6]

$$\pounds_v T^i_j = (T^{i\,+}_j|_k) v^k - (v^{i\,-}|_k) T^k_j + (v^{k\,-}|_j) T^i_k - \big(\dot{\partial}_h T^i_j\big)\big(v^{h\,-}|_k\big) \dot{x}^k. \tag{1.14}$$

Also the Lie derivative of connection parameter with non-symmetric connection with reference to ⊖ − covariant derivative (Rund [3]), we can write

$$\pounds_v\, \tilde{\Gamma}^i_{jk} = \big(v^{i\,-}|_j\big)^{-}|_k + (\dot{\partial}_r\, \tilde{\Gamma}^i_{kj})(v^{r\,-}|_h) \dot{x}^h + v^h\, \tilde{R}^i_{jkh}. \tag{1.15}$$

Differentiating (1.15) ⊖ − covariant derivative, we get

$$(\pounds_v \tilde{\Gamma}^i_{hj})^-|_k - (\pounds_v \tilde{\Gamma}^i_{hk})^-|_j = (v^{i\,-}|_h)^-|_k N^r_{kj} + \Gamma^i_{rjh} v^{r\,-}|_l \dot{x}^l + \Gamma^i_{rjh}(v^{r\,-}|_l)^-|_k \dot{x}^l + + v^{i\,-}|_h \tilde{R}^i_{hlj} + v^l \tilde{R}^i_{hlj}{}^-|_k - \Gamma^{i\,-}_{rkh}|_j v^{r\,-}|_l \dot{x}^l - \Gamma^i_{rkh}(v^{r\,-}|_l)^-|_j \dot{x}^l - v^{i\,-}|_j \tilde{R}^i_{hlk} - v^l \tilde{R}^i_{hlk}{}^-|_j - \dot{\partial}_r(v^{i\,-}|_j)\tilde{R}^r_{jk} + v^{r\,-}|_h \tilde{R}^i_{rkj} - v^{i\,-}|_r \tilde{R}^r_{hkj}. \quad (1.16)$$

From (1.14), we get

$$\pounds_v \tilde{R}^i_{hjk} = \tilde{R}^i_{hlk}{}^-|_l v^l - \tilde{R}^r_{hjk} v^{i\,-}|_r + \tilde{R}^i_{rjk} v^{r\,-}|_h + \tilde{R}^i_{hrk} v^{r\,-}|_j + + \tilde{R}^i_{hjk} v^{r\,-}|_k + (\dot{\partial}_r \tilde{R}^i_{hjk})(v^{r\,-}|_s)\dot{x}^s. \quad (1.17)$$

We can also verify the following

$$\dot{\partial}_r(v^{i\,-}|_j) = \Gamma^i_{rmh} v^m \quad (1.18)$$

$$v^{i\,+}|_j - v^{i\,-}|_j = v^m N^i_{mj}\ . \quad (1.19)$$

Using (1.17), (1.18), (1.19) and (1.15), the set of equation (1.16) becomes

$$(\pounds_v \tilde{\Gamma}^i_{hj})^-|_k - (\pounds_v \tilde{\Gamma}^i_{hk})^-|_j = \pounds_v \tilde{R}^i_{hjk} + \dot{x}^l \Gamma^i_{rjh} \pounds_v \Gamma^r_{kl} - \dot{x}^l \Gamma^i_{rkh} \pounds_v \Gamma^r_{jl} + + N^r_{kj} \pounds_v \Gamma^i_{rh} + P^i_{pjhk} v^{p\,-}|_s \dot{x}^s, \quad (1.20)$$

where

$$P^i_{pjhk} = -\dot{\partial}_p \tilde{R}^i_{hjk} + \Gamma^i_{pjh}{}^-|_k - \Gamma^i_{pkh}{}^-|_j - \Gamma^r_{pkl}\Gamma^i_{rjh}\dot{x}^l + \Gamma^r_{pjl}\Gamma^i_{rkh}\dot{x}^l - \Gamma^r_{prh} N^r_{jk}\ . \quad (1.21)$$

It can be easily be verified that the right hand member of (1.21) vanishes and hence

$$P^i_{pjhk} = 0. \quad (1.22)$$

After making use of (1.22), the set of equation (1.20) assume the following form

$$(\pounds_v \tilde{\Gamma}^i_{hj})^-|_k - (\pounds_v \tilde{\Gamma}^i_{hk})^-|_j = \pounds_v \tilde{R}^i_{hjk} + \dot{x}^l \Gamma^i_{rjh} \pounds_v \Gamma^r_{kl} - \dot{x}^l \Gamma^i_{rkh} \pounds_v \Gamma^r_{jl} + N^r_{kj} \pounds_v \Gamma^i_{rh}\ . \quad (1.23)$$

2. SPECIAL $\bar{R}$-PROJECTIVE MOTION IN A FINSLER SPACE $F_n{}^*$:

In our later discussions we will use following definitions:

DEFINITION(2.1):

A Finsler space $F_n{}^*$ equipped with non-symmetric connection is called $\bar{R}$-symmetric or special symmetric if the curvature tensor $\tilde{R}^i_{jkh}$ with respect to $\ominus$ − covariant derivative satisfying

$$\tilde{R}^i_{hjk}{}^-|_m = 0. \quad (2.1)$$

On contracting (2.1) with respect to the indices i and h, we can get

$$\tilde{R}_{jk}{}^-|_m = 0 \quad \text{where } \tilde{R}^i_{ijk} = \tilde{R}_{jk} \quad (2.2)$$

DEFINITION(2.2):

A Finsler space $F_n{}^*$ equipped with non-symmetric connection is called $\bar{R}$- affinely connected motion if

$$\dot{\partial}_l \tilde{\Gamma}^i_{jk} = 0\ . \quad (2.3)$$

DEFINITION(2.3):

The infinitesimal point transformation $\bar{x}^i = x^i + v^i(x)dt$ is said to define a special $\bar{R}$ -curvature collineation in a Finsler space $F_n{}^*$ provided there exists a field $v^i(x)$ satisfying

$$\pounds_v \tilde{R}^i_{jkh} = 0. \quad (2.4)$$

DEFINITION(2.4):

A Finsler space $F_n{}^*$ equipped with non-symmetric connection is called $\bar{R}$- Ricci collineation provided there exists a field $v^i(x)$ satisfying

$$\pounds_v \tilde{R}_{kh} = 0. \quad (2.5)$$

DEFINITION(2.5):

The infinitesimal point transformation $\bar{x}^i = x^i + v^i\,(x)dt$ defines an infinitesimal $\bar{R}$- projective transformation in a Finsler space $F_n{}^*$, if

$$\pounds_v\,\tilde{\Gamma}^i_{jk} = \delta^i_j\lambda_k + \delta^i_k\lambda_j - g_{jk}g^{il}d_l, \quad (2.6)$$

where $b_j(x,\dot{x})$ and $d_l(x,\dot{x})$ are vector fields satisfying the following

(a) $\dot{\partial}_j\lambda = \lambda_j$, (b) $\dot{\partial}_k\,\lambda_j = \lambda_{jk}$, (c) $\lambda_{jk}\,\dot{x}^k = \lambda_j$,

(d) $\lambda_j\dot{x}^j = \lambda$, (e) $\dot{\partial}_j\,d = d_j$, (f) $\dot{\partial}_k\,d_j = d_{jk}$,

(g) $d_{jk}\dot{x}^k = d_j$ and (h) $d_j\dot{x}^j =$ d. (2.7)

With the help of (1.23) the Lie-derivative of $\tilde{R}^i_{hjk}$ in an affinely connected $\bar{R}$- projective space is given by

$$\pounds_v\tilde{R}^i_{jkh} = \left(\pounds_v\,\tilde{\Gamma}^i_{kh}\right)^-|_j - \left(\pounds_v\,\tilde{\Gamma}^i_{jh}\right)^-|_k\,. \quad (2.8)$$

We now make use of (2.6) in (2.8) and get

$$\pounds_v\tilde{R}^i_{jkh} = \delta^i_k\lambda_h{}^-|_j - \delta^i_j\lambda_h{}^-|_k - g^{il}\left(g_{kh}d_l{}^-|_j - g_{jh}d_l{}^-|_k\right), \quad (2.9)$$

where, we have taken into account the fact that the covariant derivative of δ^i_j and g^{ij} vanish. At this stage, if we assume that the Finsler space F^*_n under consideration admits $\bar{R}$- projective motion as well as $\bar{R}$- curvature collineation then (2.9) gives

$$\delta^i_k\lambda_h{}^-|_j - \delta^i_j\lambda_h{}^-|_k - g^{il}\left(g_{kh}d_l{}^-|_j - g_{jh}d_l{}^-|_k\right) = 0. \quad (2.10)$$

Contracting (2.10) with respect to the indices i and j, we get

$$d_h{}^-|_k = (n-1)\,\lambda_h{}^-|_k - g^{jl}g_{kh}d_l{}^-|_j\,. \quad (2.11)$$

So, we get our first result as below:

Theorem (2.1):

If $\bar{R}$ -Projective motion in a Finsler space F^*_n is $\bar{R}$ -curvature collineation, then the set of equation (2.11) must hold.

Again Contracting equation (2.9) with respect to the indices i and j and get

$$\pounds_v\tilde{R}^*_{kh} = \lambda_h{}^-|_k - n\lambda_h{}^-|_k - g^{jl}\left(g_{kh}d_l{}^-|_j - g_{jh}d_l{}^-|_k\right) = 0. \quad (2.12)$$

At this stage, Let us now make the assumption that the Finsler space F^*_n under consideration admits $\bar{R}$- projective motion as well as $\bar{R}$- Ricci collineation then from (2.12), we get

$$\lambda_h{}^-|_k = d_h{}^-|_k - \frac{g^{jl}(g_{kl}d_h{}^-|_j - g_{jh}d_l{}^-|_k)}{n-1}. \quad (2.13)$$

Conversely, if we assume that $\lambda_h{}^-|_k$ is given by (2.13) then the relation (2.12) yields

$$\pounds_v\tilde{R}^*_{kh} = 0. \quad (2.14)$$

We now state our next results as below:

Theorem (2.2):

In an affinely connected $\bar{R}$ –projective F^*_n, the necessary and sufficient condition that $\bar{R}$ -projective motion be $\bar{R}$ -Ricci collineation is that form of function $\lambda(x,\dot{x})$ is given by (2.13).

In a symmetric Finsler space the commutation formula (1.23) may be written as

$$(\pounds_v\tilde{R}^i_{jkh})^-|_l = \tilde{R}^m_{jkh}\pounds_v\,\tilde{\Gamma}^i_{ml} - \tilde{R}^i_{mkh}\pounds_v\,\tilde{\Gamma}^m_{jk} - \tilde{R}^i_{jmh}\pounds_v\,\tilde{\Gamma}^m_{kl} - \tilde{R}^i_{jkm}\pounds_v\,\tilde{\Gamma}^m_{hl} - \left(\dot{\partial}_m\tilde{R}^i_{jkh}\right)(\pounds_v\,\tilde{\Gamma}^m_{sl})\dot{x}^s\,. \quad (2.15)$$

Making use of the equations (2.6) and (2.7) in (2.15) we get

$$(\pounds_v\tilde{R}^i_{jkh})^-|_l = \left(\delta^i_l\lambda_m - g_{ml}g^{ir}d_r\right)\tilde{R}^m_{jkh} - \tilde{R}^i_{lkh}\lambda_j + g_{jl}g^{ir}d_r\tilde{R}^i_{mkh} - 2\tilde{R}^i_{jkh}\lambda_l - \tilde{R}^i_{jlk}\lambda_h + g_{kl}g^{mr}d_r\tilde{R}^i_{mjh} - \tilde{R}^i_{jkl}\lambda_h + g_{hl}g^{mr}d_r\tilde{R}^i_{jkm} - \lambda\left(\dot{\partial}_l\tilde{R}^i_{jkh}\right) + g_{sl}g^{mr}d_r\left(\dot{\partial}_m\tilde{R}^i_{jkh}\right)\dot{x}^s. \quad (2.16)$$

Trnsvecting (2.15) successively by $\dot{x}^h$ and $\dot{x}^k$ and thereafter using (1.13), we get

$$\left(\delta_l^i \lambda_m - g_{ml} g^{ir} d_r\right) \tilde{R}_{jkh}^m \dot{x}^h \quad \dot{x}^k - \lambda_j \tilde{R}_{lkh}^i \dot{x}^h \quad \dot{x}^k + g_{jl} g^{ir} d_r \tilde{R}_{mkh}^i \dot{x}^h \quad \dot{x}^k - 2\lambda_l \tilde{R}_{jkh}^i \dot{x}^h \quad \dot{x}^k - \quad \tilde{R}_{jlk}^i \lambda_h \dot{x}^h \quad \dot{x}^k + g_{kl} g^{mr} d_r \tilde{R}_{mjh}^i \dot{x}^h \dot{x}^k - \lambda \tilde{R}_{jkl}^i \dot{x}^k + g_{hl} g^{mr} d_r \tilde{R}_{jkm}^i \dot{x}^h \dot{x}^k - \lambda\left(\dot{\partial}_l \tilde{R}_{jkh}^i\right) \dot{x}^h \dot{x}^k + g_{sl} g^{mr} d_r \left(\dot{\partial}_m \tilde{R}_{jkh}^i\right) \dot{x}^s \dot{x}^h \dot{x}^k. \tag{2.17}$$

Contracting (2.17) with respect to the indices i and j, we get

$$\tilde{R}_{ikh}^m \left(\delta_l^i \lambda_m - g_{ml} g^{ir} d_r\right) \dot{x}^h \dot{x}^k + \tilde{R}_{lkh}^i \lambda_i \dot{x}^h \dot{x}^k + d_l \tilde{R}_{mkh}^i \dot{x}^h \qquad \dot{x}^k - 2\lambda_l \tilde{R}_{kh} \dot{x}^h \qquad \dot{x}^k - p \tilde{R}_{lh} \dot{x}^h + + g_{kl} g^{mr} d_r \tilde{R}_{mh} \dot{x}^h \dot{x}^k - \lambda \tilde{R}_{kl} \dot{x}^k + g_{hl} g^{mr} d_r \tilde{R}_{km} \dot{x}^h \dot{x}^k - \lambda\left(\dot{\partial}_l \tilde{R}_{kh}\right) \dot{x}^h \dot{x}^k + + g_{sl} g^{mr} d_r \left(\dot{\partial}_m \tilde{R}_{kh}\right) \dot{x}^s \dot{x}^h \dot{x}^k. \tag{2.18}$$

where, $\tilde{R}_{ikh}^i = \tilde{R}_{kh}$.

Now our last result is stated as below:

Theorem (2.3):

In a special $\bar{R}-$ projective symmetric F_n^*, $\bar{R}$-projective motion will be a special $\bar{R}$-curvature collineation if the Ricci tensor $\tilde{R}_{kh}(x,\dot{x})$ and the scalar $\lambda(x,\dot{x})$ are connected by the equation (2.18) .

CONCLUSION:

In this paper we have studied special $\bar{R}$-Projective motion in a Finsler space $F_n{}^*$ equipped with non-symmetric connection with special reference to $\ominus -$ covariant derivative. In this context we have studied $\bar{R}$ - Curvature collineation and Ricci- Collineation. We have found new results in the form of the $\bar{R}$ -projective motion consisting of $\bar{R}$ -curvature collineation and $\bar{R}$ -Ricci collineation in an affinely connected $\bar{R}$ –projective F_n^*. We have also found that the $\bar{R}$-projective motion will be a special $\bar{R}$-curvature collineation in a special $\bar{R}$ –symmetric F_n^*.

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