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Cost Analysis of Two- Phase M/M/1 Queueing Systems with Server Dormant, Start up and Break downs

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ABSTRACT

This paper investigates an optimum strategy of two-phase M/M/1 queueing system with server dormant, start up and breakdowns. The server first starts batch service where the customers arrive according to Poisson process and in second-phase it gives individual service. The server is turned off each time the system empties. When the queue length reaches or exceeds M, the server will be in dormant state and when it reaches to N or more than N batch service starts. During both batch as well as individual services the server may breakdown at any time according to a Poisson process and repair will be immediately done. Explicit expressions for the steady state distribution of the number of customers in the system are obtained and also derived various system measures.

Keywords : Vacation, N-Policy, Two-phase Queueing System, Server Breakdowns.

I. INTRODUCTION

We consider two-phase M/M/1 queuing system with N-policy and server breakdowns that operates as follows. Customers arrive individually according to a Poisson process and receive batch service in first phase and individual service in second phase. The server is turned off each time the system empties, as and when the queue length reaches or exceeds N (threshold) batch service starts. Before the batch service, the system requires a dormant period followed by a random startup time for pre-service. When the number of customers in the queue is less than or equal to M-1, the server is in vacation, when the number of customers in the queue is greater than or equal to M and up to N-1 server is in dormant and when the number of customers become N it goes to a startup period for pre service. Arrivals during pre-service are also allowed to enter the batch. As soon as the startup period is over the server starts the batch service followed by individual service to all customers in the batch. During both batch as well as individual services, the server may breakdown at any time according to a Poisson process and if the server fails, it is immediately sent for repair. After repair the server resume service.

A practical problem related to a manufacturing system is presented for illustration purpose. Consider a production system where the items are produced on order. The orders are collected as and when their number reaches M the production process alerts and their number reaches N the production process gets initiated. Service may require two phases, such as preliminary checking of orders followed by the actual production. When there are no orders the production process is stopped and is resumed only when N orders accumulate. Before each production cycle the machine may need certain startup time and it may breakdowns due to some unforeseen problems.

Krishna and Lee (1990) first introduced the two-phase M/M/1 queueing system. Doshi (1991) studied the two-phase M/G/1 queueing system. Selvam and Sivasankaran (1994) introduced the two-phase queueing system with server vacations. Kim and Chae (1998) analyzed the two-phase queueing system with N-policy. Wang (1995) first proposed a Markovian queueing system under the N-Policy with server breakdowns. Wang (1997) and Wang et al. (1999) extended the model proposed by Wang (1995) to $M/E_k/1$ and $M/H_2/1$ queueing systems respectively. Ke (2003) presented the optimal control policy in batch arrival queue with server breakdowns and multiple vacations. Wang and Li (2008) studied a retrial queue with general retrial times, Bernoulli vacations, setup times and two-phase service. Anantha Lakshmi et. al. (2008) presented the optimal strategy analysis of an N-policy bulk arrival queueing system with a removable and non-reliable server. Jau-Chuan Ke (2006) derived the p.g.f. of the number of customers for the *(m, N)* policy M/G/1 queueing systems with an unreliable server and single vacation. He also studied other important system characteristics. Vasanta Kumar and Chandan (2007) and (2008) presented the optimal control policy of two-phase M/M/1 and $M/E_k/1$ queueing systems with N-policy. Vasanta Kumar et al. (2011) studied Two-phase N-policy $M^x/M/1$ queueing system with startup times and server breakdowns and also some of the system performance measures are derived.

This paper extends the work of Anantha Lakshmi et al. (2008) to an N-policy two-phase M/M/1 queueing system with startup times and server breakdowns.

The objectives of this paper are:

(i) to establish the state equations to obtain the steady state probability distribution of the number of units in the system.

(ii) to derive system characteristics such as expected number of units in the system when the server is in vacation, in setup, at batch service, at individual service and breakdown states respectively and expected system length.

II. THE SYSTEM AND ASSUMPTIONS

Customers are assumed to arrive according to a Poisson process with mean arrival rate λ and join the batch queue. When the batch size reaches M ($\geq$1and $\leq$N-1) the server will spend a random dormant period t_1, which is assumed to follow an exponential distribution with mean $1/\theta_1$ and when it reaches to N the server will spend a random startup time t_2 for pre-service, which is assumed to follow an exponential distribution with mean $1/\theta$. As soon as the period of startup is over, the server begins batch service in first phase. While serving in batch queue, the server may breakdown at any time with a Poisson breakdown rate ξ_1. When the server fails it is immediately repaired at a repair rate ξ_2, where the repair times are exponentially distributed. Upon completion of batch service the server proceeds to the second phase to serve all customers in the batch individually. Individual queue is served in FIFO mode. Batch service time is assumed to be exponentially distributed with mean $1/\beta$ and is independent of batch size. Individual service times are also assumed to be exponentially distributed with mean $1/\mu$. While serving in individual queue, the server may breakdown at any time with a Poisson breakdown rate α_1. When the server fails it is immediately repaired at a repair rateα_2, where the repair times are exponentially distributed. After repair the server immediately resumes service in individual queue. On completion of individual service the server returns to the batch queue to serve the customers who have arrived. If the customers are waiting, the server starts the cycle by providing them batch service followed by individual service. If no customer is waiting, the server takes a vacation and return from vacation only after M customers accumulate in the batch queue and start pre-service work.

III. STEADY – STATE ANALYSIS

In steady – state the following notations are used.

$P_{0,i,0}$ = The probability that there are i customers in the batch queue when the server is on vacation, where i = 0,1,2,3,…,M-1

$P_{1,i,0}$ = The probability that there are i customers in the batch queue when the server is in dormant period, where i= M,M+1,……N-1

$P_{2,i,0}$ = The probability that there are i customers in the batch queue when the server is doing pre-service (startup work), where i = N, N+1, N+2,…………

$P_{3,i,0}$ = The probability that there are i customers in the batch queue when the server is in batch service where i = 1,2,3,…

$P_{4,i,0}$ = The probability that there are i customers in batch queue when the server is working but found to be broken down, where i = 1,2,3,…

$P_{5,i,j}$ = The probability that there are i customers in the batch queue and j customers in individual queue when the server is in individual service, where i = 0,1,2,… and j = 1,2,3,…

$P_{6,i,j}$ = The probability that there are i customers in the batch queue and j customers in individual queue when the server is working but found to be broken down, where i = 0,1,2,… and j = 1,2,3, …

The steady-state equations satisfied by the system size probabilities are as follows:

$$\lambda P_{0,0,0} = \mu P_{5,0,1} . \tag{1}$$

$$\lambda P_{0,i,0} = \lambda P_{0,i-1,0}, \quad 1 \leq i \leq M-1. \tag{2}$$

$$(\lambda+\theta_1) P_{1,M,0} = \lambda P_{0,M-1,0} . \tag{3}$$

$$(\lambda+\theta_1) P_{1,i,0} = \lambda P_{1,i-1,0}, \quad M+1 \leq i \leq N-1. \tag{4}$$

$$(\lambda+\theta) P_{2,N,0} = \lambda P_{1,N-1,0} . \tag{5}$$

$$(\lambda+\theta) P_{2,i,0} = \lambda P_{2,i-1,0}, \quad i > N . \tag{6}$$

$$(\lambda+\beta+\xi_1) P_{3,i,0} = \xi_2 P_{4,i,0} + \lambda P_{3,i-1,0} + \mu P_{5,i,1}, \quad 1 \leq i \leq N-1 . \tag{7}$$

$$(\lambda+\beta+\xi_1) P_{3,i,0} = \xi_2 P_{4,i,0} + \lambda P_{3,i-1,0} + \mu P_{5,i,1} + \theta P_{2,i,0}, \quad i \geq N . \tag{8}$$

$$(\lambda+\xi_2) P_{4,i,0} = \lambda P_{4,i-1,0} + \xi_1 P_{3,i,0}, \quad i \geq 1. \tag{9}$$

$$(\lambda+\alpha_1+\mu) P_{5,0,j} = \mu P_{5,0,j+1} + \beta P_{3,j,0} + \alpha_2 P_{6,0,j}, \quad j \geq 1. \tag{10}$$

$$(\lambda+\alpha_1+\mu) P_{5,i,j} = \mu P_{5,i,j+1} + \lambda P_{5,i-1,j} + \alpha_2 P_{6,i,j}, \quad i,j \geq 1. \tag{11}$$

$(\lambda + \alpha_2) P_{6,0,j} = \alpha_1 P_{5,0,j}$, j≥1. (12)

$(\lambda + \alpha_2) P_{6,i,j} = \alpha_1 P_{5,i,j} + \lambda P_{6,i-1,j}$, i,j≥1 . (13)

The following probability generating functions are defined

$G_0(z) = \sum_{i=0}^{M-1} P_{0,i,0} z^i$, $|z| \le 1$, $G_1(z) = \sum_{i=M}^{N-1} P_{1,i,0} z^i$, $|z| \le 1$,

$G_2(z) = \sum_{i=N}^{\infty} P_{2,i,0} z^i$, $|z| \le 1$,

$G_3(z) = \sum_{i=1}^{\infty} P_{3,i,0} z^i$, $|z| \le 1$,

$G_4(z) = \sum_{i=1}^{\infty} P_{4,i,0} z^i$, $|z| \le 1$,

$G_5(z, y) = \sum_{i=0}^{\infty} \sum_{j=1}^{\infty} P_{5,i,j} z^i y^j$, $|z| \le 1$, and $|y| \le$,

$G_6(z, y) = \sum_{i=0}^{\infty} \sum_{j=1}^{\infty} P_{6,i,j} z^i y^j$, $|z| \le 1$ and $|y| \le 1$,

$R_j(z) = \sum_{i=0}^{\infty} P_{5,i,j} z^i$, $|z| \le 1$ and $S_j(z) = \sum_{i=0}^{\infty} P_{6,i,j} z^i$, $|z| \le 1$.

Multiplication of equation (1.2) by z^i and adding over i (1 ≤i≤M-1) gives

$$G_o(z) = \frac{(1-z^M)}{(1-z)} P_{0,0,0}. \quad (14)$$

Multiplication of equations (1.3) and (1.4) by z^i and adding over i (i≥N) gives

$$G_1(z) = \frac{\lambda\left(Z^M - Z^N\left(\frac{\lambda}{\lambda+\theta 1}\right)^{N-M}\right)}{(\lambda(1-Z)+\theta 1)} P_{0,0,0}. \quad (15)$$

Multiplication of equations (1.5) and (1.6) by z^i and adding over i (i ≥N) gives

$$(\lambda(1-z) + \theta\, G_2(z) = \lambda \left(\frac{\lambda}{\lambda+\theta 1}\right)^{N-M} Z^N P_{0,0,0} . \quad (16)$$

Multiplication of equations (1.7) and (1.8) by z^i and adding over i (i ≥1) gives

$$(\lambda(1-z) + \beta + \xi_1) G_3(z) = \xi_2 G_4(z) + \mu S_1(z) + \theta G_2(z) - \lambda P_{0,0,0}. \quad (17)$$

Multiplication of equation (1.9) by z^i and adding over i (i≥1) gives

$(\lambda(1-z) + \xi_2) G_4(z) = \xi_1 G_3(z)$. (18)

Multiplication of equation (1.11) by z^i and adding over i (i≥1)
and using (1.10) gives

$(\lambda(1-z) + \alpha_1 + \mu) R_j(z) = \mu R_{j+1}(z) + \alpha_2 S_j(z) + \beta P_{3,j,0.}$ (19)

Multiplication of this equation by y^j and adding over j (j≥1) gives

$[\lambda y(1-z) + \alpha_1 y - \mu(1-y)] G_5(z, y) = y\alpha_2 G_6(z,y) + \beta y G_3(y) - \mu y R_1(z)$...(20)

Multiplication of equation (1.13) by z^i and adding over i (i≥1)
and using (1.12) gives

$$(\lambda(1-z) + \alpha_2) S_j(z) = \alpha_1 S_j(z). \quad (21)$$

Multiplication of this equation by y^j and adding over j (j≥1) gives

$$(\lambda(1-z) + \alpha_2) G_6(z, y) = \alpha_1 G_5(z, y). \quad (22)$$

The total probability generating function G(z, y) is given by

$$G(z, y) = G_0(z) + G_1(z) + G_2(z) + G_3(z) + G_4(z) + G_5(z, y) + G_6(z, y)$$

The normalizing condition is

$G(1,1) = G_0(1) + G_1(1) + G_2(1) + G_3(1) + G_4(1) + G_5(1,1) + G_6(1,1) = 1$. (23)

From equations (1.14) to (1.22)

$G_0(1) = M P_{0,0,0}$, (24)

$G_1(1) = \frac{\lambda}{\theta 1}\left(1 - \left(\frac{\lambda}{\lambda+\theta 1}\right)^{N-M}\right) P_{0,0,0}$, (25)

$$G_2(1) = \frac{\lambda}{\theta}\left(\frac{\lambda}{\lambda+\theta 1}\right)^{N-M} P_{0,0,0}\,. \tag{26}$$

$$G_3(1) = \frac{1}{\beta}\left[\mu R_1(1) + \left(\left(\frac{\lambda}{\lambda+\theta 1}\right)^{N-M} - 1\right)\lambda P0,0,0\right]. \tag{27}$$

$$G_4(1) = \frac{\xi 1}{\xi 2} G_3(1)\,. \tag{28}$$

$$G_5(1,1) = \frac{\left(\alpha 2\beta G3^1(1) - \lambda\alpha 2\mu {R_1}^1(1)\right)}{\left(\mu\alpha 2 - \lambda(\alpha 1+\alpha 2)\right)}\,. \tag{29}$$

and $G_6(1,1) = \frac{\alpha 1}{\alpha 2} G_5(1,1)$, (30)

where $P_{0,0,0} = \dfrac{\left[1 - \frac{\lambda}{\mu}\left(1+\frac{\alpha 1}{\alpha 2}\right) - \frac{\lambda}{\beta}\left(1+\frac{\xi 1}{\xi 2}\right)\right]}{\left(M + \frac{\lambda}{\theta 1} - \frac{\lambda}{\theta 1}\left(\frac{\lambda}{\lambda+\theta 1}\right)^{N-M} + \frac{\lambda}{\theta}\left(\frac{\lambda}{\lambda+\theta 1}\right)^{N-M}\right)}$.

Normalizing condition (1.23) gives

$$R_1(1) = \frac{\left\{\begin{array}{l} \frac{\lambda}{\mu}\left(1+\frac{\alpha 1}{\alpha 2}\right) + \frac{\lambda}{\beta}\left(1+\frac{\xi 1}{\xi 2}\right) - \frac{\lambda^2}{\theta}\frac{(\alpha 1+\alpha 2)}{(\mu\alpha 2 - \lambda(\alpha 1+\alpha 2))}\left(1-\frac{\theta}{\xi 2}\right)\left(\frac{\lambda}{\lambda+\theta 1}\right)^{N-M} P0,0,0 - \\ \frac{N\lambda(\alpha 1+\alpha 2)}{(\mu\alpha 2 - \lambda(\alpha 1+\alpha 2))}\left(\frac{\lambda}{\lambda+\theta 1}\right)^{N-M} P0,0,0 - \frac{\lambda^2}{\xi 2}\frac{(\alpha 1+\alpha 2)}{(\mu\alpha 2 - \lambda(\alpha 1+\alpha 2))} P0,0,0 - \\ \frac{\lambda}{\beta}\left[\left(\left(\frac{\lambda}{\lambda+\theta 1}\right)^{N-M} - 1\right)\left[\left(1+\frac{\xi 1}{\xi 2}\right) + \frac{\left(1+\frac{\alpha 1}{\alpha 2}\right)\left(1+\frac{\beta}{\xi 2}+\frac{\xi 1}{\xi 2}\right)}{(\mu\alpha 2 - \lambda(\alpha 1+\alpha 2))}\lambda\alpha 2\right]\right] \end{array}\right\}}{\left[\frac{\mu}{\beta}\left[\left(1+\frac{\xi 1}{\xi 2}\right) + \frac{\left(1+\frac{\alpha 1}{\alpha 2}\right)}{(\mu\alpha 2 - \lambda(\alpha 1+\alpha 2))}\left(1+\frac{\beta}{\xi 2}+\frac{\xi 1}{\xi 2}\right)\lambda\alpha 2\right] - \frac{\mu\lambda(\alpha 1+\alpha 2)}{\xi 2(\mu\alpha 2 - \lambda(\alpha 1+\alpha 2))}\right]}.$$

Substituting the value of $R_1(1)$ in (1.27), (1.28), (1.29) and (1.30) gives $G_2(1)$, $G_3(1)$, $G_4(1)$, $G_5(1,1)$, $G_6(1,1)$.

Under steady state conditions, let P_0, P_1, P_2, P_3, P_4 , P_5 and P_6 be the probabilities that the server is in vacation, Dormant, startup, in batch service, in batch service with break down, in individual service and breakdown states respectively. Then,

$$P_0 = G_0(1) = M\,P_{0,0,0},$$

$$P_1 = \frac{\lambda}{\theta 1}\left(1 - \left(\frac{\lambda}{\lambda+\theta 1}\right)^{N-M}\right) P_{0,0,0}\,.$$

$$P_2 = \frac{\lambda}{\theta}\left(\frac{\lambda}{\lambda+\theta 1}\right)^{N-M} P_{0,0,0}\,.$$

$$P_3 = \frac{1}{\beta}\left[\mu R_1(1) + \left(\left(\frac{\lambda}{\lambda+\theta 1}\right)^{N-M} - 1\right)\lambda P0,0,0\right].$$

$$P_4 = \frac{\xi 1}{\xi 2} G_3(1).$$

$$P_5 = \frac{\left(\begin{array}{l} G3(1)\left(\lambda\alpha 2 + \frac{\lambda\alpha 2\beta}{\xi 2} + \frac{\lambda\alpha 2\xi 1}{\xi 2}\right) + G2(1)\left(\lambda\alpha 2 - \frac{\lambda\alpha 2\theta}{\xi 2}\right) - \\ \frac{\mu\alpha 2\lambda}{\xi 2}R_1(1) + N\alpha 2\lambda\left(\frac{\lambda}{\lambda+\theta 1}\right)^{N-M} P0,0,0 + \lambda^2\frac{\alpha 2}{\xi 2}P0,0,0 \end{array}\right)}{(\mu\alpha 2 - \lambda(\alpha 1+\alpha 2))}\text{.and}$$

$$P_6 = \frac{\alpha 1}{\alpha 2} G5(1,1).$$

IV. EXPECTED NUMBER OF CUSTOMERS IN THE SYSTEM

Using the probability generating functions expected number of customers in the system at different states are presented below.

Let L_0, L_1, L_2, L_3, L_4, L_5 and L_6 be the expected number of customers in the system when the server is in idle, Dormant, startup, batch service, batch service, break down in batch service, individual service and breakdown states respectively.

Then

$$L_0 = \sum_{i=0}^{M-1} i\, P_{0,i,0} = G_0^1(1) = \frac{M(M-1)}{2} P_{0,0,0}\ . \qquad (31)$$

$$L_1 = \sum_{i=M}^{N-1} i P_{1,i,0} = G_1^1(1) = \left[\frac{M\lambda 1}{\theta 1} + \left(\frac{\lambda}{\theta 1}\right)^2 \left(1 - \left(\frac{\lambda}{\lambda+\theta 1}\right)\right)^{N-M-1} - \frac{\lambda}{\theta 1}(N-1)\left(\frac{\lambda}{\lambda+\theta 1}\right)^{N-M}\right] P_{0,0,0}\ . \qquad (32)$$

$$L_2 = \sum_{i=N}^{\infty} i P_{2,i,0} = G_2^1(1)$$

$$= \frac{\lambda}{\theta}\left(\frac{\lambda}{\lambda+\theta 1}\right)^{N-M}\left(\frac{\lambda+N\theta}{\theta}\right) P_{0,0,0}. \qquad (33)$$

$$L_3 = \sum_{i=N}^{\infty} i P_{3,i,0} = G_3^1(1)$$

$$= \frac{\frac{\lambda\mu}{\beta^2}\left(1+\frac{\xi_1}{\xi_2}\right)R_1(1) + \frac{\lambda}{\beta}\left(\frac{\lambda}{\lambda+\theta_1}\right)^{N-M}\left(\frac{\lambda+N\theta}{\theta}\right)P_{0,0,0} - \frac{\lambda\theta}{\beta\xi_2}G_2(1) + \frac{\lambda^2}{\beta^2}P_{0,0,0}\left[\left[\left(\frac{\lambda}{\lambda+\theta 1}\right)^{N-M} - 1\right]\left[1+\frac{\xi_1}{\xi_2}+\frac{\beta}{\xi_2}\right]+\frac{\beta}{\xi_2}\right]}{\left(1-\frac{\lambda}{\mu\alpha_2}(\alpha_1+\alpha_2)\right)}\ .$$

$$L_4 = \sum_{i=1}^{\infty} i P_{4,i,0} = G_4^1(1)$$

$$= \frac{\xi_1}{\xi_2} G_3^1(1) + \frac{\lambda\xi_1}{\xi_2^2} G_3(1) \qquad (34)$$

$$L_5 = \sum_{i=0}^{\infty}\sum_{j=1}^{\infty} (i+j) P_{5,i,j} = G_5^1(1,1) =$$

$$\frac{\left[\begin{array}{l} 2\left(\lambda(\alpha_1+\alpha_2+\mu)-\lambda^2\right)\alpha_2 \frac{\left(\beta G_3^1(1)-\mu S_1^1(1)\right)}{(\mu\alpha_2-\lambda(\alpha_1+\alpha_2))} \\ +2(\alpha_2-\lambda)\left(\beta G_3^1(1)-\mu S_1^1(1)\right)+\alpha_2\left\{\begin{array}{l} \frac{1}{\xi_2}[2\lambda(\beta+\xi_1+\xi_2)]-2\lambda\beta\left(\frac{\alpha_1+\alpha_2}{\mu\alpha_2}\right)G_3^1(1) \\ +\frac{\lambda}{\theta}\left(\frac{\lambda+N\theta}{\theta}\right)\left(\frac{\lambda}{\lambda+\theta_1}\right)^{N-M}\left(2\lambda P_{0,0,0}-\frac{2\lambda\theta}{\xi_2}\right) \\ +\lambda N(N-1)\left(\frac{\lambda}{\lambda+\theta_1}\right)^{N-M} P_{0,0,0}-\frac{2\lambda^2\mu}{\xi_2\ \beta}S_1(1) \\ -\frac{2\lambda^3}{\xi_2}\left[\left(\frac{\lambda}{\lambda+\theta_1}\right)^{N-M}-1\right]P_{0,0,0} \end{array}\right\} \end{array}\right]}{\left(2\left(\mu\alpha_2-\lambda(\alpha_1+\alpha_2)\right)\right)}$$

(35)

$$L_6 = \sum_{i=0}^{\infty}\sum_{j=1}^{\infty} (i+j) P_{6,i,j} = G_6^1(1,1)$$

$$= \frac{\alpha_1}{\alpha_2} G_5^1(1,1) + \frac{\lambda\alpha_1}{\alpha_2^2} G_5(1,1)\ . \qquad (36)$$

Thus the expected number of units in the system

$$L(N^*) = L_0 + L_1 + L_2 + L_3 + L_4 + L_5 + L_6$$

$$= \frac{M(M-1)}{2} P_{0,0,0} + \left[\frac{M\lambda 1}{\theta 1} + \left(\frac{\lambda}{\theta 1}\right)^2\left(1-\left(\frac{\lambda}{\lambda+\theta 1}\right)\right)^{N-M-1} - \frac{\lambda}{\theta 1}(N-1)\left(\frac{\lambda}{\lambda+\theta 1}\right)^{N-M}\right] P_{0,0,0} +$$

$$\frac{\lambda}{\theta}\left(\frac{\lambda}{\lambda+\theta 1}\right)^{N-M}\left(\frac{\lambda+N\theta}{\theta}\right)P_{0,0,0}.+$$

$$\frac{\frac{\lambda\mu}{\beta^2}\left(1+\frac{\xi_1}{\xi_2}\right)R_1(1)+\frac{\lambda}{\beta}\left(\frac{\lambda}{\lambda+\theta_1}\right)^{N-M}\left(\frac{\lambda+N\theta}{\theta}\right)P_{0,0,0}-\frac{\lambda\theta}{\beta\xi_2}G_2(1)+\frac{\lambda^2}{\beta^2}P_{0,0,0}\left[\left[\left(\frac{\lambda}{\lambda+\theta 1}\right)^{N-M}-1\right]\left[1+\frac{\xi_1}{\xi_2}+\frac{\beta}{\xi_2}\right]+\frac{\beta}{\xi_2}\right]}{\left(1-\frac{\lambda}{\mu\alpha_2}(\alpha_1+\alpha_2)\right)}+$$

$$\frac{\xi_1}{\xi_2}G_3{}^1(1)+\frac{\lambda\xi_1}{\xi_2{}^2}G_3(1)+$$

$$\frac{\left[\begin{array}{l}2(\lambda(\alpha_1+\alpha_2+\mu)-\lambda^2)\alpha_2\dfrac{\left(\beta G_3{}^1(1)-\mu S_1{}^1(1)\right)}{(\mu\alpha_2-\lambda(\alpha_1+\alpha_2))}\\ +2(\alpha_2-\lambda)\left(\beta G_3{}^1(1)-\mu S_1{}^1(1)\right)+\alpha_2\left\{\begin{array}{l}\frac{1}{\xi_2}[2\lambda(\beta+\xi_1+\xi_2)]-2\lambda\beta\left(\frac{\alpha_1+\alpha_2}{\mu\alpha_2}\right)G_3{}^1(1)\\ +\frac{\lambda}{\theta}\left(\frac{\lambda+N\theta}{\theta}\right)\left(\frac{\lambda}{\lambda+\theta_1}\right)^{N-M}\left(2\lambda P_{0,0,0}-\frac{2\lambda\theta}{\xi_2}\right)\\ +\lambda N(N-1)\left(\frac{\lambda}{\lambda+\theta_1}\right)^{N-M}P_{0,0,0}-\frac{2\lambda^2}{\xi_2}\frac{\mu}{\beta}S_1(1)\\ -\frac{2\lambda^3}{\xi_2}\left[\left(\frac{\lambda}{\lambda+\theta_1}\right)^{N-M}-1\right]P_{0,0,0}\end{array}\right\}\end{array}\right]}{\left(2\left(\mu\alpha_2-\lambda(\alpha_1+\alpha_2)\right)\right)}$$

$$+\frac{\alpha_1}{\alpha_2}G_5{}^1(1,1)+\frac{\lambda\alpha_1}{\alpha_2{}^2}G_5(1,1)\quad . \tag{37}$$

V. CHARACTERISTIC FEATURES OF THE SYSTEM

In this section, we obtain the expected system length when the server is in different states. Let E_0, E_1, E_2, E_3, E_4 *and* E_5denote the expected length of vacation period, dormant period, startup period, batch service period , batch service breakdown period, individual service period, and waiting period for repair during individual service respectively. Then the expected length of a busy cycle is given by

$E_c = E_0 + E_1 + E_2 + E_3 + E_4 + E_5 + E_6$.

The long run fractions of time the server is in different states are as follows:

$\frac{E_0}{E_c} = p_0$,

$\frac{E_1}{E_c} = p_1$,

$\frac{E_2}{E_c} = p_2$,

$\frac{E_3}{E_c} = p_3$,

$\frac{E_4}{E_c} = p_4$,

$\frac{E_5}{E_c} = p_5$,

$\frac{E_6}{E_c} = p_6$

Expected length of vacation period is given by

$E_0 = \frac{N}{\lambda}$.

Hence,

$E_c = \frac{1}{(\lambda p_{0,0,0})}$.

VI. COST FUNCTION

In this section, we determine the long run average cost function for the two- phase M/M/1, N-policy queue with server break downs. It is as follows

Let T (N*) be the average cost per unit of time, then

$$T(N*) = C_h L(N) + C_o\left(\frac{E_3}{E_c}+\frac{E_5}{E_c}\right) + C_m\left(\frac{E_S}{E_c}\right)+C_{b1}\left(\frac{E_4}{E_c}\right)+C_{b2}\left(\frac{E_6}{E_c}\right)+C_s\left(\frac{1}{E_c}\right) -C_r\left(\frac{E_0}{E_c}\right).$$

Where
C_h= Holding cost per unit time for each customer present in the system,
C_o = Cost per unit time for keeping the server on and in operation,
C_m= Startup cost per unit time,
C_s = Setup cost per cycle,
C_{b1} = Break down cost per unit time for the unavailable server in batch service mode,
C_{b2} = Break down cost per unit time for the unavailable server in individual service mode,
C_r= Reward per unit time as the server is doing secondary work in vacation.

A computational algorithm translated in MATLAB is used to obtain the numerical values.

VII. SENSITIVITY ANALYSIS

In order to verify the efficiency of our analytical results, we perform numerical experiment by using MATLAB. The variations of different parameters (both monetary and non-monetary) on the mean number of jobs in the system and total expected cost are shown.
Parameters for which the model is relatively sensitive would require more attention of researchers, as compared to the parameters for which the model is relatively insensitive or less sensitive.
We perform the sensitivity analysis by fixing
Non –monetary parameters as
λ=0.5, μ=8,α_1=0.2,α_2=0.5,ξ_1=0.2,ξ_2=0.3,θ=6,β=12 and monetary parameters as
C_r=15,C_{b1}=50,C_{b2}=75,C_m=200,C_h=5,C_s=1000;
The values are shown in tables 1-15 in Appendix and summary is stated here

- With increase in values of λ:Mean number of customers in the system and expected costs are increasing.
- With increase in values of μ:Mean number of customers in the system are increasing, expected cost is decreasing.
- With increase in values of α_1:Mean number of customers in the system is increasing and expected cost is decreasing.
- With increase in values of α_2.: both mean number of customers in the system and expected cost are decreasing.
- With increase in values of ξ_1, :, Mean number of customers in the system is decreasing and expected cost is increasing.
- With increase in values of ξ_2: Mean number of customers in the system is increasing and expected cost is also increasing.
- With increase in values of θ :Mean number of customers in the system and expected cost are decreasing.
- With increase in values of β: Mean number of customers in the system is increasing and expected cost is decreasing.
- With increase in values of C_r: Mean number of customers in the system is slightly increasing and expected cost is decreasing.
- With increase in values ofC_{b1}: Mean number of customers in the system is increasing and expected cost is insensitive.
- With increase in values ofC_{b2}: mean number of customers in the system are decreasing and expected cost is slightly increasing.
- With increase in values of C_m : mean number of customers in the system and expected cost are increasing.
- With increase in values ofC_o: Mean number of customers in the system is decreasing and expected cost is increasing.
- With increase in values of C_h :mean number of customers in the system are decreasing and expected cost is increasing.
- With increase in values of C_s : mean number of customers in the system and expected cost are increasing.

VIII. CONCLUSION

Two-phase (M, N) -Policy of M/M/1 queueing systems with server dormant, start up and breakdowns is studied. Explicit expressions for the steady state distribution of the number of customers in the system are obtained and also explored the impact of various parameters on system constants.

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Appendix

Effect of variation in the non-monetary parameters

(i)Variation in λ

Table 1: Effect of **λ** on expected system length and expected cost

λ	0.9	1.3	1.7	2.1	2.5	2.9
L(N*)	6	7	7	8	9	9
T(N*)	53.58	69.7	83.97	96.12	107.39	117.67

(ii)Variation in μ

Table 2: Effect of **μ** on expected system length and expected cost

M	9	10	11	12	13	14
L(N*)	4	4	4	4	4	4
T(N*)	33.71	33.58	33.48	33.39	33.31	33.25

(iii) Variation in α_1

Table 3: Effect of α_1 on expected system length and expected cost

α_1	0.205	0.21	0.215	0.220	0.225	0.230
L(N*)	4	4	4	4	4	4
T(N*)	33.92	33.92	33.92	33.93	33.93	33.93

(iv)Variation in α_2

Table 4: Effect of α_2 on expected system length and expected cost

α_2	3.1	3.2	3.3	3.4	3.5	3.6
L(N*)	4	4	4	4	4	4
T(N*)	33.87	33.87	33.86	33.86	33.86	33.85

(v)Variation in ξ_1

Table 5: Effect of ξ_1 on expected system length and expected cost

ξ_1	0.3	0.4	0.5	0.6	0.7	0.8
L(N*)	4	4	4	4	4	4
T(N*)	33.87	33.91	34.00	34.16	34.39	34.7

(vi)Variation in ξ_2

Table 6: Effect of ξ_2 on expected system length and expected cost

ξ_2	0.305	0.31	0.315	0.320	0.325	0.330
L(N*)	4	4	4	4	4	4
T(N*)	33.92	33.91	33.91	33.91	33.91	33.91

(vii) Variation in θ

Table 7: Effect of **θ** on expected system length and expected cost

θ	7	8	9	10	11	12
L(N*)	4	4	4	4	4	4
T(N*)	33.72	33.64	33.58	33.54	33.51	32.89

viii)Variation in β

Table 8: Effect of **β** on expected system length and expected cost

B	12.05	12.10	12.15	12.20	12.25	12.30
L(N*)	4	4	4	4	4	4
T(N*)	33.92	33.91	33.91	33.91	33.91	33.91

Effect of variation in the monetary parameters

ix)Variation in C_r

Table 9: Effect of C_r on expected system length and expected cost

Cr	17	19	21	23	25	27
L(N*)	4	4	4	4	4	4
T(N*)	32.06	29.97	28.04	26.02	24.03	22.36

x)Variation in C_{b1}

Table 10: Effect of C_{b1} on expected system length and expected cost

C_{b1}	52	54	56	58	60	62
L(N*)	4	4	4	4	4	4
T(N*)	33.88	33.88	33.88	33.88	33.88	33.88

Xi) Variation in C_{b2}

Table 11: Effect of C_{b2}on expected system length and expected cost

C_{b2}	80	85	90	95	100	105
L(N*)	4	4	4	4	4	4
T(N*)	33.87	33.89	33.91	33.92	33.92	33.93

xii) Variation in C_m

Table 12: Effect of C_m on expected system length and expected cost

C_m	215	230	245	260	275	290
L(N*)	4	4	4	4	4	4
T(N*)	33.93	33.98	34.04	34.09	34.15	34.20

xiii)Variation in C_o

Table 13: Effect of $C_o$on expected system length and expected cost

Co	55	60	65	70	75	80
L(N*)	4	4	4	4	4	4
T(N*)	34.00	34.12	34.15	34.17	34.50	34.62

xiv)Variation in C_h

Table 14: Effect of $\mathbf{C_h}$ on expected system length and expected cost

C_h	6	7	8	9	10	11
L(N*)	4	3	3	3	3	3
T(N*)	39.70	41.18	44.38	49.36	50.16	52.8

xv)Variation in C_s

Table 15: Effect of $\mathbf{C_s}$ on expected system length and expected cost

Cs	1100	1200	1300	1400	1500	1600
L(N*)	4	4	5	5	6	6
T(N*)	32.23	36.02	39.06	41.83	43.64	45.39