

# Transient analysis of $M/E_k/1$ Queueing System with Vacation and Two types of Repair facilities

V. N. Rama Devi

Associate Professor, Department of Mathematics, GRIET, Hyderabad, Telangana, India

## ABSTRACT

Here we presented a queueing model with Two types of Repair facilities (TRF) and server timeout. The server does the service for the waiting customer exhaustively in  $k$ - phases for each customer. After service is completed, the server waits for the fixed time 'c' for the customer arrive called server Time-out, otherwise go for vacation. During Time-out, server provides service if at least one customer enters else it goes for vacation. If the system encounters breakdowns, repair process will be initiated instantly. Here failure server is expedited with two types of Repair facilities (TRF) "where Type -I service facility is used with a probability of  $1-q$  to resume the interrupted service when the server fails during a service and customer stays in service facility to get the balance service whereas Type-II service facility is done with a probability  $q$  when the server fails before the starting of service to a new customer who joins the head of the queue". Explicit expressions are derived for various constants of queueing system and also illustrated numerical results.

Keywords: Vacation, length of the system, breakdowns, Two types of Repair facilities and Server timeout.

## I. INTRODUCTION

Queues with server vacations have been studied over a few decennary and also being adapted in many areas such as manufacturing, computer communication network models. In vacation queueing model the server completely stops service when it is on vacation. Levy and Yechiali [5], B.T. Doshi [1], Zhe George Zhang et.al [3] did excellent work on queueing systems with vacation. Y. Praby Loit et.al [2] did lot of work on server timeout concept.

In many circumstances, the server may face unforeseen failures. Understanding the nature of the such server in terms of unforeseen failures is vital as it influence system's efficiency. Kailash C. Madan [4] and PF. Jose et.al [6] did on breakdown and repair facilities.

To make these concepts realistic at any point of time, we are presenting our paper with breakdown, server timeout and two types of repair facilities based on the stage of server failure in transient mode.

The main objectives of this model are:

- To evaluate transient state probability distribution of the number of customers in the system in each state
- To compute system constants expected length and waiting time at time  $t$ .
- To carry out sensitivity analysis to explain the influence of various system parameters on system constants through numerical experiments.

## II. Model Description

Consider single server where arrival of units in to the system as Poisson process with rate ' $\lambda$ ' and be served individually. Each customer in the system is to be served in  $k$  independent and identically distributed exponential phases in the order in which they arrive. Whenever the system becomes empty the server waits for certain time ' $c$ ', which is called server timeout. If the server fails, repair process will be initiated

immediately. Here failure server is facilitated with two TRF ("where Type -I service mode is given with a probability of  $1-q$  to resume the interrupted service when the server fails amidst a service and customer stays in service facility to complete the balance service and Type-II service mode is used with a probability  $q$  when the server fails before the starting of service to a new customer who is just in front of the window"). light intensity of the polyhouse is controlled by the system to the given value.

## III. TRANSIENT STATE ANALYSIS

Steady state probabilities of the system are shown below:

$$P_{00} = p(\text{server is in vacation stage})$$

$$P_{10} = p(\text{server is in Time - out stage})$$

$$P_{2i} = p(\text{server is providing service}), i = k, 2k, 3k, \dots$$

$$P_{3i} = p(\text{server is in failure stage due to Break down}), i = k, 2k, 3k, \dots$$

The system can be modeled as

$$\frac{dp_{0,0}}{dt} = -\lambda p_{0,0} + Cp_{1,0} \quad (1)$$

$$\frac{dp_{1,0}}{dt} = -(\lambda + C)p_{1,0} + \mu kp_{2,k} \quad (2)$$

$$\frac{dp_{2,k}}{dt} = -(\lambda + \mu k + \alpha_1 + \xi_1)p_{2,k} + \lambda p_{1,0} + \mu kp_{2,k} + \alpha_2 p_{3,k} + \xi_2 p_{3,k} + \lambda p_{0,0} \quad (3)$$

$$\frac{dp_{2,i}}{dt} = -(\lambda + \mu k + \alpha_1 + \xi_1)p_{2,i} + \lambda p_{2,i-k} + \mu kp_{2,i+k} + \alpha_2 p_{3,i} + \xi_2 p_{3,i}; 2k \leq i \leq (s-1)k \quad (4)$$

$$\frac{dp_{2,sk}}{dt} = -(\mu k + \alpha_1 + \xi_1)p_{2,sk} + \lambda p_{2,sk-k} + \alpha_2 p_{3,sk} + \xi_2 p_{3,sk} \quad (5)$$

$$\frac{dp_{3,i}}{dt} = -(\lambda + \alpha_2 + \xi_2)p_{3,i} + \lambda p_{3,i-k} + \alpha_1(1-q)p_{2,i} + \xi_1 qp_{2,i}; k \leq i \leq (s-1)k \quad (6)$$

$$\frac{dp_{3,sk}}{dt} = -(\alpha_2 + \xi_2)p_{3,sk} + \lambda p_{3,sk-k} + \alpha_1(1-q)p_{2,sk} + \xi_1 qp_{2,sk} \quad (7)$$

## IV. PERFORMANCE MEASURES

Some performance measures are calculated to predict the system behaviour using the probabilities obtained through Runge-Kutta method:

1.  $P(\text{server being idle at time } t) = I(t)$
2.  $P(\text{server being timeout at time } t) = T(t)$

3.  $P(\text{server being busy at time } t) = S(t)$
4.  $P(\text{server being broken down at time } t) = B(t)$
5.  $L(t) = \text{Expected number of customers in the system at time } t = \sum n * p_n$
6.  $W(t) = \text{waiting time in the system at time } t = \frac{L(t)}{(\lambda * (1 - p_{\text{max.customers at time } t}))}$

*In this paper, we have presented length and waiting times only at time period of 1.5 where we have taken total of 2 minutes with an increment of 0.5*

## V. NUMERICAL RESULTS

MATLAB software is used to develop the computational program to find out system performance measures by giving numeric values to all the parameters. And also the effect of various parameters on the system performance measures is studied. The effect of different parameters in the system on performance measures (length and waiting time) is summarized in Tables 1-9.

In all numerical computations, the model parameters are taken as

$$s = 60, \lambda = 2, \mu = 4, C = 0.1, \alpha_1 = 0.25, \alpha_2 = 0.5, \xi_1 = 0.35, \xi_2 = 0.75, p = .85 \text{ and } k = 2$$

## VI. CONCLUSIONS AND FURTHER SCOPE OF STUDY

In this paper we have detailed transient analysis of a M/Ek/1 Queueing System with N-Policy, Server Failure, timeout and two types of repair facilities. Sensitivity analysis is also performed to know the influence of various parameters on system performance measures. Wrt some features like arrival rate, breakdown rate etc, there is an increase in length and waiting times whereas wrt some constants service rate, repair rate etc. they are decreasing. This study can be extended as Steady State analysis by considering the general distribution for service times with cost analysis can be done for optimum solution.

## VII. REFERENCES

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## Appendix

	2	3	4	5	6
L(t)	11.22	14.24	17.46	28.45	47.23
W(t)	4.65	6.54	7.86	26.42	38.45

	4	5	6	7	8
L(t)	11.22	9.65	8.84	6.47	4.13
W(t)	4.65	4.2	3.46	3.24	2.87

	0.1	0.11	0.12	0.13	0.14
L(t)	11.22	12.45	18.65	28.46	29.32
W(t)	4.65	4.8	4.86	5.63	5.97

	0.25	0.26	0.27	0.28	0.29
L(t)	11.22	12.24	18.64	19.87	21.46
W(t)	4.65	5.26	5.68	6.12	6.45

	0.5	0.51	0.52	0.53	0.54
L(t)	11.22	10.26	10.14	8.68	8.12
W(t)	4.65	4.54	4.52	4.12	4.08

	0.35	0.36	0.37	0.38	0.39
T					
L(t)	11.22	12.26	12.89	13.65	13.48
W(t)	4.65	4.68	4.68	5.2	5.3

	0.75	0.76	0.77	0.78	0.79
T					
L(t)	11.22	10.24	10.13	9.89	9.64
W(t)	4.65	4.24	3.87	3.63	3.23

	0.85	0.851	0.852	0.852	0.853
T					
L(t)	11.22	11.14	10.25	9.6	8.56
W(t)	4.65	4.12	3.87	3.65	3.12

Table 9: Effect of k					
T	2	3	4	5	6
L(t)	11.22	14.46	24.56	28.56	34.56
W(t)	4.65	5.12	5.36	5.88	6.12