© 2017 IJSRSET | Volume 3 | Issue 8 | Print ISSN: 2395-1990 | Online ISSN : 2394-4099
Themed Section : Engineering and Technology
doi : https://doi.org/10.32628/IJSRSET1173828

Studies on the Nonlinear Dynamics of Two Coupled Microwave Oscillators

Manaj Dandapathak
Department of Physics, Ramkrishna Mahato Government Engineering College,Purulia, Agharpur, Purulia, West Bengal, India email: phymanoj@yahoo.co.in

ABSTRACT

The nonlinear dynamics of two bilaterally coupled Gunn oscillators (BCGO) has been tried to explain theoretically by considering the cubic type nonlinear model of Gunn oscillator. To explain the dynamics, Melnikov's technique is used. From analytical point of view it is possible to predict the values of coupling factors and set of parameters zone for which the system dynamics may be chaotic. Effect of coupling factors on the two systems have also been calculated numerically and it is observed that numerical results are nearly similar with analytical prediction.
Keyword : Ultrasonic velocity, Binary mixtures, FLT and Intermolecular interaction.

I. INTRODUCTION

Microwave and millimeter wave oscillators are essential components in all microwave and millimeter wave transceiver for wireless LAN, radar system etc. Single solid state oscillator has limited output power in the microwave and millimeter region [1]. It has some problems also like spectral broadening, frequency drifting etc. To overcome these problems there are many ways and one of that has been considered by many authors is the combining of numerous synchronized oscillators [2-5] by unilateral or bilateral coupling. Bilateral coupling is more effective than unilateral coupling to overcome the problems [4]. In bilateral coupling, fraction of output of one oscillator is injected to another and vice versa. Now depending on the coupling parameters the system dynamics may be stable or chaotic. The experimental results of two bilaterally coupled Gunn oscillators already have been reported in literature [4]. Due to its high nonlinear character the detailed analytical study is very difficult. In this paper we have tried to analyze the behavior of BCGO by Melnikov's technique [6-9]. By studying the Melnikov's function the values of parameters can be predicted in which the oscillator system become chaotic. This analytical study is helpful to find the stable or synchronized mode of oscillation and also to find the regions of chaotic oscillation, which can be used to generate chaotic signals.

The whole study is divided into the following sections. In section-II system equation equations have been formulated. In section III and section-IV analytical and numerical results are discussed and finally in section-V, some conclusions are made.

II. DESCRIPTION OF THE SYSTEM AND FORMULATION OF SYSTEM EQUATIONS

Microwave Gunn oscillator is designed by mounting a Gunn diode inside a cavity with proper dc biasing so that the diode is in negative resistance region. We consider the equivalent circuit representation of two coupled Gunn oscillator systems as shown in figure 1. Here Gunn diode is replaced by the series combination of two nonlinear voltage sources of cubic type nonlinearity as written in equation (1), (2) and the cavity is replaced by lumped inductance (L_i) and capacitance (C_i) and resistance R_i. Total resistance is the combination of load resistance and lumped resistance.

$$v_r = -\beta_1 i - \beta_3 i^3 \quad (1)$$

$$v_c = -\alpha_1 q + \alpha_3 q^3 \quad (2)$$

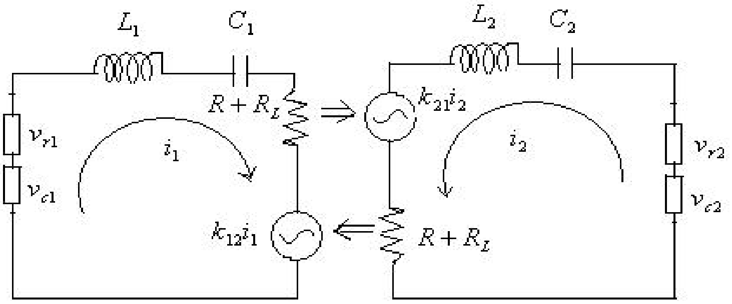


Figure 1. Equivalent circuit of BCGO.

Using the Kirchoff's mesh law and equation (1) and (2) the normalized system equations can be written as

$$\frac{d^2q_1}{dt^2} = a_1q_1 - b_1q_1^3 + c_1\frac{dq_1}{dt} - d_1\left(\frac{dq_1}{dt}\right)^3 + k_{21}\left(\frac{dq_2}{dt} - \frac{dq_1}{dt}\right) \quad (3)$$

$$\frac{d^2q_2}{dt^2} = a_2q_2 - b_2q_2^3 + c_2\frac{dq_2}{dt} - d_2\left(\frac{dq_2}{dt}\right)^3 + k_{12}\left(\frac{dq_1}{dt} - \frac{dq_2}{dt}\right) \quad (4)$$

Where $a_i = \alpha_1 C_i - 1, b_i = \alpha_3 C_i$, $c_i = \frac{\beta_1 - R_i - R_{Li}}{\omega_{ri} L_i}$, $d_i = \frac{\beta_3 \omega_{ri}}{L_i}$ and the normalized time is $\omega_{ri}t$. $\omega_{ri} = \frac{1}{\sqrt{L_i C_i}}$ represent the resonant frequency. Here k_{12} and k_{21} are the coupling factors in between two oscillators. Now solving the two equations (3) and (4) analytically by Melnikov methods the dynamics of coupled oscillator can be studied.

III. ANALYTICAL PREDICTION USING MELNIKOV'S METHOD

The dynamical properties of the BCGO can be studied by solving the system equations (3) and (4). However, to get a closed form solution of these coupled differential equations is difficult, if not impossible, in general. In this respect one can employ well known Melnikov technique of nonlinear dynamics as a possible analytical tool [6]. This method is applicable in studying a perturbed dynamical system [6-9]. Obviously, by evaluating the Melnikov function for the system, one can predict about the stability of the system. This function would have negative, zero or positive values for different sets of system parameters. If the function is of zero value or changes its sign, the dynamics of the system could be predicted to be chaotic in nature.

To derive Melnikov function, first it is necessary to find total Hamiltonian of the system. We rewrite the system equations (3) and (4) as a set of four first order differential equations as in (5).

$$\dot{q}_1 = p_1 \tag{5a}$$

$$\dot{p}_1 = a_1 q_1 - b_1 q_1^3 + c_1 p_1 - d_1 p_1^3 + k_{21}(p_2 - p_1) \tag{5b}$$

$$\dot{q}_2 = p_2 \tag{5c}$$

$$\dot{p}_2 = a_2 q_2 - b_2 q_2^3 + c_2 p_2 - d_2 p_2^3 + k_{12}(p_1 - p_2) \tag{5d}$$

Now, examining the set of equations (5a) to (5d), one can write the total Hamiltonian of the system in terms of canonical parameters p_1, q_1, p_2 and q_2 as follows,

$$H = F_1(q_1, p_1) + F_2(q_2, p_2) \pm H'(q_1, q_2, p_1, p_2) \tag{6}$$

Where

$$F_1(q_1, p_1) = \frac{1}{2} p_1^2 - \frac{1}{2} a_1 q_1^2 + \frac{1}{4} b_1 q_1^4 \tag{7}$$

$$F_2(q_2, p_2) = \frac{1}{2} p_2^2 - \frac{1}{2} a_2 q_2^2 + \frac{1}{4} b_2 q_2^4 \tag{8}$$

and

$$H'(q_1, q_2, p_1, p_2) = -c_1 p_1 q_1 + d_1 p_1^3 q - c_2 p_2 q_2 + d_2 p_2^3 q_2 - k_{12}(p_2 - p_1) q_1 - k_{21}(p_1 - p_2) q_2 \tag{9}$$

Here $F_1(q_1, p_1)$ and $F_2(q_2, p_2)$ represent the two unperturbed Hamiltonian individually and $H'(q_1, q_2, p_1, p_2)$ represents perturbed term due to which the system dynamics may be chaotic.

Now for simplicity to calculate Melnikov's function we consider for $F_2(q_2, p_2)$ we have a periodic oscillation and for system $F_1(q_1, p_1)$ we have homoclinic orbit by equating it to zero. We can write the equations of homoclinic orbits as

$$q_1 = \sqrt{\frac{2a_1}{b_1}} \sec h\left(t\sqrt{a}\right) \tag{10a}$$

$$p_1 = -\sqrt{\frac{2}{b_1}} a_1 \sec h\left(t\sqrt{a_1}\right) \tanh\left(t\sqrt{a_1}\right) \tag{10b}$$

Also the periodic solution for $F_2(q_2, p_2)$ may be written as

$$q_2 = A_0 \sin(\omega_{02} t) \tag{11a}$$

$$p_2 = \omega_{02} A_0 \cos(\omega_{02} t) \tag{11b}$$

Where $A_0 = \frac{4c_2}{3d\omega_0^2}$ and $\omega_{02}^2 = \frac{a^2}{2}\left[\left(1 + \frac{4b_2 c_2}{d_2 a_2^2}\right)^{1/2} - 1\right]$ are the free running amplitude and frequency of oscillation.

Now for this type of coupled system the Melnikov's function is defined as

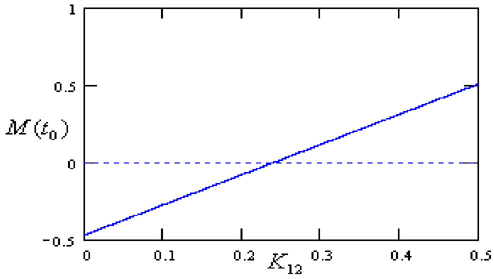
$$M(t_0) = \int_{-\infty}^{\infty} \{F_1, H'\} dt \tag{12}$$

Where $\{F_1, H''\} = \frac{\partial F_1}{\partial q_1}\frac{\partial H'}{\partial p_1} - \frac{\partial F_1}{\partial p_1}\frac{\partial H'}{\partial q_1}$ (13)

Using the equations (10), (11), (12), (13) the Melnikov's function for the system may be written as

$$M(t_0) = \frac{8}{3}\frac{a_1^{3/2}c_1}{b_1} - \frac{48}{35}\frac{a_1^{7/2}d_1}{b_1^2} - \frac{4k_{12}a_1^{3/2}}{b_1} + \frac{k_{21}A_0\pi\sqrt{2}a_1\operatorname{sech}(\pi\omega_{02}/2\sqrt{a})}{b_1^{1/2}}\sin(\omega_{02}t_0) + \frac{8}{3}\frac{k_{12}a_1^{3/2}}{b_1} - \frac{4}{3}\frac{k_{12}a_1^{3/2}}{b_1} \tag{14}$$
$$-\frac{\pi\omega k_{21}}{3b_1}\left(4a_1+\omega_{02}^2\right)\operatorname{cosech}(\pi\omega_{02}/4\sqrt{a})\operatorname{sech}(\pi\omega_{02}/4\sqrt{a})\sin(\omega_{02}t_0) \mp \sqrt{2}\pi A_0\omega_0^2\frac{k_{12}}{b_1a_1^{3/2}}\operatorname{sech}(\pi\omega_{02}/2\sqrt{a})\sin(\omega_{02}t_0)$$

The dynamics of the system can be predicted by the analysis of the Melnikov function. In figure 2 and 3 the variation of Melnikov function with coupling factor and with the frequency of the second oscillator are shown.



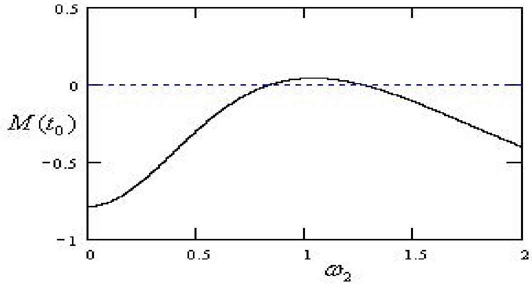


Figure.2 Variation of Melnikov function with coupling parameter **Figure.3** Variation of Melnikov function with frequency

For $a_1 = 0.73, b_1 = 0.52, c_1 = 0.2, d_1 = 0.51, k_{21} = .15$

From the plot of Melnikov function we see that the function becomes zero for some value of k_{12} (near about 0.24). The function has zero value also in some range of frequency of the second oscillator. Now zero value of the function represents the unstable or chaotic oscillation of the system. So by studying the variation of Melnikov's function with different parameters, the dynamics of the system can be explained.

IV. NUMERICAL SIMULATION RESULTS:

The system equations (3) and (4) are solved numerically by Runge-Kutta algorithm for some values of different parameters. Phase plane plot and Fourier spectrum is shown in the following figures.

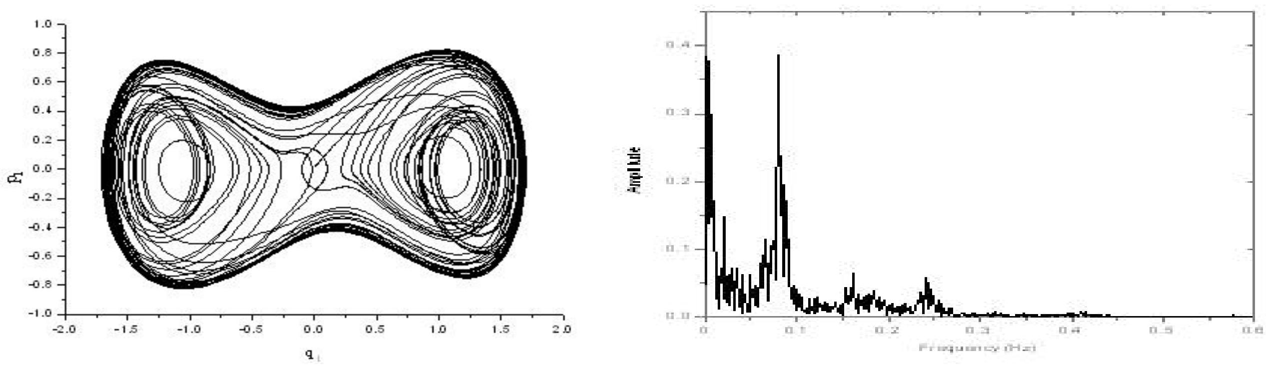


Figure.4 Phase plane plot and corresponding Fourier spectrum of first oscillator for $a_1 = 0.73, b_1 = 0.52, c_1 = 0.2, d_1 = 0.51, k_{21} = .1, k_{12} = .25$

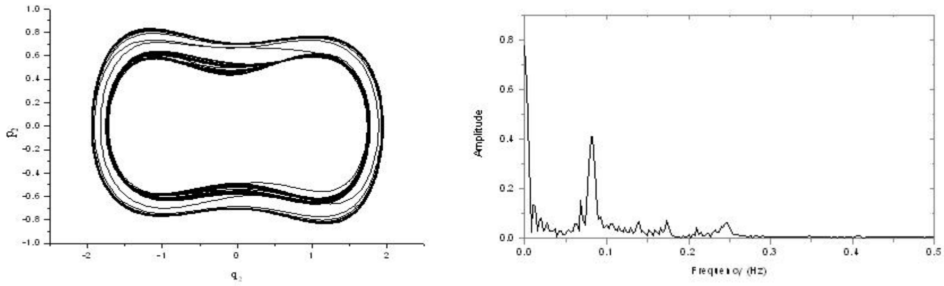


Figure.5 Phase plane plot and corresponding Fourier spectrum of second oscillator for

$$a_1 = 0.73, b_1 = 0.52, c_1 = 0.2, d_1 = 0.51, k_{21} = .1, k_{12} = .25$$

From these results we see that first oscillator is in chaotic state but second oscillator is in stable state. Now depending on the coupling factors two oscillators at a time may be in stable mode or in chaotic mode or one may be in stable mode or another in chaotic mode and vice versa. In numerical results only one set of results are shown. From analytical and numerical results it is observed that for the set of values of parameters at which chaotic oscillation is observed, Melnikov's function shows a zero value. So, analytical predictions are close agreed with numerical results.

V. CONCLUDING REMARKS

In this paper theoretical numerical studies on the behavior of two coupled Gunn oscillators have been done by calculating the Melnikov's function. By evaluating the function it is possible to predict the parameter zone and values of coupling factors for which the chaotic oscillation is observed. So the stability and behavior of two oscillators can be explained by observing the value of the function. This study may be helpful to remove unwanted chaos in many coupling systems or may be helpful to produce chaotic signals by two coupled oscillators with proper choice of coupling factors.

VI. REFERENCES

[1]. York R.A and Popovic Z.B "Active and quasi optical array for solid state power combining" Willey-1997.

[2]. Navaro J.A and Chang. K "Integrated active antennas and optical power combining" Willey-1996.

[3]. Birkeland J and Itoh T "A 16 element quasi- optical FET oscillator power combining array with external injection locking" IEEE, Trans Microwave theory Tech, 1992, Vol. 40(3), pp- 475-481.

[4]. B.C. Sarkar, D. Sarkar, S. Sarkar, and J. Chakraborty "Studies on the dynamics of bilaterally coupled X-band Gunn Oscillators" PIER-B, Vol. 32, pp- 149-167, 2011.

[5]. Anischenko. V, S. Astakhov and T. Vadivasova "Phase dynamics of two coupled oscillators under external periodic force" Europhysics Letters, Vol. 86, 2009.

[6]. M.A.Lieberman and A.J. Lichtenberg, Regular and Stochastic Motion (Springer, Berlin)

[7]. Simiu E. Melnikov process for stochastically perturbed, slowly varying oscillator: application to a model of wind driven coastal currents. J Appl Mech 1996; 63 (June) :429-35.

[8]. M. Dandapathak, B C Sarkar, Nonlinear Dynamics of Optical Costas Loop with Inherent Time Delay, Optik, Int. J. of Light Electron Opt. (2014).

[9]. M. Dandapathak, B. C. Sarkar, "Nonlinear Dynamics of Optical Costas Loop with Inherent Time Delay", Optik. 126 (24), (2015)

Cite this Article

Manaj Dandapathak, "Studies on the Nonlinear Dynamics of Two Coupled Microwave Oscillators", International Journal of Scientific Research in Science, Engineering and Technology (IJSRSET), Online ISSN : 2394-4099, Print ISSN : 2395-1990, Volume 3 Issue 8, pp. 1391-1395, November-December 2017. Available at doi : https://doi.org/10.32628/IJSRSET1173828

Journal URL : https://ijsrset.com/IJSRSET1173828