© 2017 IJSRSET | Volume 3 | Issue 8 | Print ISSN: 2395-1990 | Online ISSN : 2394-4099
Themed Section : Engineering and Technology
doi : https://doi.org/10.32628/IJSRSET1173829

Studies on Chaotic Dynamics of Rayleigh-Duffing Oscillator by Melnikov Method

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ABSTRACT

Chaotic dynamics of Rayleigh-Duffing oscillator has been described with the help of Melnikov's method of perturbation. Critical observation shows that depending on system parameters, value an external force signal chaotic oscillation can be generated in the system. Analytically, range of value of different system parameters and external signal strength required for chaotic oscillation can be determined. All the analytical predictions have been verified by solving the system equation of the oscillator numerically.
Keywords : Nonlinear Dynamics, Duffing Oscillator, Melnikov Function

I. INTRODUCTION

In recent years, researchers are interested to quantify and characterise the nonlinear behaviour of different nonlinear oscillator by theoretical, numerical, and experimental techniques. This increased interest is due to the fact that this nonlinear phenomenon appears in various fields, from mathematics, physics, chemistry, to engineering, biology, economics, medicine etc. [1]. By using the chaos theory so many phenomenon such as nature of atmospheric weather, intensity of solar activity, nonlinear oscillation in different electrical and mechanical systems, etc. can be explained properly [2].

To describe different physical system, different oscillators have been designed mathematically. Duffing oscillator (DO) is one such model, which is used to describe nonlinear behaviours of many physical systems [3-4]. The forced Duffing oscillator is used as a prototype model for various physical and engineering problems such as different electrical circuit, Josephson junctions, plasma oscillations, optical bistability, etc.[5-6]. The potential of DO is of double well type and its chaotic behaviour is highly dependent on a nonlinear velocity damping term. Such damping can, in some systems, change the sign depending on velocity or displacement values, and provide excitation energy to the examined system [7]. Now, to describe some other physical systems such as, Brusselator, Selkov, rolling response, certain micro-electromechanical systems (MEMS) a cubic power of the velocity dissipative function is introduced in the system equation of Duffing oscillator [8-10]. This cubic power of velocity damping term is known as Rayleigh dissipation and with this dissipation terms, the Duffing oscillator is known as Rayleigh-Duffing oscillator (RDO). Potential of RDO is same as the double well potential like a DO and the two systems are differing by term of Rayleigh dissipation. Chaotic responses in Duffing oscillator are very common event and so many articles have been reported on the chaotic dynamics of DO [11-12]. Dynamics of RDO has been also reported in different articles [13-15]. Studies on chaotic dynamics of this system is important, since it might be useful to model many physical and engineering systems, and in the context of chemical and biological oscillators. In [15], chaotic dynamics of RDO has been explained by the Melnikov's method of homoclinic bifurcation. How the fractal basin boundaries arise and are modified as the damping coefficient is varied has been studied here. In most of the work the mechanisms by which the strange attractors arise and are modified with the variation of system parameters have been studied. If one sees the system equation of microwave Gunn oscillator (GO) then, it is basically the system equation of RDO.

Different experimental and numerical observation on the chaotic dynamics of GO in under biased (non oscillatory) condition have been reported recently in different articles [16-19]. In [17], it has been reported that in under biased non oscillatory state presence of sync signal of very low strength makes the system dynamics chaotic. It has been also reported in [18], at the time of growth and quenching of oscillation a hysteresis occurs. Coefficient of different damping and restoring terms depends on dimension of cavity and dc biasing voltage. So, to explain different experimental evidences by the theory of RDO, one has to take the different values of damping and restoring coefficients. In this paper attempt has been made to explore the dynamics of RDO by considering both positive and negative values of damping factors. The condition for stable oscillation and how hysteresis occurs have been explained with proper analytical and numerical results. Again, by using the Melnikov theory of homoclinic bifurcation [20-24], chaotic dynamics of forced RDO has been explained in different parameter space. Although in [15], chaotic dynamics of RDO has been explained by the theory of Melnikov method, but analysis has been done for same value of cubic and linear damping coefficients. Also, the effect of only damping term has been studied. But in this paper, we have explained chaotic dynamics, by considering different values of damping coefficients. Also, we have shown the effects of strength and frequency of external periodic forced signal on the dynamics of RDO. This study could be quite helpful to explain the dynamics of different nonlinear oscillator systems like GO.

The system equation of RDO is a second order differential equation of a class of nonlinear oscillators in terms of a state variable q the generalised system equation is given as.

$$\frac{d^2q}{d\tau^2} = aq - bq^3 + c(\frac{dq}{d\tau}) - d(\frac{dq}{d\tau})^3 \qquad (1)$$

Here, τ is the normalized time and a, b, c and d are parameters used to express the relative strengths of respective restoring and dissipating forces.

II. Method and Materials

2.1. Analytical Study by Melnikov Method:

We perform an analytical study on the dynamics of the driven R-D oscillator described by (1) using well known Melnikov method [22-24]. Here an integral, called Melnikov function (MF), is calculated for a set given parameters of the system. If the MF be a negative quantity for all time within the period of perturbation, the perturbed orbit lies inside the unperturbed one. On the other hand, if the MF be a positive quantity, the perturbed orbit always lies outside of the unperturbed orbit. When the MF is simple zero value or changes its sign from positive to negative or vice versa during the perturbation period, then the intersection of perturbed and unperturbed orbits occur and it represents the chaotic oscillation in the system [24]. We rewrite (1) in the following form,

$$\dot{q} = p \qquad (2a)$$

$$\dot{p} = f(q) + \varepsilon g(q, p, \tau) \qquad (2b)$$

Where, $g(q,p,\tau)$ represents the perturbed term in terms of the velocity dependent damping forces and external forcing term. ε is a small parameter written to ensure the smallness of the perturbation. For this type of system, the MF is written as,

$$M(\tau_0) = \int_{-\infty}^{\infty} p_0(\tau-\tau_0) g[q_0(\tau-\tau_0), p_0(\tau-\tau_0), \tau] d\tau \qquad (3)$$

Here q_0 and p_0 are the state variables representing the unperturbed trajectory in the phase plane. In the unperturbed case, this system equation takes the following form,

$$\dot{q} = p \qquad (4a)$$

$$\dot{p} = aq - bq^3 \qquad (4b)$$

Using Hamilton's canonical equation, the unperturbed Hamiltonian for the system (Eq.5) is,

$$H_0(q,p) = \frac{1}{2}p^2 - \frac{1}{2}aq^2 + \frac{1}{4}bq^4 \qquad (5)$$

Equating the unperturbed Hamiltonian to zero, the parametric equation for the homoclinic orbit is obtained as,

$$q = \pm\sqrt{\frac{2a}{b}} \sec h\left(\sqrt{a}\tau\right) \qquad (6a)$$

$$p = \mp a\sqrt{\frac{2}{b}} \sec h\left(\sqrt{a}\tau\right) \tan h\left(\sqrt{a}\tau\right) \qquad (6b)$$

Using equation (9a) and (9b), the MF for this system would be written as,

$$M(\tau_0) = \int_{-\infty}^{\infty} p[cp - dp^3 + q_s \cos\Omega_s(\tau+\tau_0)] d\tau \qquad (7)$$

After simplification of the integral, the MF for the driven R-D oscillator can be written as,

$$M(\tau_0) = \frac{4c}{3b}a^{3/2} - \frac{16d}{35b^2}a^{7/2} - \sqrt{\frac{2}{b}}\pi\Omega_s q_s \sin(\Omega_s\tau_0) \sec h\left(\frac{\pi\Omega_s}{2\sqrt{a}}\right) \qquad (8)$$

Here, τ_0 is the time, at which first transverse intersection occurs between unstable and stable orbit. If for any real value of τ_0, MF shows simple zero or change of its sign, then system dynamics becomes chaotic. The effects of external sync signal and system parameters on chaotic dynamics of the system can be predicted by observing the variation of MF with different values of system parameters as well as the strength and the frequency of the external forcing signal.

2.2. Effect of the parameter c:

We have drawn a parameter space $(q_s - c)$, based on computed values of the MF for fixed values of other parameters used. The parameter space is shown in Fig-1 and it consists of three zones where the MF is negative (black region), or positive (yellow region) or zero and changes sign (combination of black and yellow region). Now from the criterion of Melnikov method, we conclude that, the region containing both colours represents chaotic oscillation zone as in this region MF shows the change of its value and which indicates the transverse intersection of unstable orbit with stable one.

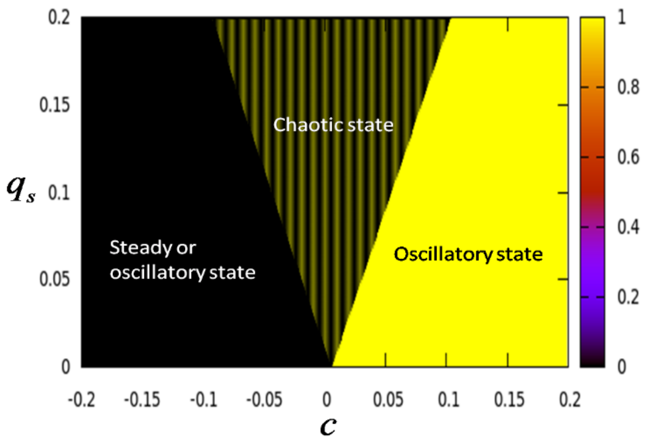

Fig.1. Parameter space $(q_s - c)$, indicating different zones (stable oscillatory and chaotic).
$[a=1, b=1, d=0.015, \Omega_s=1.2]$

Negative value of MF indicates the unstable manifold lies inside the stable homoclinic orbit. So, negative value of MF indicates steady oscillatory state or a stable point solution of the system. This behaviour is observed for slight large negative value of damping parameter c. For more large negative value of c, the oscillation would be quenched to a stable point. When value of c is increased towards the positive value, unstable orbit moves near about stable homoclinic orbit and for some range of c value, transverse intersection occurs between the two orbits. In this region MF changes its sign, which represents the chaotic oscillatory state of the system. For large value of c, unstable orbit moves out side to the stable orbit and no transverse intersection between stable and unstable orbits would occur. In this region, MF always remains positive by indicating a stable oscillatory state of the system.

From Fig-1, it is seen that for a particular value of strength and other parameters, if the value of c is increased from negative to positive, then there exists a range of c, in which MF changes its sign. In this range of c, the system dynamics would be chaotic in nature. With further increase of c, we have the zone in which MF always remains positive, indicating steady or quasi periodic oscillation. It is also seen that, the higher is the strength of the external signal, the more is the range of c, leading to chaotic oscillations. The chaotic zone is extended from negative value to positive value of c. This means that chaotic oscillations could be generated in a non-oscillatory Rayleigh-Duffing oscillator (when $c<0$), if it be driven by an external forcing signal of proper strength and frequency. Using (8), the range of c, in which the system shows chaotic oscillations, can be obtained as,

$$c \geq \frac{3b}{4a^{3/2}}\left[\frac{16d}{35b^2}a^{7/2} \mp \sqrt{\frac{2}{b}}\pi\Omega_s q_s \sin(\Omega_s\tau_0)\operatorname{sec}h\left(\frac{\pi\Omega_s}{2\sqrt{a}}\right)\right] \qquad (13)$$

Effects of other parameters on the system dynamics can also be understood by observing the variation of MF in a similar way.

2.3. Effect of the strength of forcing signal:

The variation of the MF with the strength of the external forcing signal (q_s) has been computed keeping other system parameters and the forcing signal frequency fixed. It is observed that for a negative value of c MF changes its sign from negative to positive for increasing q_s. However with positive value of c, the MF changes sign from positive to negative at some other value of q_s with all other parameters kept unchanged. The variation of the MF with q_s for c<0 and c>0 are shown in Fig.2 and Fig.3 respectively.

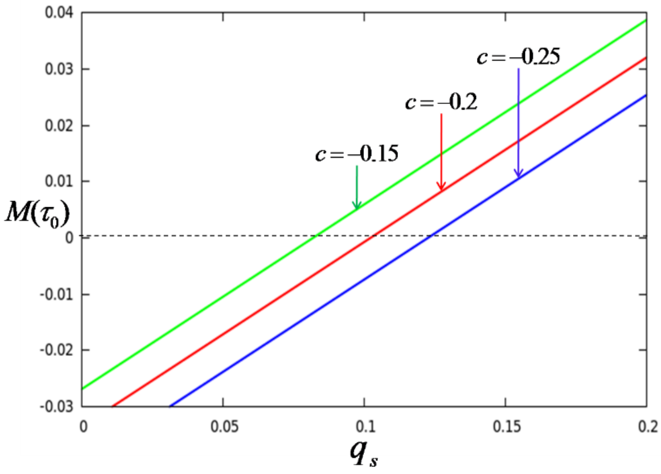

Fig.2. Variation of MF with the strength of sync signal for negative value of *c*, $[a=1, b=1, d=0.015, \Omega_s=1.2]$

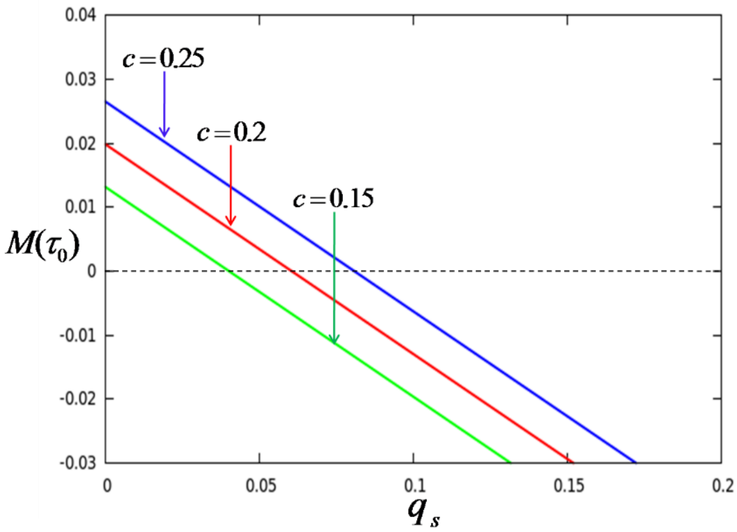

Fig.3. Variation of MF with the strength of sync signal for positive value of *c*, $[a=1, b=1, d=0.015, \Omega_s=1.2]$

So applying Melnikov's theory, it can be concluded that the oscillation would be chaotic, when strength of the external signal is greater than some critical value. The required critical strength is different for non-oscillating and oscillating R-D oscillator. The minimum value of the required strength (considering $\sin(\omega\tau_0)=1$) for chaotic oscillation can be obtained from (8) as,

$$|q_s| \geq \frac{\frac{4c}{3b}a^{3/2} - \frac{16d}{35b^2}a^{7/2}}{\sqrt{2/b}\,\pi\Omega_s \sec h\left(\frac{\pi\Omega_s}{2\sqrt{a}}\right)} \tag{14}$$

III. Numerical Analysis of the System

Varying the parameter *c* from a negative to positive value with other parameters fixed, we note the (*q*-*p*) phase plane plot and the frequency spectrum of *q* variable. In phase plane plot we have also plotted the phase plane unperturbed homoclinic orbit. Numerically simulated phase plane plot along with unperturbed homoclinic path of the RDO and corresponding power spectrum are shown in Fig.4 to Fig.5. It is seen that after a critical value of *c* the output is chaotic and finally becomes periodic at some frequency and strength of the driving RF signal through sequence of quasi-periodic states. Fig 4(a) and 4(b) show the phase plane plot and frequency spectrum for periodic oscillation.

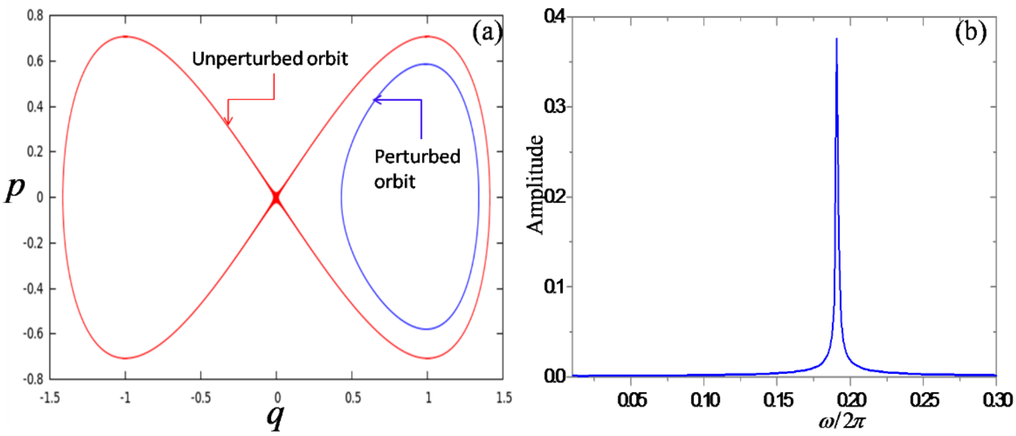

Fig. 4: Numerically simulated (a) Phase plane, and (b) Power spectrum

$$[a = 1, b = 1, c = -0.05, d = 0.015, \Omega_s = 1.2, q_s = 0.2]$$

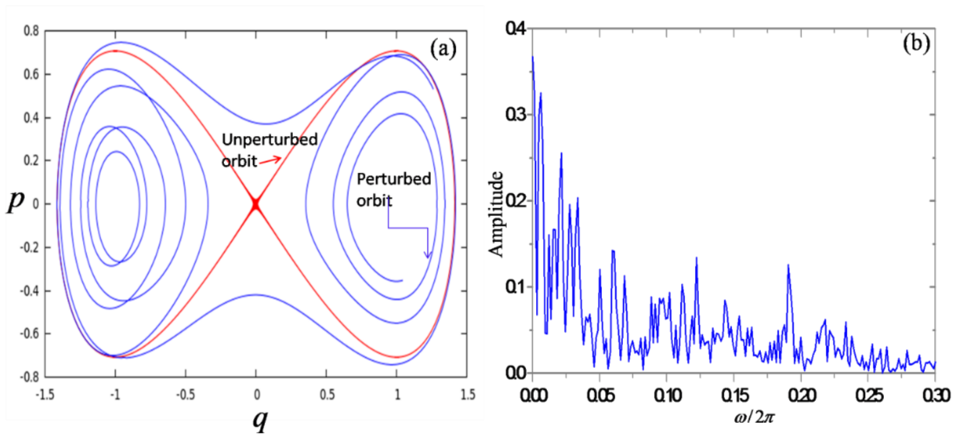

Fig. 5: Numerically simulated (a) Phase plane, and (b) Power spectrum

$$[a = 1, b = 1, c = -0.02, d = 0.015, \Omega_s = 1.2, q_s = 0.2]$$

In the figure (Fig.4) the orbit shown by the red colour represents the unperturbed homoclinic orbit and the orbit shown by blue colour represents the perturbed orbit. From this figure it is observed that the perturbed orbit lies inside the unperturbed one and there is no intersection of perturbed and unperturbed orbit. So, there is no occurrence of chaotic oscillation and the corresponding frequency spectrum shows one peak representing period-1 oscillation. In Fig.5, perturbed and unperturbed orbits intersect each other and corresponding frequency spectrum is broad in nature representing a chaotic oscillation. For some higher value of c $(= 0.025)$ phase plane plot and frequency spectrum are shown in Fig.6.

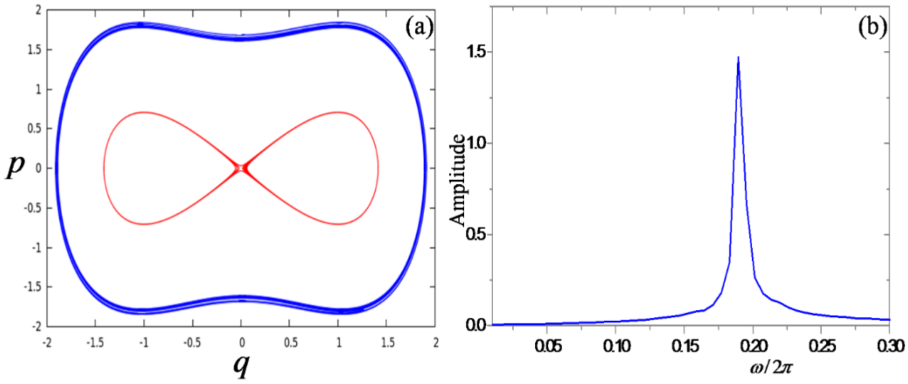

Fig. 6: Numerically simulated (a) Phase plane, and (b) Power spectrum
$[a = 1, b = 1, c = 0.025, d = 0.015, \Omega_s = 1.2, q_s = 0.2]$

From this figure, one observes that the perturbed orbit lies outside the stable unperturbed orbit and there is no intersection of the two types of orbits. The corresponding frequency spectrum shows one peak representing stable period-1 oscillation again.

IV. CONCLUSION

Chaotic dynamics of Rayleigh Duffing oscillator has been analysed in this paper. By using the linear technique of stability to produce self-oscillation role of different damping terms has been discussed. Analytical result shows that in non-oscillatory state by the presence of any external periodic force the system dynamics may be chaotic. Numerical results also agree with the analytical prediction. This study is helpful to estimate the value of amplitude and frequency range of the external force in which system shows chaotic oscillation. This study has two fold applications. From designer point of view, this study is important to design such type of a system, which is free from chaos and can be used in stable oscillatory condition. So, generation of chaotic signal is another important field of research. Particularly, to generate microwave carrier signal from Gunn oscillator, the theory of RDO is suitable to choose the proper values of system parameters and external sync signal so that chaotic carrier signal can be generated in a control manner.

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Cite this Article

Manaj Dandapathak, "Studies on Chaotic Dynamics of Rayleigh-Duffing Oscillator by Melnikov Method", International Journal of Scientific Research in Science, Engineering and Technology (IJSRSET), Online ISSN : 2394-4099, Print ISSN : 2395-1990, Volume 3 Issue 8, pp. 1396-1403, November-December 2017. Available at doi : https://doi.org/10.32628/IJSRSET1173829

Journal URL : https://ijsrset.com/IJSRSET1173829