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On Strong Generalised Derivations in Semi-Prime Rings

G.Shakila Chitra Selvi[1], Dr. V. Thiripurasundari[2], Dr. G. Gopala Krishnamoorthy[3]
[1]Research Scholar, PG and Research, Department of Mathematics, Sri S. Ramasamy Naidu Memorial College, Sattur, Tamil Nadu, India,
[2]Assistant Professor, PG and Research, Department of Mathematics, Sri S. Ramasamy Naidu Memorial College, Sattur, Tamil Nadu, India,
[3]Academic Advisor, P.S.N.L. College of Education, Mettam Alai, Tamil Nadu, India

ABSTRACT

The concept of generalized derivations is a ring was generalized as strong generalized derivations by the authors. The properties of strong generalized derivations is semi-prime rings are studied and more generalized results are obtained.

Keywords: - Semi-prime rings, Strong generalized derivation, commutativity.

I. INTRODUCTION

Let R be an arbitrary ring. An additive mapping $d : R \rightarrow R$ is called a derivation of R if $d(xy) = d(x)y + xd(y)$ for all $x, y \in R$. Following Bresar [3] an additive mapping $D : R \rightarrow R$ is called a generalized derivation on R if there exists a derivation on various algebraic structures. H.E.Bell [1] proved that if N be a 3-prime, 2-torsion free near ring admitting a non-zero generalized derivation f such that $f(N) \subset Z$, then N is a commutative ring. He also proved that if N is a 3- prime 2 – torsion – free near – ring admitting a generalized derivation f associated with a non-zero derivation D of N satisfying $f(x)f(y) = f(y)f(x)$ for all $x, y \in N$, then N is a commutative ring. H.E.Bell and N.U.Rehman [2], G.Gopalakrishnamoorthy, G .Shakila Chitra Selvi and V.Thirupurasundari [5] N.U. Rehmann [7] and many others have studied and published many results on generalized derivations. Mehsin Jabel Atteya and Dalal Ibrahim Reshan [6] have proved many results regarding generalized derivations in semi-prime rings assuming that the semi-prime has cancellation properties. It is noted that the property of semi-prime is not at all used is their proof and logically the proof seems to be not good.

In this paper we investigate all those results and prove more stronger results by omitting the condition. " The ring has to satisfy cancellations Laws". Throughout this paper R denote an arbitrary ring, Z its multiplicative center. A ring R is said to be prime if $aRb = 0$ implies either $a = 0\ (or) b = 0$. It is said to be semi prime if $aRa = 0$ implies $a = 0$, Every prime ring is semi-prime.

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II. PRELIMINARIES

In this section we shall see some definitions and results which we use in our proof.

Definition 2.1

Let R be any ring for $x, y \in R. define\ [x, y] = xy - yx$; called commutator of $x\ and\ y$.

Lemma 2.2

Let R be any ring.

(i) $[x, y] = -[y, x] \quad \forall\, x, y \in R$

(ii) $[x, y + z] = [x, y] + [x, z] \quad \forall\, x, y\, z \in R$

(iii) $[x, yz] = y[x, z] + [x, y]z \quad \forall\, x, y, z \in R$

(iv) $[x, y] = 0 \quad \forall\, x, y \in R$ iff R is Commutative

(v) $[x, x] = 0 \quad \forall\, x \in R$

Definition 2.3

Let R be any ring for $x, y \in R$ define $x\, y = xy + yx$ called the anti-commutator of $x\ and\ y$

Definition 2.4

Let R be any ring. Then $z = \left\{x \in \frac{R}{xy=yx}\ \forall\, y \in R\right\}$ is called the centre of R. R is commutative iff Z=R.

III. MAIN RESULTS

Theorem 3.1

Let R be a semi-prime ring admitting a non-zero strong generalized derivation F associated with a non-zero additive map $f: R \rightarrow R$ and a non-zero derivation d of R. If $F([x, y]) = f([x, y]) = [x, y]$ for all $x, y \in R$, then R is commutative if d is an onto map.

Proof :

By the hypothesis

$F([x, y]) = f([x, y]) = [x, y] \quad \forall\, x, y \in R$ ………………(1)

Replace x by xz we get

$$F([x\,z, y]) = [xz, y] \quad \forall\, x, y, z \in R$$

$$F(x[z, y] + [x, y]z) = x[z, y] + [x, y]Z$$

$$F\,(x[z, y]) + F([x, y]z) = x[z, y] + [x, y]z$$

$$f(x)[z, y] + x\,d([z, y]) + f([x, y])z + [x, y]d(z) = x[z, y] + [x, y]z$$

using (1) we get

$$f(x)[z, y] + xd([z, y]) + [x, y]\,d(z) = x[z, y]$$

Replacing z by y we get

$$[x, y]d(y) = 0 \quad \forall\, x, y \in R$$

Since d is onto we have

$[x,y]u = 0 \ \forall x,y,u \in R$

(ie) $[x,y]u[x,y] = 0 \ \forall x,y,u \in R$

Since R is semi prime, $[x,y] = 0 \ \forall \ x,y \in R$

(ie) R is commutative.

Remark 3.2

Taking f=F, we get Theorem 3.1[8]

Theorem 3.3

Let R be a semi-prime ring admitting a non-zero strong generalized derivation F associated with a non-zero additative map $f : R \to R$ and a non-zero derivation d of R. If $F([x,y]) = f([x,y]) = -[x,y]$ for all $x,y \in R$ then R is commutative provided d is an onto map.

Proof :

By the hypothesis

$F([x,y]) = f([x,y]) = -[x,y] \ \forall x,y \in R$

(ie) $F([x,y]) = f([x,y]) = [y,x] \ \forall x,y \in R$ ……………….(1)

Replacing y by yz we get

$F[x,yz] = [yz,x]$

$F(y[x,z] + [x,y]z) = y[z,x] + [y,x]z$

$f(y)[x,z] + yd([x,z]) + f([x,y])z + [x,y]d(z) = y[z,x] + [y,x]z$

using (1) we get

$f(y)[x,z] + yd[x,z]) + [x,y]d(z) = y[z,x]$

Replacing z by x we get

$[x,y]d(x) = 0 \ \forall x,y \in R$

Since d is onto we get

$[x,y]u = 0 \ \forall x,y,u \in R$

(ie) $[x,y]u[x,y] = 0 \ \forall x,y\,u \in R$

Since R is semi-prime, we get

$[x,y] = 0 \ \forall x,y \in R$

(ie) R is commutative.

Remark 3.4

Taking f=F, we get Theorem 3.2[8]

Theorem 3.5

Let R be a semi-prime ring. If R admits a non-zero strong generalized derivation F associated with a non-zero additive map $f: R \to R$ and a non-zero derivation d of R such that $F(x \circ y) = f(x \circ y) = x \circ y$ for all $x,y \in R$, then R is anti-commutative provided d is an onto map.

Proof :-

By the hypothesis

$F(x \circ y) = f(x \circ y) = x \circ y$

(ie) $F(xy + yx) = f(xy + yx) = xy + yx \ \forall \ x,y \in R$

$F(xy) + F(yx) = f(xy) + f(yx) = xy + yx \ \forall x,y \in R$ ……………(1)

Replace x by xy we get

$$F(xy^2) + F(yxy) = xy^2 + yxy$$

$$f(xy)y + xyd(y) + f(yx)y + yxd(y) = xy^2 + yxy$$

$$\big(f(xy) + f(yx)\big)y + (xy + yx)d(y) = xy^2 + yxy$$

using (1) we get

$$(xy + yx)d(y) = 0 \qquad \forall\, x, y \in R$$

Since d is onto

$(xy + yx)u = 0 \quad \forall\, x, y, u \in R$

(ie) $(xy + yx)u(xy + yx) = 0 \quad \forall\, x, y, u \in R$

Since R is semi-prime, $xy + yx = 0 \quad \forall\, x, y \in R$

(ie) R is a anti-commutative.

Remark 3.6

Taking f=F, we get Theorem 3.3[8]

Theorem 3.7

Let R be a semi-prime ring. If R admits a non-zero strong generalized derivation associated with a non-zero additive map $f: R \to R$ and a non-zero derivation d of R such that

$F(x \circ y) = f(x \circ y) = -(x \circ y) \quad \forall\, x, y \in R$, then R is anti-commutative provided d is an onto map.

Proof:

By the hypothesis

$F(x \circ y) = f(x \circ y) = -(x \circ y) \quad \forall\, x, y \in R$

$F(xy) + F(yx) = f(xy) + f(yx) = -(xy + yx) \quad \forall\, x, y \in R$ ....(1)

Replace *x by xy we get*

$$F(xy^2) + F(yxy) = f(xy^2) + f(yxy) = -(xy^2 + yxy) \quad \forall\, x, y \in R$$

$$f(xy)y + xyd(y) + f(yx)y + yxd(y) = -(xy^2 + yxy) \quad \forall\, x, y \in R$$

$$f(xy)y + xy\, d(y) + f(yx)y + yxd(y) = -(xy + yx)y$$

(ie) $\big(f(xy) + f(yx)\big)y + (xy + yx)d(y) = -(xy + yx)y$

using (1) we get

$$(xy + yx)d(y) = 0 \quad \forall\, x, y \in R$$

Since d is onto

$$(xy + yx)u = 0 \quad \forall\, x, y, u \in R$$

(ie) $(xy + yx)u(xy + yx) = 0 \quad \forall\, x, y, u \in R$

Since R is semi-prime, $xy + yx = 0, \quad \forall\, x, y \in R$

(ie) R is anti-commutative.

Remark 3.8

Taking f=F, we get Theorem 3.4[8]

Theorem 3.9

Let R be a semi-prime ring. If R admits a non-zero strong generalized derivation F associated with a non-zero additive map $f: R \to R$ and a non-zero derivation d of R such that

$F([x,y]) = f([x,y]) = xy + yx \quad \forall\, x, y \in R$, then R is commutative provided d is an onto map.

Proof:

By the hypothesis

$F([x,y]) = f([x,y]) = xy + yx \quad \forall\, x, y \in R$

(ie) $F(xy) - F(yx) = f(xy) - f(yx) = xy + yx \quad \forall\, x, y \in R$ ……………(1)

Replacing x *by* xy *we get*

$$F(xy^2) - F(yxy) = xy^2 + yxy$$

(ie) $f(xy)y + xyd(y) - f(yx)y - yxd(y) = (xy + yx)y$

(ie) $\big(f(xy) - f(yx)\big)y + (xy - yx)d(y) = (xy + yx)y$

using (1) we get

$$(xy - yx)d(y) = 0 \quad \forall\, x, y \in R$$

Since d is onto, $(xy - yx)u = 0 \quad \forall\, x, y, u \in R$

(ie) $(xy - yx)u(xy - yx) = 0 \quad \forall\, x, y, u \in R$

Since R is semi-prime, $xy - yx = 0 \quad \forall\, x, y \in R$

Remark 3.10

Taking f=F, we get Theorem 3.5[8]

Theorem 3.11

Let R be a semi-prime ring. If R admits a non-zero strong generalized derivation F associated with a non-zero additive map $f: R \to R$ and a non-zero derivation d of R such that $F(xy + yx) = f(xy + yx) = xy - yx$ for all $x, y \in R$, then R is anti-commutative provided d is an onto map.

Proof:

By the hypothsis

$F(xy + yx) = f(xy + yx = xy - yx \quad \forall\, x, y \in R$

(ie) $F(xy) + F(yx) = f(xy) + f(yx) = xy - yx \quad \forall\, x, y \in R$

Replace x *by* xy *we get*

$$F(xy^2) + F(yxy) = xy^2 - yxy$$

$$f(xy)y + xyd(y) + f(yx)y + yxd(y) = (xy - yx)y$$

$$\big(f(xy) + f(yx)\big)y + (xy + yx)d(y) = (xy - yx)y$$

using (1) we get

$$(xy + yx)d(y) = 0 \quad \forall\, x, y \in R$$

Since d is onto, we have

$$(xy+yx)u=0 \quad \forall\ x,y,u\ \in R$$

(ie) $(xy+yx)u(xy+yx)=0 \quad \forall\ x,y,u\ \in R$

Since R is semi-prime, $xy+yx=0 \quad \forall\, x,y\ \in R$

(ie) R is anti-commutative.

Remark 3.12

Taking f=F we get Theorem 3.6[8]

Theorem 3.13

Let R be a semi-prime ring admitting a non-zero strong generalized derivation F associated with a non-zero additive map $f: R\ \to R$ and a non-zero derivation d of R such that $[d(x),F(y)]=[d(x),f(y)]=0\ \ \forall\, x,y\ \in R$. Then R is commutative.

Proof:

By the hypothesis $[d(x),F(y)]=[d(x),f(y)]=0 \quad \forall\ x,y\ \in R$ ……………(1)

Replacing y by $yd(x) we\ get$

$$\left[d(x),F\big(yd(x)\big)\right]=0 \quad \forall\, x,y\ \in R$$

$$[d(x),f(y)d(x)+yd^2(x)]=0 \quad \forall\ x,y\ \in R$$

(ie) $[d(x),f(y)d(x)]+[d(x),yd^2(x)]=0 \quad \forall\, x,y\ \in R$

$$f(y)[d(x),d(x)]+[d(x),f(y)]d(x)+y[d(x),d^2(x)]+[d(x),y]d^2(x)=0 \quad \forall\, x,y\ \in R$$

using (1) we get

$$y[d(x),d^2(x)]+[d(x),y]d^2(x)=0 \quad \forall\, x,y\ \in R$$

Replacing y by $xy\ \ we\ get$

$$xy[d(x),d^2(x)]+[d(x),xy]d^2(x)=0$$

$$xy[d(x),d^2(x)+x[d(x),y]d^2(x)+[d(x),x]y\,d^2(x)=0$$

using (2) we get

$$[d(x),x]yd^2(x)=0 \quad \forall\, x,y\ \in R$$

$[d(x),x]Rd^2(x)=0 \quad \forall\, x\ \in R$ ……………………..(3)

Since R is semi-prime, it must contain a family $p=\{p_\propto/\propto\in\wedge\}$ of non-zero prime ideals

With $\wedge\ p_\propto=\{0\}$ Then (3) shows that for each $\propto\in\wedge$ either

$d^2(x)\ \in p_\propto\ (or)[d(x),x]\in\ p_\propto \quad \forall\ x\ \in R$ ………………………(4)

Suppose that $d^2(x)\ \in\ p_\propto$

Now $d[d(x),x]=d(d(x)x-d\big(xd(x)\big)$

$=d^2(x)x+d(x)d(x)-d(x)d(x)-xd^2(x)$

$$d([d(x),x]) \ = \ d^2(x)x-x\,d^2(x) \quad \in\ p_\propto \quad \forall\propto$$

$$\Rightarrow d([d(x),x])\ \in\wedge\, p_\propto=\{0\}$$

(ie) $d([d(x),x])=0$

Since $d \neq 0, [d(x), x] = 0 \quad \forall\ x\ \in R$

If $[d(x), x] \in p_{\propto}, then\ [d(x), x]\ \in \wedge\ p_{\propto} = \{0\}$

and so $[d(x), x] = 0 \quad \forall\ x\ \in R$ ……………………(5)

Either of there conditions implies $[d(x), x] = 0 \quad \forall\ x\ \in R$

$$[d(x + y), x + y] = 0 \quad \forall\ x, y\ \in R$$

(ie) $[d(x), x] + [d(x), y] + [d(x), x] + [d(x), y] = 0$

using (5) we get

$[d(x(x), y] + [d(y), x] = 0 \quad \forall\ x, y\ \in R$

(ie) $[d(x), y] = -[d(y), x] = [x, d(y)] \quad \forall\ x, y\ \in R$ ……………….(6)

Replacing $y\ by\ xy\ we\ get$

$$[d(x), xy] = [x, d(xy)]$$

$$[d(x), xy] = [x, d(x)y + xd(y)]$$

$$x[d(x), y] + [d(x), x]y = d(x)[x, y] + [x, d(x)]y + x[x, d(y)] + [x, x]d(y)$$

using (5) we get

$$x[d(x), y] = d(x)[x, y] + x[x, d(y)$$

using (6) we get

$$x[d(x), y] = d(x)[x, y] + x[d(x), y]$$

(ie) $d(x)[x, y] = 0 \quad \forall\ x, y\ \in R$ ………………..(7)

Replacing $y\ by\ yz\ we\ get$

$$d(x)[x\ \ yz] = 0 \quad \forall\ x, y, z\ \in R$$

$$d(x)y[x, z] + d(x)\ [x, y]z = 0$$

using (7) we get

$$d(x)y[x, z] = 0 \quad \forall\ x, y, z\ \in R$$

$d(x)R[x, z] = 0 \quad \forall\ x, z\ \in R$ …………………(8)

Since R is semi-prime, it must contain a family $p = \{p_{\propto}/\propto \in \wedge\}$ of non-zero prime ideals such that $\wedge\ p_{\propto} = \{0\}$

Then (8) shows that either $d(x) \in\ p_{\propto}\ (or)[x, z]\ \in\ p_{\propto}$

If $d(x) \in\ p_{\propto} \quad \forall \propto \in \wedge$, then $d(x) \in \cap\ p_{\propto} = \{0\}$

Since $d\ \neq 0, we\ get\ [x, y] = 0$

If $[x, z]\ \in\ p_{\propto}\ \ \forall\ \propto, we\ get$

$$[x, z] \in \cap\ p_{\propto} = \{0\} \quad \forall\ x, z\ \in R$$

(ie) $[x, z] = \{0\} \quad \forall\ z\ \in R$

Either of the conditions shows that

$$[x, z] = 0 \quad \forall\ x, z\ \in R$$

(ie) R is commutative.

Remark 3.14

Taking f=F, we get Theorem 3.7[8].

IV.REFERENCES

[1]. H.E.Bell On prime Near-rings with generalized derivation, Int.Jou.of Mathematics and Mathematical sciences vol(2008) (2008) 1-5.

[2]. H.E.Bell and N.U.Rehman, Generalised derivations with commutativity and anti-commutativity conditions, Maths.Jour of Okayama University, vol49, No1,(2007) 139-147.

[3]. M.Bresar, Centralising mappings and derivations in prime rings, J.Algebra, 156,(1993),385-394.

[4]. O.Golbasi, Notes on prime Near-rings with generalized derivation, south east Asian Bulleting of Mathematics, vol30, No1(2006), 49-54.

[5]. G.Gopalakrishnamoorthy, G.Shakila Chitra Selvi, and V.Thiripurasundari, On commutativity of prime near-rings with generalized derivations, Advances in Mathematics, scientific.Jour vol8, No3(2019) 371-376.

[6]. Mehsin Jabel Atteya and Ddalal Ibrahim Resan, commuting Derivations of semi-prime rings. Int J.Contemp.Maths.Sciences.Vol6,NO24,(2011) 1151-1158.

[7]. N.U.Rehman, on commutativity of rings with generalized derivations, Math.J.Okayama University, Vol.44(2002) 43-49.

[8]. G.Shakila Chitra Selvi, G.Gopalakrishnamoorthy and V.Thiripurasundari, On strong Generalised derivations in Prime-Near Rings.

[9]. G.Shakila Chitra Selvi, G.Gopalakrishnamoorthy, and V.Thiripurasundari, On Generalised derivations in semi-prime Rings.