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Modified and Non-Reducible Farey Matrix

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ABSTRACT

In this paper, we define Row Modified Farey Matrix, Column Modified Farey Matrix, Non-Reducible Farey Matrix from Farey matrix and discuss some of its properties.

Keywords: Farey Matrix, Row Modified Farey Matrix; Column Modified Farey Matrix, Non-Reducible Farey Matrix, Farey Incidence Matrix.

I. INTRODUCTION

A Farey sequence of order N is a set of irreducible fractions between 0 and 1 arranged in an increasing order, the denominators of which do not exceed N. F_N could be obtained from F_{N-1} by calculating the mediant between the two values from which it was derived. In [1] Farey graph and Farey matrix have been constructed for a Farey sequence of order N. From Farey Matrix we again construct new matrices namely, Row Modified Farey Matrix $[MFM_R]$ and Column Modified Farey Matrix $[MFM_C]$, Farey Incidence Matrix$[FIM]$, Non-Reducible Farey Matrix$[NRFM]$ and discuss some properties related to the above matrices.

II. FAREY GRAPH

2.1 Farey sequence

The sequence of all reduced fractions with denominators not exceeding N listed in order of their size is called the Farey sequence of order N.

For example:- The Farey Sequence of order 5 is

$$F_5 = \left[\frac{0}{1} < \frac{1}{5} < \frac{1}{4} < \frac{1}{3} < \frac{2}{5} < \frac{1}{2} < \frac{3}{5} < \frac{2}{3} < \frac{3}{4} < \frac{4}{5} < \frac{1}{1}\right]$$

2.2 Non Reducible Farey Sequence

The Sequence of non-reduced fractions with denominators not exceeding N listed in order of their size is called Non Reducible Farey Sequence of order N.

For Example :- The Non Reducible Farey Sequence of order 6 is

$$\widetilde{F_6} = \left\{\frac{0}{1} = \frac{0}{6}, \frac{1}{6}, \frac{1}{5}, \frac{1}{4}, \frac{2}{6}, \frac{2}{5}, \frac{3}{6}, \frac{3}{5}, \frac{4}{6}, \frac{3}{4}, \frac{4}{5}, \frac{5}{6}, \frac{6}{6} = \frac{1}{1}\right\}$$

2.3 Non Reducible Farey$N$-Subsequence

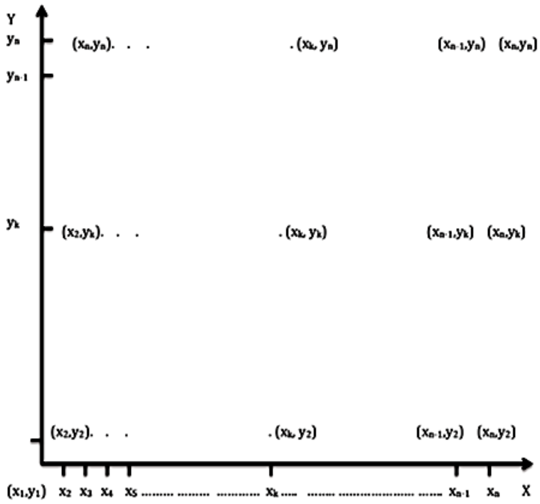
The sequence of non-reduced Farey fractions with denominators equal to the order of the size N is called Non Reducible Farey N-Subsequence.

For Example: - The Non Reducible Farey N -Subsequence of order 6 is

$$\tilde{F}_6 = \left\{\frac{1}{6}, \frac{2}{6}, \frac{3}{6}, \frac{4}{6}, \frac{5}{6}\right\}$$

2.4 Farey Graph:

$X = (x_1, x_2, x_3, \dots, x_n)$, $Y = (y_1, y_2, y_3, \dots, y_n)$ where x_i , $y_j \in F_N[0,1]$. The Farey graph is a graph of vertices (x_i, y_j) and it forms a grid whose graphical representation is as given below.



FAREY GRAPH

III. FAREY MATRICES

3.1 Farey Matrix:

A Farey matrix denoted by FM is defined as the square matrix of order n, whose elements are the sum of Farey fractions in the Farey graph. In other words,

$$[FM] = \begin{bmatrix} x_1+y_1 & x_1+y_2 & \cdots & x_1+y_{k-1} & x_1+y_k & x_1+y_{k+1} & \cdots & x_1+y_n \\ x_2+y_1 & x_2+y_2 & \cdots & x_2+y_{k-1} & x_2+y_k & x_2+y_{k+1} & \cdots & x_2+y_n \\ \vdots & \vdots & & & & \vdots & & \\ x_{k-1}+y_1 & x_{k-1}+y_2 & \cdots & x_{k-1}+y_{k-1} & x_{k-1}+y_k & x_{k-1}+y_{k+1} & \cdots & x_{k-1}+y_n \\ x_k+y_1 & x_k+y_2 & \cdots & x_k+y_{k-1} & x_k+y_k & x_k+y_{k+1} & \cdots & x_k+y_n \\ x_{k+1}+y_1 & x_{k+1}+y_2 & \cdots & x_{k+1}+y_{k-1} & x_{k+1}+y_k & x_{k+1}+y_{k+1} & \cdots & x_{k+1}+y_n \\ \vdots & \vdots & \cdots & & & \vdots & \cdots & \vdots \\ x_n+y_1 & x_n+y_2 & & x_n+y_{k-1} & x_n+y_k & x_n+y_{k+1} & \cdots & x_n+y_n \end{bmatrix} = [x_i+y_j]_{n\times n}$$

3.2 Row Modified Farey Matrix:$[MFM_R]$

In Farey Matrix, remove the middle column and find the ratio of terms in each row of Farey matrix which are equidistant from the ends. It is called the Row Modified Farey Matrix and is denoted by$[MFM_R]$.

$$[MFM_R] = \begin{bmatrix} \frac{x_1+y_1}{x_1+y_n} & \frac{x_1+y_2}{x_1+y_{n-1}} & \cdots & \frac{x_1+y_{k-1}}{x_1+y_{k+1}} \\ \frac{x_2+y_1}{x_2+y_n} & \frac{x_2+y_2}{x_2+y_{n-1}} & \cdots & \frac{x_2+y_{k-1}}{x_2+y_{k+1}} \\ \vdots & \vdots & \vdots & \vdots \\ \frac{x_{n-1}+y_1}{x_{n-1}+y_n} & \frac{x_{n-1}+y_2}{x_{n-1}+y_{n-1}} & \cdots & \frac{x_{n-1}+y_{k-1}}{x_{n-1}+y_{k+1}} \\ \frac{x_n+y_1}{x_n+y_n} & \frac{x_n+y_2}{x_n+y_{n-1}} & \cdots & \frac{x_n+y_{k-1}}{x_n+y_{k+1}} \end{bmatrix}$$

3.3 Column Modified Farey Matrix:$[MFM_C]$

In Farey Matrix, remove the middle row and find the ratio of terms in each column of Farey matrix which are equidistant from the ends. It is called the Column Modified FareyMatrix $[MFM_C]$.

$$[MFM_C] = \begin{bmatrix} \frac{x_1+y_1}{x_n+y_1} & \frac{x_1+y_2}{x_n+y_2} & \cdots & \frac{x_1+y_n}{x_n+y_n} \\ \frac{x_2+y_1}{x_{n-1}+y_1} & \frac{x_2+y_2}{x_n+y_2} & \cdots & \frac{x_2+y_n}{x_{n-1}+y_n} \\ \vdots & \vdots & \vdots & \vdots \\ \frac{x_{k-1}+y_1}{x_{k+1}+y_1} & \frac{x_{k-1}+y_2}{x_{k+1}+y_2} & \cdots & \frac{x_{k-1}+y_n}{x_{k+1}+y_n} \end{bmatrix}$$

3.4 Theorem

The product of the Modified Farey matrices $[MFM_R]$ and $[MFM_C]$ is symmetric.

Proof:
We know that the Farey matrix is always symmetric. Consider the matrices $[MFM_R]$ and $[MFM_C]$ derived from $[FM]$.

$$\Delta = [MFM_R] \times [MFM_C] = \begin{bmatrix} \frac{x_1+y_1}{x_1+y_n} & \frac{x_1+y_2}{x_1+y_{n-1}} & \cdots & \frac{x_1+y_{k-1}}{x_1+y_{k+1}} \\ \frac{x_2+y_1}{x_2+y_n} & \frac{x_2+y_2}{x_2+y_{n-1}} & \cdots & \frac{x_2+y_{k-1}}{x_2+y_{k+1}} \\ \vdots & \vdots & & \vdots \\ \frac{x_{n-1}+y_1}{x_{n-1}+y_n} & \frac{x_{n-1}+y_2}{x_{n-1}+y_{n-1}} & \vdots \cdots & \frac{x_{n-1}+y_{k-1}}{x_{n-1}+y_{k+1}} \\ \frac{x_n+y_1}{x_n+y_n} & \frac{x_n+y_2}{x_n+y_{n-1}} & \cdots & \frac{x_n+y_{k-1}}{x_n+y_{k+1}} \end{bmatrix} \times \begin{bmatrix} \frac{x_1+y_1}{x_n+y_1} & \frac{x_1+y_2}{x_n+y_2} & \cdots & \frac{x_1+y_n}{x_n+y_n} \\ \frac{x_2+y_1}{x_{n-1}+y_1} & \frac{x_2+y_2}{x_n+y_2} & \cdots & \frac{x_2+y_n}{x_{n-1}+y_n} \\ \vdots & \vdots & \vdots & \vdots \\ \frac{x_{k-1}+y_1}{x_{k+1}+y_1} & \frac{x_{k-1}+y_2}{x_{k+1}+y_2} & \cdots & \frac{x_{k-1}+y_n}{x_{k+1}+y_n} \end{bmatrix}$$

$$= \begin{bmatrix} \left(\frac{x_1+y_1}{x_1+y_n}\right)\left(\frac{x_1+y_1}{x_n+y_1}\right) + \left(\frac{x_1+y_2}{x_1+y_{n-1}}\right)\left(\frac{x_2+y_1}{x_{n-1}+y_1}\right) + \cdots + \left(\frac{x_1+y_{k-1}}{x_1+y_{k+1}}\right)\left(\frac{x_{k-1}+y_1}{x_{k+1}+y_1}\right) & \cdots & \left(\frac{x_1+y_1}{x_1+y_n}\right)\left(\frac{x_1+y_n}{x_n+y_n}\right) + \cdots + \left(\frac{x_1+y_{k-1}}{x_1+y_{k+1}}\right)\left(\frac{x_{k-1}+y_n}{x_{k+1}+y_n}\right) \\ \left(\frac{x_2+y_1}{x_2+y_n}\right)\left(\frac{x_1+y_1}{x_n+y_1}\right) + \left(\frac{x_2+y_2}{x_2+y_{n-1}}\right)\left(\frac{x_2+y_1}{x_{n-1}+y_1}\right) + \cdots + \left(\frac{x_2+y_{k-1}}{x_2+y_{k+1}}\right)\left(\frac{x_{k-1}+y_1}{x_{k+1}+y_1}\right) & \cdots & \left(\frac{x_2+y_1}{x_2+y_n}\right)\left(\frac{x_1+y_n}{x_n+y_n}\right) + \cdots + \left(\frac{x_2+y_{k-1}}{x_2+y_{k+1}}\right)\left(\frac{x_{k-1}+y_n}{x_{k+1}+y_n}\right) \\ \vdots & \cdots & \vdots \\ \left(\frac{x_n+y_1}{x_n+y_n}\right)\left(\frac{x_1+y_1}{x_n+y_1}\right) + \left(\frac{x_n+y_2}{x_n+y_{n-1}}\right)\left(\frac{x_2+y_1}{x_{n-1}+y_1}\right) + \cdots + \left(\frac{x_n+y_{k-1}}{x_n+y_{k+1}}\right)\left(\frac{x_{k-1}+y_1}{x_{k+1}+y_1}\right) & \cdots & \left(\frac{x_n+y_1}{x_n+y_n}\right)\left(\frac{x_1+y_n}{x_n+y_n}\right) + \cdots + \left(\frac{x_n+y_{k-1}}{x_n+y_{k+1}}\right)\left(\frac{x_{k-1}+y_n}{x_{k+1}+y_n}\right) \end{bmatrix}$$

In Farey graph, for each $i = j$, $x_i = y_j$

$\Delta =$

$$\begin{bmatrix} \frac{4x_1^2}{(x_1+x_n)^2} + \frac{(x_1+x_2)^2}{(x_1+x_{n-1})^2} + \cdots + \frac{(x_1+x_{k-1})^2}{(x_1+x_{k+1})^2} & \cdots & \frac{x_1}{x_n} + \left(\frac{x_1+x_2}{x_1+x_{n-1}}\right)\left(\frac{x_2+x_n}{x_{n-1}+x_n}\right) + \cdots + \left(\frac{x_1+x_{k-1}}{x_1+x_{k+1}}\right)\left(\frac{x_{k-1}+x_n}{x_{k+1}+x_n}\right) \\ \frac{(x_1+x_2)2x_1}{(x_2+x_n)(x_1+x_n)} + \frac{(x_1+x_2)2x_2}{(x_2+x_{n-1})(x_{n-1}+x_1)} + \cdots + \left(\frac{x_1+x_{k-1}}{x_1+x_{k+1}}\right)\left(\frac{x_{k-1}+x_n}{x_{k+1}+x_n}\right) & \cdots & \left(\frac{x_1+x_2}{x_2+x_n}\right)\left(\frac{x_1+x_n}{2x_n}\right) + \left(\frac{2x_2}{x_2+x_{n-1}}\right)\left(\frac{x_2+x_n}{x_{n-1}+x_n}\right) + \cdots + \left(\frac{x_2+x_{k-1}}{x_2+x_{k+1}}\right)\left(\frac{x_{k-1}+x_n}{x_{k+1}+x_n}\right) \\ \vdots & \cdots & \vdots \\ \frac{x_1}{x_n} + \left(\frac{x_1+x_2}{x_1+x_{n-1}}\right)\left(\frac{x_2+x_n}{x_{n-1}+x_n}\right) + \cdots + \left(\frac{x_1+x_{k-1}}{x_1+x_{k+1}}\right)\left(\frac{x_{k-1}+x_n}{x_{k+1}+x_n}\right) & \cdots & \left(\frac{x_1+x_n}{2x_n}\right)^2 + \left(\frac{x_2+x_n}{x_{n-1}+x_n}\right)^2 + \cdots + \left(\frac{x_{k-1}+x_n}{x_{k+1}+x_n}\right)^2 \end{bmatrix}$$

Here $\Delta^T = \Delta$

Therefore, the product of modified Farey matrices $[MFM_R]$ and $[MFM_C]$ is symmetric.

3.4.1 Example:

Consider Farey matrix of order 5

$$[FM] = \begin{bmatrix} 0 & \frac{1}{3} & \frac{1}{2} & \frac{2}{3} & 1 \\ \frac{1}{3} & \frac{2}{3} & \frac{5}{6} & 1 & \frac{4}{3} \\ \frac{1}{2} & \frac{5}{6} & 1 & \frac{7}{6} & \frac{3}{2} \\ \frac{2}{3} & 1 & \frac{7}{6} & \frac{4}{3} & \frac{5}{3} \\ 1 & \frac{4}{3} & \frac{3}{2} & \frac{5}{3} & 2 \end{bmatrix}$$

$$[MFM_R]_{5\times 2} = \begin{bmatrix} 0 & \frac{1}{2} \\ \frac{1}{4} & \frac{2}{3} \\ \frac{1}{3} & \frac{5}{7} \\ \frac{2}{5} & \frac{3}{4} \\ \frac{1}{2} & \frac{4}{5} \end{bmatrix} \text{ and } [MFM_c]_{2\times 5} = \begin{bmatrix} 0 & \frac{1}{4} & \frac{1}{3} & \frac{2}{5} & \frac{1}{2} \\ \frac{1}{2} & \frac{2}{3} & \frac{5}{7} & \frac{3}{4} & \frac{4}{5} \end{bmatrix}$$

The product of $[MFM_R]_{5\times 2}$ and $[MFM_c]_{2\times 5}$ is

$$\Delta = [MFM_R]_{5\times 2} \times [MFM_c]_{2\times 5}$$

$$= \begin{bmatrix} 0 & \frac{1}{2} \\ \frac{1}{4} & \frac{2}{3} \\ \frac{1}{3} & \frac{5}{7} \\ \frac{2}{5} & \frac{3}{4} \\ \frac{1}{2} & \frac{4}{5} \end{bmatrix} \begin{bmatrix} 0 & \frac{1}{4} & \frac{1}{3} & \frac{2}{5} & \frac{1}{2} \\ \frac{1}{2} & \frac{2}{3} & \frac{5}{7} & \frac{3}{4} & \frac{4}{5} \end{bmatrix}$$

$$\Delta = \begin{bmatrix} \frac{1}{4} & \frac{1}{3} & \frac{5}{14} & \frac{3}{8} & \frac{2}{5} \\ \frac{1}{3} & \frac{73}{144} & \frac{141}{252} & \frac{3}{5} & \frac{79}{120} \\ \frac{5}{14} & \frac{141}{252} & \frac{274}{441} & \frac{281}{420} & \frac{31}{42} \\ \frac{3}{8} & \frac{3}{5} & \frac{281}{420} & \frac{289}{400} & \frac{4}{5} \\ \frac{2}{5} & \frac{79}{120} & \frac{31}{42} & \frac{4}{5} & \frac{89}{100} \end{bmatrix} = \Delta^T$$

Hence, the product of Modified Farey matrices $[MFM_R]$ and $[MFM_C]$ is symmetry.

3.5 Farey Incidence Matrix:

The Farey Incidence Matrix $[FIM]$ of Farey graph is

$$[FIM] = \begin{cases} 1, & if\ x_i = y_j \\ 0, & if\ x_i \neq y_j \end{cases}$$

It forms an Identity matrix. The order of $[FIM] = |F_N| = 1 + \sum_{k=1}^{N} \phi(k)$.

3.5.1 Illustration:

The Farey Incidence Matrix of order 5 is formed from Farey Sequence of order 3.

The order of

$$[FIM] = |F_3| = 1 + \sum_{k=1}^{3} \phi(k) = 1 + \phi(1) + \phi(2) + \phi(3)$$
$$= 1 + 1 + 1 + 2 = 5$$

$$[FIM] = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

3.6 Non-Reducible Farey Matrix:

In Farey graph, consider Non-Reducible Farey N -Subsequence fractions including boundary fractions $\left\{\frac{0}{N}, \frac{N}{N}\right\}$ .From this grid we form Non-Reducible Farey Matrix. The order of Non-Reducible Farey Matrix$m = N + 1$.

$$[NRFM] = \frac{1}{N^{N+1}} \begin{bmatrix} u_1 + v_1 & u_1 + v_2 & \cdots & u_1 + v_m \\ u_2 + v_1 & u_2 + v_2 & \cdots & u_2 + v_m \\ \vdots & \vdots & \cdots & \vdots \\ u_m + v_1 & u_m + v_2 & \cdots & u_m + v_m \end{bmatrix}$$

3.7 Theorem:

The square of Non-Reducible Farey Matrix of order m is

$$\frac{1}{(N^{(N+1)})^2} \begin{bmatrix} \sum_{m=0}^{N} m^2 & \sum_{m=0}^{N} m(m+1) & \cdots & \sum_{m=0}^{N} m(m+N) \\ \sum_{m=0}^{N} m(m+1) & \sum_{m=0}^{N} (m+1)^2 & \cdots & \sum_{m=0}^{N} (m+1)(m+N) \\ \vdots & \vdots & & \vdots \\ \sum_{m=0}^{N} m(m+N) & \sum_{m=0}^{N} (m+1)(m+N) & \cdots & \sum_{m=0}^{N} (m+N)^2 \end{bmatrix}$$

Proof:

Consider the Non-Reducible Farey Matrix

$$[NRFM] = \frac{1}{N^{N+1}} \begin{bmatrix} u_1 + v_1 & u_1 + v_2 & \cdots & u_1 + v_m \\ u_2 + v_1 & u_2 + v_2 & \cdots & u_2 + v_m \\ \vdots & \vdots & \cdots & \vdots \\ u_m + v_1 & u_m + v_2 & \cdots & u_m + v_m \end{bmatrix}$$

Let $a_{ij} = u_i + v_j$,

$$[NRFM] = \frac{1}{N^{(N+1)}} \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1m} \\ a_{21} & a_{22} & \cdots & a_{2m} \\ \vdots & \vdots & \cdots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mm} \end{bmatrix}$$

The square of Non-Reducible Farey Matrix

$$[NRFM]^2 = \frac{1}{(N^{(N+1)})^2} \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1m} \\ a_{21} & a_{22} & \cdots & a_{2m} \\ \vdots & \vdots & \cdots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mm} \end{bmatrix} \times \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1m} \\ a_{21} & a_{22} & \cdots & a_{2m} \\ \vdots & \vdots & \cdots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mm} \end{bmatrix}$$

$$= \frac{1}{(N^{(N+1)})^2} \begin{bmatrix} a_{11}^2 + \cdots + a_{1m}a_{m1} & a_{11}a_{12} + \cdots + a_{1m}a_{m2} & \cdots & a_{11}a_{1m} + \cdots + a_{1m}a_{mm} \\ a_{21}a_{11} + \cdots + a_{2m}a_{m1} & a_{21}a_{12} + \cdots + a_{2m}a_{m2} & \cdots & a_{21}a_{1m} + \cdots + a_{2m}a_{mm} \\ \vdots & \vdots & \cdots & \vdots \\ a_{m1}a_{11} + \cdots + a_{mm}a_{m1} & a_{m1}a_{12} + \cdots + a_{mm}a_{m2} & \cdots & a_{m1}a_{1m} + \cdots + a_{mm}^2 \end{bmatrix}$$

$$= \frac{1}{(N^{(N+1)})^2}\begin{bmatrix} a_{11}^2 + \cdots + {a_{1m}}^2 & a_{11}a_{12} + \cdots + a_{1m}a_{m2} & \dots & a_{11}a_{1m} + \cdots + a_{1m}a_{mm} \\ a_{21}a_{11} + \cdots + a_{2m}a_{m1} & {a_{21}}^2 + \cdots + {a_{2m}}^2 & \dots & a_{21}a_{1m} + \cdots + a_{2m}a_{mm} \\ \vdots & \vdots & \dots & \vdots \\ a_{m1}a_{11} + \cdots + a_{mm}a_{m1} & a_{m1}a_{12} + \cdots + a_{mm}a_{m2} & \dots & {a_{m1}}^2 + \cdots + a_{mm}^2 \end{bmatrix}$$

$\because a_{ij} = a_{ji}$

$$= \frac{1}{(N^{(N+1)})^2}\begin{bmatrix} \sum_{m=0}^{N} m^2 & \sum_{m=0}^{N} m(m+1) & \dots & \sum_{m=0}^{N} m(m+N) \\ \sum_{m=0}^{N} m(m+1) & \sum_{m=0}^{N} (m+1)^2 & \dots & \sum_{m=0}^{N} (m+1)(m+N) \\ \vdots & \vdots & & \vdots \\ \sum_{m=0}^{N} m(m+N) & \sum_{m=0}^{N} (m+1)(m+N) & \dots & \sum_{m=0}^{N} (m+N)^2 \end{bmatrix}$$

3.7.1 Illustration:

The square of Non-Reducible Farey Matrix of order 6 is

$$\frac{1}{(5^6)^2}\begin{bmatrix} \sum_{m=0}^{5} m^2 & \sum_{m=0}^{5} m(m+1) & \dots & \sum_{m=0}^{5} m(m+5) \\ \sum_{m=0}^{5} m(m+1) & \sum_{m=0}^{5} (m+1)^2 & \dots & \sum_{m=0}^{5} (m+1)(m+5) \\ \vdots & \vdots & & \vdots \\ \sum_{m=0}^{5} m(m+5) & \sum_{m=0}^{N} (m+1)(m+5) & \dots & \sum_{m=0}^{N} (m+5)^2 \end{bmatrix}$$

Solution:

Consider the Non-Reducible Farey Matrix of order 6,

$$[NRFM] = \frac{1}{(5^6)}\begin{bmatrix} 0 & 1 & 2 & 3 & 4 & 5 \\ 1 & 2 & 3 & 4 & 5 & 6 \\ 2 & 3 & 4 & 5 & 6 & 7 \\ 3 & 4 & 5 & 6 & 7 & 8 \\ 4 & 5 & 6 & 7 & 8 & 9 \\ 5 & 6 & 7 & 8 & 9 & 10 \end{bmatrix}$$

Then the square of $[NRFM]$ is

$$[NRFM]^2 = \frac{1}{(5^6)}\begin{bmatrix} 0 & 1 & 2 & 3 & 4 & 5 \\ 1 & 2 & 3 & 4 & 5 & 6 \\ 2 & 3 & 4 & 5 & 6 & 7 \\ 3 & 4 & 5 & 6 & 7 & 8 \\ 4 & 5 & 6 & 7 & 8 & 9 \\ 5 & 6 & 7 & 8 & 9 & 10 \end{bmatrix} \times \frac{1}{(5^6)}\begin{bmatrix} 0 & 1 & 2 & 3 & 4 & 5 \\ 1 & 2 & 3 & 4 & 5 & 6 \\ 2 & 3 & 4 & 5 & 6 & 7 \\ 3 & 4 & 5 & 6 & 7 & 8 \\ 4 & 5 & 6 & 7 & 8 & 9 \\ 5 & 6 & 7 & 8 & 9 & 10 \end{bmatrix}$$

$$= \frac{1}{(5^6)^2}\begin{bmatrix} \sum_{m=0}^{5} m^2 & \sum_{m=0}^{5} m(m+1) & \dots & \sum_{m=0}^{5} m(m+5) \\ \sum_{m=0}^{5} m(m+1) & \sum_{m=0}^{5} (m+1)^2 & \dots & \sum_{m=0}^{5} (m+1)(m+5) \\ \vdots & \vdots & & \vdots \\ \sum_{m=0}^{5} m(m+5) & \sum_{m=0}^{N} (m+1)(m+5) & \dots & \sum_{m=0}^{N} (m+5)^2 \end{bmatrix}$$

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