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The Rainbow-Vertex Connection Number [RVCN] of Subdivision of Certain Graphs

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ABSTRACT

Rainbow-Vertex Connection Number [rvcn] is computed for some graphs by the researchers. Here we have considered the subdivision graphs of certain graph classes. The rainbow edge connection number of subdivision of Triangular snake graph was already found[1]. Using the definition of rainbow-vertex connection number [5], which is the smallest positive integer k such that the graph is rainbow-vertex connected, we find the rainbow vertex number of subdivision graph of Friendship graph $rvc(S(F_n)) = n + 2\ \forall\, n \geq 2$,, Triangular snake graph $vc(S(T_n)) = 2n - 1\ \forall n \geq 3$ and Comb graph $rvc(S(P_n \odot K_1)) = 3n - 1\ \forall n \geq 2$.

Keywords: Rainbow Vertex Connected Graph and Number, Friendship Graph, Triangular Graph, Comb Graph, Subdivision Graph.

I. INTRODUCTION

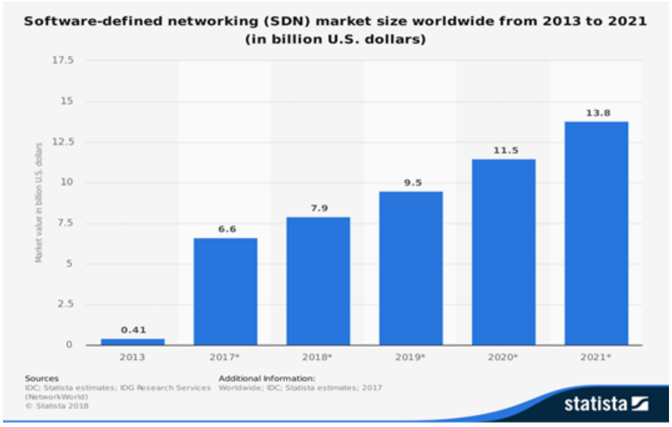
We consider simple, finite, connected, and undirected subdivision graphs. A graph is a set of vertices and edges (v_i, e_j) , $v_i's$ are non-empty. Krivelevich and Yuster [5] introduced the rainbow-vertex connection number, and Li and Shi investigated it. The lower bound was stated by Krivelevich and Yuster as $rvc(G) \geq diam(G) - 1$.

Definition 1.1: Graph Colouring: Proper colouring is the process of colouring each vertex of a graph so that no two neighbouring vertices have the same colour.

Definition 1.2: Rainbow colouring: A path in an edge-coloured graph is claimed to be rainbow coloured if no colour repeats thereon path.

Definition 1.3: Rainbow vertex connected graph: If all of a graph's internal vertices have unique colours, the graph is said to have a rainbow vertex path. If there is a path connecting every pair of vertices in the network or graph, it is said to be rainbow vertex connected.

Definition 1.4: Friendship graph: It is a planar graph with $2n + 1$ vertices and $3n$ edges. This graph is constructed by joining n-copies of the cycle graph C_3 with a common vertex, which is called as universal vertex for the graph. It is also called as Dutch windmill or n -fan graph.



Definition 1.5: Triangular snake graph: This graph is obtained by starting with a path graph P_{n-1} and adding edges $(2i-1, 2i+1))$ for $i = 1,2,3, \ldots, n-1$.
Definition 1.6: Comb graph: A path with n vertices is called path graph. Then, $P_n \odot K_1$ with $2n$ vertices and $2n-1$ edges is said to be a comb graph.
Definition 1.7: Subdivision graph: Subdivision graph is obtained by deleting an edge [u, v] from the graph G and addition of two edges [u, w] and [w, v] along with the new vertex V∪{w}.

II. RESULTS AND DISCUSSION

Rainbow-vertex connection number [rvcn] of subdivision of friendship graph

Theorem 1

If F_n is the Friendship graph of order $2n+1$ and $S(F_n)$ is the subdivision graph of F_n, then $rvcn(S(F_n)) = n+2 \quad \forall\, n \geq 2$.

Proof: Let v_1 be the universal vertex (common vertex) for the graph. Now join the $n-$ copies of the cycle graph C_3 to the universal vertex. Let $\{v_1, w_1, w_1', \ldots, w_n, w_n'\}$ be the vertices of n cycles. Now insert new vertices $\{u_1, y_1, u_1', \ldots u_n'\}$ to the $n-$cycles to form the subdivision graph of F_n.

Vertex colouring algorithm:

1. The vertex v_1 is assigned the colour c_i , $1 \leq i \leq n$.
2. The vertices $\{w_1, w_1', \ldots, w_n^1\}$ is assigned the colour c_i,
   $2 \leq i \leq n$.
3. The vertices $\{u_1, y_1, u_1'\}$, $\{u_2, y_2, u_2'\}$, …, $\{u_n, y_n, u_n'\}$
   is assigned colour $c_3, c_4, \ldots, c_{n+2}$.

Now let us consider any path of $S(F_n)$

Case-i: For (v_1 , w_t), when $1 \leq t \leq n$ then the rainbow- vertex path is $v_1 u_t w_t$.

Case-ii: For (v_1, w_t') when $1 \leq t \leq n$ then the rainbow-vertex path is $v_1 u_t' w_t'$.

Case-iii: If (v_1, y_t), then the shortest path will be $v_1 u_t w_t y_t$ which is a rainbow- vertex path for $1 \leq t \leq n$.

Case-iv: If u_s and w_t are the end vertices of the path, then $u_s v_1 u_t w_t$ will be the shortest path and is the rainbow-vertex path for $s \neq t, 1 \leq s,t \leq n$.

Case-v: If u_s and w_t' are the end vertices of the path, then $u_s v_1 u_t' w_t'$ will be the shortest path and is the rainbow-vertex path for $1 \leq s,t \leq n$.

Case-vi: For (u_s, y_t), when $1 \leq s,t \leq n$, the shortest path is $\begin{cases} u_s w_s y_s, & s = t \\ u_s v_1 u_t w_t y_t, & s \neq t \end{cases}$. This is the rainbow-vertex path.

Case-vii: For the end vertices u_s, u_t,when$1 \leq s,t \leq n$ the shortest path is $u_s v_1 u_t$, which will be a rainbow-vertex path.

Case-viii: For the end vertices u_s,u_t',when$1 \leq s,t \leq n$, the rainbow-vertex path is $u_s v_1 u_t'$.

Case-ix: If the end vertices are u_s', u_t',when $s \neq t$ and $1 \leq s,t \leq n$ the rainbow-vertex path is $u_s' v_1 u_t'$.

Case-x: If u_s' and w_t are the end vertices of a path then the rainbow-vertex path is given by $u_s v_1 u_t w_t$ for $s \neq t, 1 \leq s,t \leq n$.

Case-xi: If $u_s'$and w_t' are the end vertices then the rainbow-vertex path is given by $u_s' v_1 u_t' w_t'$, $s \neq t, 1 \leq s,t \leq n$.

Case-xii: The rainbow-vertex path for the end vertices $(u_s'$,$y_t)$ will be $\begin{cases} u_s' w_s' y_s, & s = t \\ u_s' v_1 u_t w_t y_t, & s \neq t \end{cases}$ for $1 \leq s,t \leq n$.

Case-xiii: The rainbow-vertex path for the end vertices (w_s, w_t) is $w_s u_s v_1 u_t w_t$, for $1 \leq s,t \leq n, s \neq t$.

Case-xiv: The shortest path between the vertices w_s and w_t' is $\begin{cases} w_s y_s w_s' , & s = t \\ w_s u_s v_1 u_t' w_t', & s \neq t \end{cases}$ for $1 \leq s,t \leq n$ which is a rainbow-vertex path.

Case-xv: The shortest path between the vertices w_s and y_t is $w_s u_s v_1 u_t w_t y_t$,for $1 \leq s,t \leq n, s \neq t$ which is a rainbow-vertex path.

Case-xvi: For the end vertices (w_s', y_t), $1 \leq s,t \leq n, s \neq t$ the shortest path will be $w_s' u_s' v_1 u_t w_t y_t$ which is a rainbow vertex path.

Between each and every vertex, there is a rainbow-vertex path.

Therefore, $rvcn(S(F_n)) = n + 2,\ \forall\, n \geq 2$.

Example 1.1

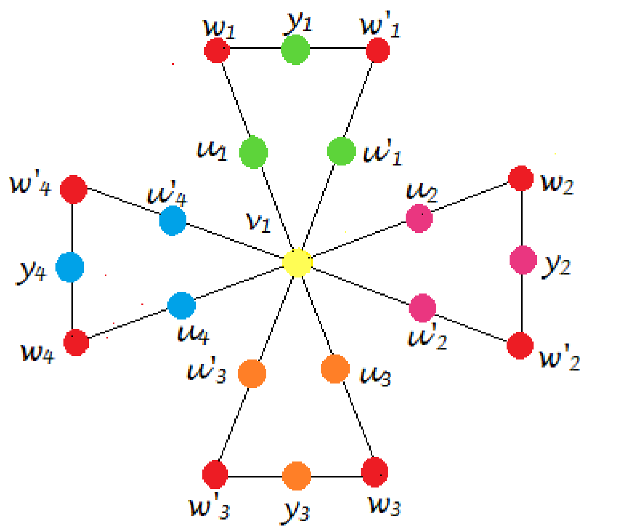


Figure 1: Rainbow -vertex colouring of subdivision graph of Friendship graph F_4

The rainbow-vertex connection number $rvcn(S(F_4)) = 6$.

Rainbow-vertex connection number [rvcn] of subdivision of triangular snake graph [1]

Theorem 2

If T_n is the triangular snake graph [6] of order n and $S(T_n)$ is subdivision of the triangular snake graph, then $rvc(S(T_n)) = 2n - 1$, $\forall\, n \geq 3$.

Proof: Consider a Triangular snake graph T_n starting with the path graph $v_1 - v_n$. Construct a triangular snake graph by adding new vertices $\{u_1, u_2, \dots, u_{n-1}\}$. Now insert new vertices $\{w_1, w'_1, y_1, \dots, w'_{n-1}\}$ and construct the subdivision graph of triangular snake graph $S(T_n)$.

Vertex colouring algorithm

1. Assign $c_1, c_2, \dots, c_{2n-1}$ colours to the path graph $v_1 - v_n$.
2. Assign colour c_1 to w_i and u_i, colour c_2 to w'_i, $1 \leq i \leq n-1$.

We now consider the any path of $S(T_n)$

Case-i: For the end vertices (v_s, v_t), when $1 \leq s, t \leq n, s \neq t$ the shortest path is $v_s y_s v_{s+1} \dots y_{t-1} v_t$ which is a rainbow-vertex path.

Case-ii: For the end vertices v_s and y_t, when $1 \leq s, t \leq n-1$ the rainbow-vertex path is $v_s y_s v_{s+1} \dots v_t y_t$

Case-iii: If v_s and w_t are the end vertices, $1 \leq s, t \leq n-1$ and $s \neq t$ then the shortest path between them is $\begin{cases} v_s y_s v_{s+1} \dots v_t w_t, & s < t \\ v_s y_{s-1} v_{s-1} \dots v_t w_t, & s > t \end{cases}$ which is the rainbow-vertex path.

Case-iv: If v_s and u_t are the end vertices with $1 \leq s \leq n, 1 \leq t \leq n-1$, then the shortest path is $\begin{cases} v_s w_s u_s, & s = t \\ v_s y_s v_{s+1} \dots w_t u_t, & s < t \\ v_s y_{s-1} v_{s-1} \dots w'_t u_t, & s > t \end{cases}$ is the rainbow-vertex path.

Case-v: For the end vertices (v_s, w'_t) , $1 \leq s, t \leq n-1$, the shortest path is $\begin{cases} v_s y_s v_{s+1} \dots v_{t+1} w'_t, & s \leq t \\ v_s y_{s-1} v_{s-1} \dots v_{t+1} w'_t, & s > t \end{cases}$ which is a rainbow-vertex path between these vertices.

Case-vi: If u_s and w_t are the end vertices, $1 \leq s, t \leq n-1$ and s$\neq t$then the rainbow-vertex path is $\begin{cases} u_s w'_s v_{s+1} y_{s+1} \dots v_t w_t, & s < t \\ u_s w_s v_s y_{s-1} \dots v_t w_t, & s > t \end{cases}$

Case-vii: For the end vertices u_s and u_t, $1 \leq s, t \leq n-1$ and s$\neq t$ the rainbow-vertex path is $u_s w'_s v_{s+1} y_{s+1} \dots v_t w_t u_t$.

Case-viii: For the end vertices u_s and w'_t, $1 \leq s, t \leq n-1$ and s$\neq t$, then the shortest path is $\begin{cases} u_s w'_s v_{s+1} y_{s+1} \dots v_{t+1} w'_t, & s < t \\ u_s w_s v_s y_{s-1} \dots v_{t+1} w'_t, & s > t \end{cases}$ and is the rainbow-vertex path.

Case-ix: If (u_s, y_t) are the end vertices, $1 \leq s, t \leq n$,then the shortest path which is the rainbow-vertex path is $\begin{cases} u_s w'_s v_{s+1} y_s, s = t \\ u_s w'_s v_{s+1} y_{s+1} \dots v_t y_t, s < t \\ u_s w_s v_s y_{s-1} \dots v_{t+1} y_t, s > t \end{cases}$ Case-x: If (w_s, w'_t) are the end vertices of a path, $1 \le s, t \le n-1$, then the rainbow -vertex path is

$$\begin{cases} w_s u_s w'_s, s = t \\ w_s v_s y_s v_{s+1} \dots v_{t+1} w'_t, s < t \\ w_s v_s y_{s-1} v_{s+1} \dots v_{t+1} w'_t, s > t \end{cases}$$

Case-xi: For the end vertices w_s and y_t, $1 \le s, t \le n-1$, the rainbow-vertex path is

$$\begin{cases} w_s v_s y_s, s = t \\ w_s v_s y_s v_{s+1} \dots v_t y_t, s < t \\ w_s v_s y_{s-1} v_{s-1} \dots v_{t+1} y_t, s > t \end{cases}$$

Case-xii: For the end vertices w_s and w_t, $1 \le s, t \le n-1$ and $s \ne t$, then the rainbow- vertex path is $w_s v_s y_s v_{s+1} \dots v_t w_t$.

Case-xiii: If the end vertices are w'_s and w'_t, for $1 \le s, t \le n-1$ and $s \ne t$, the rainbow-vertex path is given by $w'_s v_{s+1} y_{s+1} \dots . v_{t+1} w'_t$.

Case-xiv: The shortest path between the end points w'_s and y_t, $1 \le s, t \le n-1$, is $\begin{cases} w'_s v_{s+1} y_s, & s = t \\ w'_s v_{s+1} y_s, & s < t \\ w'_s v_{s+1} y_s v_s y_{s-1} \dots v_{t+1} y_t, & s > t \end{cases}$ and is the rainbow-vertex path.

Case-xv: The shortest path between the end points y_s and y_t, $1 \le s, t \le n-1$ and $s \ne t$, is $y_s v_s y_{s+1} \dots v_t y_t$ which is a rainbow-vertex path.

Between each and every vertex, there is a rainbow - vertex path.

Therefore, $rvcn(S(T_n)) = 2n - 1, \forall\, n \ge 3$.

Example 2.1

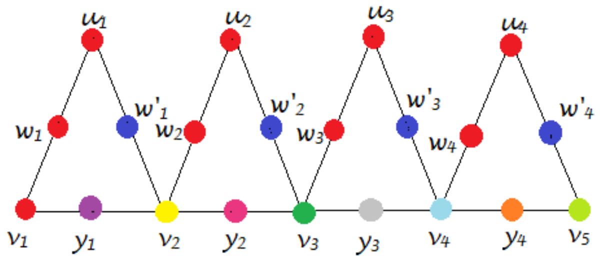


Figure 2: Rainbow-vertex colouring of subdivision graph of triangular snake graph T_5

The $rvcn(S(T_5)) = 9$.

Rainbow-vertex connection number [rvcn] of subdivision of comb graph[3]

Theorem 3

If $P_n \odot K_1$ is the comb graph of order $2n$ and $S(P_n \odot K_1)$ is the subdivision graph of comb graph, then $rvcn(S(P_n \odot K_1)) = 3n - 1, \forall\, n \ge 2$.

Proof: Let us consider the comb graph $P_n \odot K_1$ with vertices $\{u_1, u_2, \dots, u_n, v_1, v_2, \dots, v_n\}$. Now insert the new vertices $\{w_1, w_2, \dots, w_n, y_1, y_2, \dots, y_{n-1}\}$ and construct the subdivision graph of $P_n \odot K_1$ i.e., $S(P_n \odot K_1)$.

Vertex colouring algorithm

1. Assign colour c_i, $1 \le i, j \le n$ for the vertices $\{v_1, v_2, \dots, v_n\}$.
2. Assign colour $c_1, c_2, c_3, \dots, c_{3n-1}$ for the remaining vertices.

We now consider any path of $S(P_n \odot K_1)$

Case-i: The shortest path between the vertices (v_s, v_t), for $1 \le s, t \le n$, and $s \ne t$, $s < t$ is $v_s w_s u_s y_s u_{s+1} \dots u_t w_t v_t$ which is rainbow-vertex path.

Case-ii: If v_s and w_t are the end vertices with $1 \le s, t \le n$ and $s \ne t$, then the rainbow-vertex path is $\begin{cases} v_s w_s u_s y_s u_{s+1} y_{s+1} \dots u_t w_t, & s < t \\ v_s w_s u_s y_{s-1} u_{s-1} \dots y_t u_t w_t, & s > t \end{cases}$

Case-iii: The shortest distance between the end points v_s and u_t for $1 \le s, t \le n$, is $\begin{cases} v_s w_s u_s, s = t \\ v_s w_s u_s y_s u_{s+1} \dots y_{t-1} u_t, s < t \\ v_s w_s u_s y_{s-1} u_{s-1} \dots y_t u_t, s > t \end{cases}$ which is a rainbow-vertex path.

Case-iv: The shortest distance between the vertices v_s and y_t for $1 \le s \le n, 1 \le t \le n-1$ is $\begin{cases} v_s w_s u_s y_s, s = t \\ v_s w_s u_s y_s u_{s+1} y_{s+1} \dots u_t y_t, s < t \\ v_s w_s u_s y_{s-1} u_{s-1} \dots u_{t+1} y_t, s > t \end{cases}$ and is a rainbow-vertex path.

Case-v: For the end points $u_s$and w_t, $1 \leq s,t \leq n$ and $s \neq t$, the rainbow-vertex path is

$$\begin{cases} u_s y_s u_{s+1} y_{s+1} \dots u_t w_t, s < t \\ u_s y_{s-1} u_{s-1} \dots u_t w_t, \ s > t \end{cases}$$

Case-vi: For the end points u_s and u_t , $1 \leq s,t \leq n, s \neq t$and $s < t$, then the rainbow-vertex path is given by $u_s y_s u_{s+1} y_{s+1} \dots y_{t-1} u_t$.

Case-vii: If $u_s$and y_t are the end nodes of the path with $1 \leq s \leq n, 1 \leq t \leq n-1$ and $s \neq t$ then the rainbow-vertex path is

$$\begin{cases} u_s y_s u_{s+1} y_{s+1} \dots u_t y_t, \ s < t \\ u_s y_{s-1} u_{s-1} y_{s-2} \dots u_{t+1} u_t, \ s > t \end{cases}$$

Case-viii: The shortest path between the end nodes w_s and w_t, $1 \leq s,t \leq n, s \neq t$ and $s < t$, is $w_s u_s y_s u_{s+1} y_{s+1} \dots u_t w_t$. This is the rainbow-vertex path between these vertices.

Case-ix: The shortest path between the nodes $(w_s, y_t), 1 \leq s \leq n, 1 \leq t \leq n-1$ is

$$\begin{cases} w_s u_s y_s, & s = t \\ w_s u_s y_s u_{s+1} \dots u_t y_t, & s < t \\ w_s u_s y_{s-1} u_{s-1} \dots u_{t+1} y_t, s > t \end{cases}$$

. This is the rainbow-vertex path.

Case-x: The shortest path between the end points (y_s, y_t) , $1 \leq s,t \leq n-1, s \neq t$ and $s < t$ is $y_s u_{s+1} y_{s+1} \dots . u_t y_t$. This is the rainbow-vertex path.

Between each and every vertex, there is a rainbow-vertex path.

Therefore, $rvcn(S(P_n \odot K_1)) = 3n - 1, \forall\, n \geq 2$.

Example 3.1

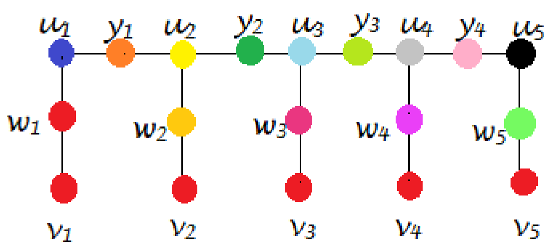


Figure 3: Rainbow-vertex colouring of subdivision graph of Comb

graph $P_5 \odot K_1$

The $rvc(S(P_5 \odot K_1)) = 14$

III.CONCLUSION

In this study, we discovered the rainbow vertex colouring number(rvcn) for the subdivision graph of comb, triangular snake, and friendship graphs.

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