© 2019 IJSRSET | Volume 6 | Issue 4 | Print ISSN: 2395-1990 | Online ISSN : 2394-4099
Themed Section : Science

Flow of Second Order Fluid in Presence of Magnetic Field

Dr. Vishal Saxena
Assistant prof. (Physics) Upadhi P. G. College, Pilibhit, U.P., India

ABSTRACT
In the presence paper, when a uniform transverse magnetic field id applied parallel to axis of a cylinder, the flow of a second order fluid through a porous medium has been discussed. The effect of different types of pressure is considered on the flow of fluid.
Keywords : Second order fluid , porous medium, pressure Guwahati Journal.

I. INTRODUCTION

Hele-Shaw flow is named after Prof. Hele-Shaw who experimentally established that The flow of fluid in a narrow gap between two parallel planes can be represented by a velocity potential. In such flows, the inertia terms of the type u $\partial u/\partial x$ etc. are neglected. The important of this flow in mid twentieth century got highlight in Bio-Mechanics when Sobin and Fung suggest that the capillary blood vessels in the alveoli of the lung are best described not as tubes, but as forming a 'Sheet', so that the alveolar blood flow is a flow between two parallel membrances interposed with posts. The Reynolds number for such a flow in the pulmonary alveoli is of the order of 10^{-4} to 10^{-2}. Hence the convective inertia force may justifiably be neglected. Many research workers have paid their attention towards this flow. The steady fluid flows have been discussed by Lamb, Reighels, Thompson, Lee and Fung and Buckmaster. In most of these investigations, the pressure gradient was assumed to be constant and the flow in the cell to be steady. The unsteady flow of non-Newtonian fluid has been discussed by Gupta et.al when the cylinder is circular; the pressure gradient being a function of time. In this paper, we have discussed the problem of unsteady flow of second-order fluid under the influence if a transverse magnetic field.

We consider the flow of a second order fluid through a porous medium between two parallel horizontal planes z = +a and z = -a,

The equations of motion and continuity of unsteady flow of second order fluid In presence of uniform transvers magnetic field B_0 are

$$\frac{\partial u}{\partial t} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \left(\nu_1 + \nu_2 \frac{\partial}{\partial t}\right)\frac{\partial^2 u}{\partial z^2} - \frac{\nu_1}{k}u - \frac{\sigma_e}{\rho}B_0^{\,2}u \quad (1)$$

$$\frac{\partial v}{\partial t} = -\frac{1}{\rho}\frac{\partial p}{\partial y} + \left(\nu_1 + \nu_2 \frac{\partial}{\partial t}\right)\frac{\partial^2 v}{\partial z^2} - \frac{\nu_1}{k}u - \frac{\sigma_e}{\rho}B_0^{\,2}v \quad (2)$$

$$0 = -\frac{1}{\rho}\frac{\partial p}{\partial z} \quad (3)$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (4)$$

Here ρ is the density of fluid and $\nu_1 = \mu_1/\rho$ and $\nu_2 = \mu_2/\rho$ are coefficient of viscosity and visco-elasticity of the fluid respectively

Using non-dimensional parameters

$$ax^* = x, \quad ay^* = y, \quad az^* = z, \quad \nu_1^2 \rho p^* = a^2\rho$$
$$a^2t^* = \nu_1 t, \quad \nu_1 u^* = au, \quad \nu_1 v^* = av$$

The equations 1 to 4 will be

$\sigma = a/\sqrt{k}$, $M = B_0 a\sqrt{\sigma/\rho\nu_1}$ (Hartmann number)

$$\frac{\partial}{\partial t} = -\frac{}{\partial x} + \left(1 + \alpha\frac{}{\partial t}\right)\frac{}{\partial z^2} - (\sigma \quad + M \quad)u \quad (5)$$

$\alpha = \dfrac{\nu_2}{a^2}$ (the non-dimensional viscoelastic parameter).

(6)

$\partial t \quad \partial y \quad (\quad \partial t) \, \partial z$

$$0 = -\frac{\partial p}{\partial z} \quad (7)$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (8)$$

Initially the fluid is at rest then using initial boundary conditions, which are

The initial and boundary conditions are :

$t \leq o : u = o = v$, everywhere in the channel (9)

$t > o : u = o = v$ at $z = \pm 1$. (10)

Using (8), the equations (5) and (6) give

$$\frac{\partial^2 \rho}{\partial x^2} + \frac{\partial^2 \rho}{\partial y^2} + 0 \tag{11}$$

Equation (7) implies that 'p' is independent of z. Therefore, 'p' is a function of x, y, t. Assume

$$p(x,y,t) = p(x,y)\, g(t) \tag{12}$$

$$u(x,y,z,t) = f(z,t)\, \phi_x \tag{13}$$

and $$v(x,y,z,t) = f(z,t)\, \phi_y \tag{14}$$

where ϕ is some function x and y.

Using (12), (13) and (14) in the equations (5) and (6)

we have

$$\left(1 + \alpha \frac{\partial}{\partial t}\right)\frac{\partial^2 f}{\partial z^2} - \frac{\partial f}{\partial t} - \left(\sigma^2 + M^2\right)f = g(t) \tag{15}$$

where $p(x,y) = \phi(x,y) + C$, C being a constant (16)

The initial and boundary conditions are

$t \leq o : f(z,t) = o$ everywhere in the channel (17)

$t > o : f(z,t) = o$ at $z = \pm 1$ (18)

Due to symmetric consideration the flow region for z = 0 is considered and then equation (18) becomes

$t > o : f(z,t) = o$ at $z = 1$, $\partial f/\partial z = o$ at $z = o$ (19)

Solution of above problem

Here we solve the above problem with help of integral transformations for this we define the finite Fourier cosine transform with range over the interval (0,1) as

$$u_c(n,t) = f_c[f(z,t); z > n]$$

$$= \int_0^1 f(z,t)\cos\theta_n z\,dz \quad (20)$$

and its inversion formula is given by

$$f(z,t) = 2\sum_{n=0}^{\infty} u_c(n,t)\cos\theta_n z \quad (21)$$

As f(z,t) = o at z = 1, therefore,

$$\theta_n = (2n+1)\ \pi/2 \quad (22)$$

Multiplying (15) by $\cos\theta_n z$ and integrating between limits 0 to 1 and using equation (19), we get

$$\frac{\partial u_c}{\partial t} + Au_c = Bg(t) \quad (23)$$

$$A = \theta_n^2 + \sigma^2 + M^2/1 + \theta_n^2\alpha, \qquad B = (-1)^{n+1}/\theta_n(1 + \theta_n^2\alpha) \quad (24)$$

using Laplace transform under condition u = 0 at t = 0, equation (23), gives

$$u_c(s) = -\frac{B}{s+A}g(s) \quad (25)$$

now using convolution theorem on (25) and applying inversion formula for the finite Fourior cosine transform, we get

$$u_c(t) = B\int_0^1 e^{-A\tau}g(t-\tau)d\tau \quad (26)$$

from (21) and (26)

$$f(z,t) = 2\sum_{n=0}^{\infty} B\cos\theta_n z\int_0^t e^{-A\tau}g(t-\tau)d\tau \tag{27}$$

using equation (12) and (16) the equation (11) can be written as

$$\frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial y^2} = 0 \tag{28}$$

by solving (28) the unknown function φ is obtained in conditions

$$\frac{\partial\phi}{\partial r} = 0 \quad \text{on} \quad r = a \tag{29}$$

where

$x = r\cos\theta$, $y = r\sin\theta$ and $\phi_x \to 1$, $\phi_y \to o$ as $|x|, |x| \to \propto$,

Thus,

$$\phi(x,y) = (r + a^2/r)\cos\theta \tag{30}$$

Using the value of f(z,t) from (27) and φ (x,y) from (30), we get the expressions for the velocity components from equation (13) and (14) as

$$u = 2\sum_{n=0}^{\infty} B\left[\cos\theta_n z\int_0^t e^{-A\tau}g(t-\tau)d\tau\right].\left[1 - \frac{x^2 - y^2}{(x^2+y^2)^2}\right] \tag{31}$$

$$v = 2\sum_{n=0}^{\infty}\left[B\cos\theta_n z\int_0^t e^{-A\tau}g(t-\tau)d\tau\right].\left[\frac{-2xy}{(x^2+y^2)^2}\right] \tag{32}$$

The pressure of fluid (12) can be obtained by using the results of (16) and (30)

Result and Discussion

The velocity components of second oreder fluid are given by equation (31) and (32). We get that u and v both are maximum at central plane (z=o) of the gap and velocity u parallel to the x-axis is much greater than the velocity v parallel to the y-axis. Also we observe that u and v both decrease with the increasing values of 'σ'. This shows that the velocity in the porous medium increase with the increase of the permeability of the medium. We observe that the non-Newtonian parameter is to decrease the magnitudes of these velocity components. And u and v both increase with increasing values of time 't'.

In table we have tabulated u and v both increase values of Hartmann number 'M'. The table shows that, both the velocity components decrease with the increasing values of 'M'.

Z	M = 0.0	M = 1	M = 2
0.0	0.1567	0.1409	0.1198
0.2	0.1551	0.1399	0.1195
0.4	0.1505	0.1371	0.1189
0.6	0.1419	0.1317	0.1173
0.8	0.1257	0.1200	0.1119
1.0	0.1000	0.1000	0.1000

REFERENCES

1. Buckmaster, j. (1970) : "Some remarks on Hele-Show flow and viscous tails", J. Fluid Mech., vol. 41, Part 3, pp. 523 – 530.
2. Gupta, M. Kant, R. and Sharma, S. (1979) : "Unsteady Hele-Show flow of non-Newtonian fluid," J. of Maths., Phy. Sci. Vol. 13, no. 2 pp. 189-197
3. Hele-Shaw, H.S. (1897) : Trans. Roy. Int. Nav. Arch. Vol. 41, p. 21
4. Lee, J.S. and Fung, Y.C. (1969) : "Stokes flow around a circular Cylindrical post confined between two parallel plates", L. Fluid Mech., Vol. 37, pp.657-670
5. Thompson, B.W. (1968) : "Secondary flow in a Hele-Show Cell", J.Fluid Mech., Vol. 31 pp. 379-395