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Approximate solution of Fredholm Integro Differential equation using Quadrature Formulas methods

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ABSTRACT
There are two reasons for this research, the first which is the main was to clarify the use of a closed quadrature formulas included (Trapizoidal , Simpson's 1/3 rule and Simpson's 3/8 rule)which are the most familiar formula of numerical integration ,to evaluating the integral part to find the approximate solution of the 2nd kind of FIDE's of the 1st order and reducing it to linear system of (n) equation with n unknowns of the solution sample value y(ti) ,i=0,1,2,3,…,n .The other reason was to explain the differences between three Quadrature formulas in solving equation according to the specified period , has been clarified through examples.
Finally, Acomparison was made between the three methods ,programs for methods were written in MATLAB language and examples with satisfactory results are given .
Keywords: General 2nd kind of linear Fredholm integro differential equation, Newton cote's formula ,Trapezoidal ,Simpson's1/3 ,and Simpson's 3/8 rule.

I. INTRODUCTION

Integro differential equations plays a very important role in modern science and technology applications such as heat transfer equ.,diffusion processes neutron diffusion and biological speices which are an essential elements in biology ,problems of mathematical physics ,theory of elasticity , and engineering applications. [1, 2] quadrature formulas are an extremely useful and straightforward family of numerical integration techniques used by engineers and scientists to find approximate solution for definite integral equation that can't be solved analytically . The simplicity of these methods makes it perfect for many applications which leads some researchers to relied on quadrature formula in their research as a basis to solve integral and integro differential equation such as Saadati and Raftari(2008),used trapezoidal role to solve integro differential equation , Elayaraja and Jumat (2012) apply iterative method namely Half-Sweep Gauss-Seidel (HSGS) method for solving high order closed repeated Newton-Cotes (CRNC) quadrature approximation equations associated with numerical solution of LFIDE's . Two different order of CRNC i.e.

repeated Simpson's 1/3 and Simpson's 3/8, Pramod(2015) introduced Simpson's rule to approximate linear FIDE's , Bashir and Sirajo introduced 1/3 Simpson's approach to solve linear FIDE's. in this research , we used this feature to evaluate the integrated part of our problem in three methods to reduce our problem to linear system and then solve it by Gauss elimination method ,we also made a comparison between these method to find the appropriate method that achieves better results.

The previous technique applied to evaluate the approximate solution for linear Fredholm integro

$$y'(x)+P(x)y(x)=g(x)+\int_a^b k(x,t)y(t)dt, t\in I=[a,b]..........(1)$$

diff.equ. of the 2nd kind has the form:-

II. Classification of Fredholm Integro Diff.equ. 2nd kind

Integro –differential equation is an equation involving one or more unknown function ,together with both differential and integral operations on t .the general n^th^ order of IDE form is:

$$y^{(n)}(x)+\sum_{i=0}^{n-1}P_i(x)y^{(i)}(x)=g(x)+\lambda\int_a^t k(x,t)y(t)dt$$

Where , $g(x), P_i(x)(i=0,1,\ldots,n-1)$ are continues on I ,and denotes a function of two variables x and t called the kernel ,a and b are the limits of integration that may be both variables, constant or mixed $P(x), g(x)$, $k(x,t)$ and functions are known $y(x)$ is the unknown function that appears under and outside integral sign , $y'(x)$ is the derivative of $y(x)$ will be determined [7] .

III. Definition of Quadrature Rule:

A quadrature rule is a generic name given to any numerical approximation method of definite integral of a function , usually stated as a weight sum of function values at specified points within the domain of integration ,in other words to integrate a function $f(x)$ over some interval [a,b],dived it into n equal parts i.e $f_n=f(x_n)$ and $h=\frac{b-a}{n}$, then find polynomials that approxima

te the tabulated function ,and integrate them to approximate the area under the curve , in order to find the fitting polynomials implement Lagrange interpolating polynomial. The resulting formulas called Newton –Cotes formula or (n+1)-point quadrature basic formulas which has the form: [8]

$$\int_a^b f(x)dt=\sum_{i=0}^{n}W_i f(x_i)+E_n(f)..................(3)$$

Where x_i *(i=0,1,2…,n)* are the integration nodes which are lying in the interval[a,b] and W_i *(i=0,1,2,…,n)* are constants which are called quadrature weights, with $E_n(f)$ approximation error .

IV. The Open and Closed Newton Cote's Method

There are two classes of newton cotes quadrature method

1) If the interval [a, b] included on the fit i.e x_0=a and x_n=b ,it called "closed" and for closed formula x_i=a+ ih, h=b-a/n.

2) If they do not use function values on endpoints ,i.e. x_0>a ,x_n<b ,it called "open" and for open formula xi= a+(i+1)h, h= b-a/n+2.[4]

Theorem 3-1 (closed Newton Cotes Quadrature Formula) [9]

Assume that x_i= x_0+ih are equally spaced nodes and $f_i=f(x_i)$ then the first three Closed Newton Cote's Quadrature Formulas are:

$$\int_{x_0}^{x_1} f(x)dx\approx\frac{h}{2}(f_0+f_1)........(4)$$

(trapezoidal rule)

$$\int_{x_0}^{x_2} f(x)dx \approx \frac{h}{3}(f_0 + 4f_1 + f_2)\ldots\ldots(5)$$

(Simpson's1/3 rule)

$$\int_{x_0}^{x_3} f(x)dx \approx \frac{3h}{8}(f_0 + 3f_1 + 3f_2 + f_3)\ldots\ldots.(6)$$

(Simpson's 3/8 rule

Corollary 3-2 :(Newton-Cotes Quadrature Precision)[10]

Assume that $f(x)$ is sufficiently differentiable; then E[f] for Newton-Cotes Quadrature involves an appropriate higher derivatives.

Trapizoidal rule has a degree of precision m=1,if $f \in C^2$ [a,b],then

V. METHODS AND MATERIAL

$$\int_{x_0}^{x_1} f(x)dx \approx \frac{h}{2}(f_0 + f_1) - \frac{h^3}{12} f^2(c)\ldots\ldots\ldots\ldots\ldots\ldots..(7)$$

1) Simpson's 1/3 rule has a degree of precision m=3,if $f \in C^4$ [a,b],then

$$\int_{x_0}^{x_2} f(x)dx \approx \frac{h}{3}(f_0 + 4f_1 + f_2) - \frac{h^5}{90}.f^4(c)\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots..(8)$$

2) Simpson's 3/8 rule has a degree of precision m=3,if $f \in C^4$ [a,b],then

$$\int_{x_0}^{x_3} f(x)dx \approx \frac{3h}{8}(f_0 + 3f_1 + 3f_2 + f_3) - \frac{3h^5}{80}.f^4(c)\ldots\ldots..(9)$$

<u>Numerical solution of FIDEs using Quadratures Rule :-</u>

In this section ,Closed Newton Cote's quadrature formulas are applied to find the approximate solution of 2nd order linear FIDE in equation (1), with the initial condition $y(a) = y_0$

Now,we defin n as a finite points of the interval [a,b] of equ(1) where $a = t_0 < t_1 < t_2,\ldots < t_{n-1} < t_n = b$, with uniform step length h=b-a/n, such that $t_j = a + j*h$,j=0,1,2,…,n, we set $x_i = t_i$,i=0,1,2,…,n $y'(t_i) = y'_i, P(x_i) = P_i, g(x_i) = g_i, y(x_i) = y_i$ and $k(x_i, t_j) = k_{ij}$,

A. Using Trapizoidal Rula

In order to applied trapezoidal rule with n-subintervals to find the approximate solution of eq(1) ,we have

$$y'(x) + P_i y_i = g_i + \frac{h}{2}\left[k_{i0}y_0 + 2k_{i1}y_1 + \ldots + 2k_{ij-1}y_{i-1} + k_{ij}y_i\right]..$$

Since

$$y'(t_i) = \frac{y_i(x) - y_{i-1}(x)}{h}\ldots\ldots\ldots\ldots\ldots(11)$$

Substitute equ(11)in equ(10) we have,

$$(1 + hP_i - \frac{h^2}{2}k_{ij})y_i - y_{i-1} = hg_i + \frac{h^2}{2}\left[k_{i0}y_0 + 2k_{i1}y_1 + \ldots + 2k_{ij-1}y_{i-1}\right]\ldots.(12)$$

From equ(12) we can generate a systems of equations for $y_1, y_2, \ldots, y_n$

For ij=1 substituting in equation (12) ,get

$$(1 + hP_i - \frac{h^2}{2}k_{11})y_1 - y_0 = hg_1 + \frac{h^2}{2}\left[k_{10}y_0\right]$$

$$(1 + hP_i - \frac{h^2}{2}k_{11})y_1 = hg_1 + \frac{h^2}{2}\left[k_{10}y_0\right] + y_0\ldots..(13)$$

For i=2

$$(1 + hP_2 - \frac{h^2}{2}k_{22})y_2 - y_1 = hg_2 + \frac{h^2}{2}\left[k_{20}y_0 + 2k_{21}y_1\right]$$

$$(1 + hP_2 - \frac{h^2}{2}k_{22})y_2 - (1 + h^2k_{21})y_1 = hg_2 + \frac{h^2}{2}k_{20}y_0\ldots..(1.4)$$

For i=3

$$(1+hP_3-\frac{h^2}{2}k_{33})y_3-y_2=hg_3+\frac{h^2}{2}\left[k_{30}y_0+2k_{31}y_1+\ldots+2k_{32}y_2\right]$$

$$(1+hP_3-\frac{h^2}{2}k_{33})y_3-(1+h^2k_{32})y_2-h^2k_{31}=hg_3+\frac{h^2}{2}k_{30}y_0 \ldots\ldots\ldots(15)$$

And so on …

which we can be represent in a matrix form : *Ay=B*……..(16)

Where A is n × n matrix:

$$a_{ij}=\begin{cases} 1+hp_i-\dfrac{h^2}{2}k_{ij}, & \text{If } i=j \text{ and } j\neq n \\ -1-h^2k_{ij} & \text{If } j\text{-}i=-1 \\ -h^2k_{ij} & \text{If } i>j \\ -h^2k_{ij} & \text{If } i<j \\ 1+hp_i-\dfrac{h^2}{2}k_{ij} & \text{If } i=j \text{ and } j=n \end{cases}$$

And

$$b_i=\begin{cases} y_0+hg_i+\dfrac{h^2}{2}k_{i0}y_0, & \text{for } i=1 \\ hg_i+\dfrac{h^2}{2}k_{i0}y_0 & \text{for } i\neq 1 \end{cases}$$

Finally, Gauss- Elemination procedure is used to solve the system of equ(16) for each value of $i=1,2,\ldots,n$ to find y_i, which is the approximate solution of equ(1).

B. Using Simpson's 1/3 Rula

A composite simpson's 1/3 rule is used with n-subintervals to approximate integrals in equ(1),with $x=x_i$,i=1,2,…,n ,give as the form

$$y'(x)+P_iy_i=g_i+\frac{h}{3}\left[k_{i0}y_0+4k_{i1}y_1+2k_{i2}y_2+\ldots+2k_{ii-2}y_{i-2}+4k_{ii-1}y_{i-1}+k_{ii}y_i\right]\ldots\ldots\ldots\ldots\ldots(17)$$

$$(1+hP_i-\frac{h^2}{3}k_{ij})y_i-y_{i-1}=hg_i+\frac{h^2}{3}\left[k_{i0}y_0+4k_{i1}y_1+2k_{i2}y_2+4k_3y_3+\ldots+2k_{ij-2}y_{i-2}+4k_{ij-1}y_{i-1}\right]\ldots\ldots(18)$$

And by susitute $y'(t_i)=\dfrac{y_i(x)-y_{i-1}(x)}{h}$. **in equ(17) yields**

if i is even

$$(1+hP_i-\frac{h^2}{3}k_{ij})y_i-y_{i-1}=hg_i+\frac{h^2}{3}\left[k_{i0}y_0+4k_{i1}y_1+2k_{i2}y_2+4k_3y_3+\ldots+2k_{ij-2}y_{i-2}+4k_{ij-1}y_{i-1}\right]\ldots\ldots(19)$$

if i is odd , then we have acombination between trapezoidal and simpson's 1/3 rule

$$(1+hP_i)y_i-y_{i-1}=hg_i+h^2\left[\frac{1}{2}k_{i0}y_0+\frac{5}{6}k_{i1}y_1+\frac{4}{3}k_{i2}y_2+\frac{2}{3}k_3y_3+\ldots+\frac{1}{3}k_{ij}y_i\right]\ldots\ldots(20)$$

Which give as asystem of n equations of y_i , that represents the approximate solution of equation(1) at $x_i=a+ih$,$i=1,2,,\ldots,n$,that is in matrix form: *AY=B*

which gives as a system forms: *Ay=B* ……………………….(19) ,where A is a n ×n matrix:

$$a_{ij}=\begin{cases} 1+hp_i-\dfrac{h^2}{2} & \text{If } i=j \quad ,i=1 \\ -1-\dfrac{4}{3}h^2k_{ij} & \text{If } j\text{-}i=-1 \\ 1+hp_i-\dfrac{h^2}{3} & \text{If } i=j \quad ,i>1 \\ -\dfrac{5}{6}h^2k_{ij} & \text{If } j=1 \text{ and } I \text{ is} \\ -\dfrac{4}{3}h^2k_{ij} & \text{If } i>j \text{ and } i\text{-}j=3 \\ -\dfrac{2}{3}h^2k_{ij} & \text{If } i>j \text{ and } i\text{-}j=2 \end{cases}$$

$$b_i=\begin{cases} y_0+hg_i+\dfrac{h^2}{2}k_i & \text{for } i=1 \\ hg_i+\dfrac{h^2}{2}k_{i0}y_0 & \text{for } I \text{ is odd} \\ hg_i+\dfrac{h^2}{3}k_{i0}y_0 & \text{for } I \text{ is even} \end{cases}$$

C. Using Simpson's 3/8 Rules

Now a combination of 3/8 Simpso's rules in equation (6) was applied to find the approximate solution of equ(1

$$y'(x)+P_i y_i = g_i + \frac{3h}{8}\left[k_{i0}y_0 + 3k_{i1}y_1 + 3k_{i2}y_2 + \ldots + 3k_{ii-1}y_{i-1} + k_{ii}y_i\right] \ldots\ldots\ldots\ldots(21)$$

Substituting eau(11) in equ(21) ,we obtain

$$(1+hP_i - \frac{3h^2}{8}k_{ii})y_i - y_{i-1} = hg_i + \frac{3h^2}{8}\left[k_{i0}y_0 + 3k_{i1}y_1 + 3k_{i2}y_2 + \ldots + 3k_{ii-1}y_{i-1}\right] \ldots\ldots(22)$$

Simpson's 3/8 method depends on the number of subintervals n are a multiple of three which give us three cases ,a first case if n divided by 3 and reminder is 0 term (denoted by rem(n,3)=0)we have

$$(1+hP_i - \frac{3h^2}{8}k_{ii})y_i - y_{i-1} = hg_i + h^2\left[\frac{1}{2}k_{i0}y_0 + \frac{7}{8}k_{i1}y_1 + \frac{9}{8}k_{i2}y_2 + \ldots + \frac{9}{8}k_{ii-1}y_{i-1}\right] \ldots\ldots(24)$$

The second case when n divided by 3 reminder 1 term (denoted by rem(n,3)=1) then we get a combination betwwen trapezoidal and simpson's 3/8 rules

$$(1+hP_i - \frac{3h^2}{8}k_{ii})y_i - y_{i-1} = hg_i + h^2\left[\frac{1}{2}k_{i0}y_0 + \frac{7}{8}k_{i1}y_1 + \frac{9}{8}k_{i2}y_2 + \ldots + \frac{9}{8}k_{ii-1}y_{i-1}\right] \ldots\ldots(24)$$

And, the third case when n divided 3 reminder 2 term (denoted by rem(n,3)=2) then we have a combination between simposn;s 1/3 and simpson's 3/8 and our equation becomes

$$(1+hP_i - \frac{3h^2}{8}k_{ii})y_i - y_{i-1} = hg_i + h^2\left[\frac{1}{3}k_{i0}y_0 + \frac{4}{3}k_{i1}y_1 + \frac{17}{24}k_{i2}y_2 + \ldots + \frac{9}{8}k_{ii-1}y_{i-1}\right] \ldots\ldots(25)$$

The equations (23) ,(24) and (25) represent the approximate solution of equation (1) at $x=x_i=a+ih$,for $i=0,1,2,\ldots,n,$ which give as a matrix form :Ay=B

Where A is a matrix n×n

$$a_{ij} = \begin{cases}
1+hp_i - \frac{h^2}{2} & \text{If } i=1 \text{ and } i=i \\
-1-\frac{4h^2}{3} & \text{If } i=1 \text{ and } i\text{-}i=1 \\
-\frac{9h^2}{8}k & \text{If } i>j \text{ and } i\text{-}j=2 \text{ or} \\
 & \text{if } i>j \text{ and } rem(i,3)=0 \\
 & \text{If } i=1 \text{ and } rem(i,3)=1 \\
-\frac{7h^2}{8}k & \\
 & \text{If } j=1 \text{ and } rem(i,3)=2 \text{ or} \\
-\frac{4h^2}{3} & \text{If } i>j \\
 & rem(i,3)=0 \text{ ,} rem(j,3)=0 \\
1-hp_i - \frac{}{3} & \text{If } i=2 \text{ and } i=i \\
-1- & \text{If } j=2 \text{ and } i\text{-}j=1 \text{ or} \\
 & \text{If } i\text{-}j=1 \text{ and } rem(j,3)=0 \text{ or} \\
 & \text{If } i\text{-}j=1 \text{ and } rem(j,3)=1 \text{ or} \\
 & \text{If } i\text{-}j=1 \text{ and } rem(j,3)=2 \\
-\frac{17h^2}{24}k_{ij} & \text{If } i=2 \text{ and } rem(i,3)=2 \\
1+hp_i - & \text{If } j=i \text{ and } rem(j,3)=0 \text{ or} \\
 & \text{If } j=i \text{ and } rem(j,3)=1 \text{ ,} rem(j,3)=2 \\
 & \text{or} \\
 & \text{If } j=I \text{ and } j=n \\
 & \text{If } i>j \text{ and} \\
-\frac{6h^2}{8}k_{ij} & rem(j,3)=0 \text{ ,} rem(I,3)=0 \text{ or} \\
 & \text{If } i>j \text{ and} \\
 & rem(j,3)=1 \text{ ,} rem(I,3)=1 \text{ or}
\end{cases}$$

$$b_i = \begin{cases} y_0 + hg_i + \frac{h^2}{2} k_{i0} & If \quad i=1 \\ hg_i + \frac{3h^2}{8} k_{i0} y_0 & If \quad rem(i.3)=0 \\ hg_i + \frac{h^2}{2} k_{i0} y_0 & If \quad rem(i.3)=1 \\ hg_i + \frac{h^2}{3} k_{i0} y_0 & If \quad rem(i.3)=2 \end{cases}$$

The algorithm:

A numerical solution of 1st order FIDE's of 2nd kind ,by using Quadrature methods included(Trapezoidal ,Simpson's 1/3 and Simpson's 3/8 rule),are obtained as follows:

Step1:

Put h=(b-a) /n , where $n \in N$ and $y_0 = y(a)$ (the initial condition gave)

Step2:

Set $x_i = a + ih$, $i=0,1,2,\dots,n$

Step3:

Compute y_i' by $y_i' = \frac{y_i - y_{i-1}}{h}$

__Case Trapezoidal method used then__

Step4:

Used step(1,2 and 3) in equation (12) to find y_i $_{,(i=1,2,\dots,n)}$

__Case simpson's 1/3 rule used then__

a) If (number of subintervals n is even)

Step 4:

Used step(1,2 and 3) in equation (19) to compute y_i $_{,(i=1,2,\dots,n)}$

b) If (number of subintervals n is odd)

Step4:

Used step(1,2 and 3) in equation (20) to compute y_i $_{,(i=1,2,\dots,n)}$

__Case simpson's 3/8 rule used then__

If the number of subintervals is multiple of three then

Step4:

Compute y_i $_{,(i=1,2,3,\dots,n)}$ by using equ(23) with step(1,2,and 3)

If the number of subintervals is multiple of three+1 then

Step 4:

Compute y_i $_{,(i=1,2,3,\dots,n)}$ by using equ(24) with step(1,2,and 3)

If the number of subintervals is multiple of three +2 then

Step4:

Compute y_i $_{,(i=1,2,3,\dots,n)}$ by using equ(25) with step(1,2,and 3)

Step5:

Use Gauss-elimination procedures to solve a resulting system

Numeriacl Examples:-

__**Example 1:-**__

Consider the 1st order of FIDE's of 2nd kind problem:-

$$y'(t) - y(t) = e^t - t + \int_0^1 y(x)dx : 0 \le t \le 1$$ With

initial condition $y(0) = 0$ and $g(t)=e^t - t$

And the exact solution is $y(t) = te^t$

Table(1): the comparison between the exact and numerical Quadrature formula solutions dependence on the least square error and running time

metho d nodes	Exact	Trapezoida l	simp1/3	simp3/8
0	0.0000000 00	0.0000000 00	0.0000000 00	0.0000000 00
0.1	0.1105170 92	0.1088364 74	0.1088364 74	0.1088364 74
0.2	0.2442805 52	0.2430225 35	0.2430225 35	0.2430921 34
0.3	0.4049576 42	0.4017429 98	0.4017115 65	0.4019152 65
0.4	0.5967298 79	0.5936406 22	0.5936535 84	0.5940363 09
0.5	0.8243606 35	0.8189936 48	0.8189605 02	0.8196654 65
0.6	1.0932712 8	1.0876816 67	1.0876556 92	1.0887923 55
0.7	1.4096268 95	1.4013812 64	1.4013606 39	1.4029982 24

0.8	1.780432743	1.771552115	1.77153496	1.773878495
0.9	2.2136428	2.201655458	2.20157494	2.204775353
1	2.718281828	2.705165691	2.705172406	2.709327407
L.S.E.		0.180514850	0.175566346	0.175521475
R.T.		0.300000000	0.320000000	0.320000000

Example 2:-

Consider the following FIDE problem:

$$y'(t) = 3e^{3t} - \frac{1}{3}(2e^{3}+1)t + \int_{0}^{1} 3txy(x)dx : 0 \le t \le 1$$

Where the initial condition $y(0)=1$,

$$g(t) = 3e^{3t} - \frac{1}{3}(2e^{3}+1)t$$ **With the exact solution**

$$y(t) = e^{3t}$$

Table(2) : the comparison between the exact and numerical Quadrature formula solutions over the interval x0=0 to xn=1 with h=0.1

method nodes	exact exp3x	trap	simp1/3	simp3/8
0	1	1.00000000	1.00000000	1.00000000
0.1	1.349858808	1.34817819	1.34817819	1.34817819
0.2	1.8221188	1.820860783	1.820860783	1.820930382
0.3	2.459603111	2.456388467	2.456357034	2.456560734
0.4	3.320116923	3.317027666	3.317040628	3.317423353
0.5	4.48168907	4.476322082	4.476288936	4.476993899
0.6	6.049647464	6.044057851	6.044031876	6.048168539
0.7	8.166169913	8.157924281	8.157903656	8.165541241
0.8	11.02317638	11.01429575	11.0142786	11.01662213
0.9	14.87973172	14.86774438	14.86766386	14.87086428
1	20.08553692	20.07242079	20.0724275	20.0765825

VI. Discussion

In this our research , we introduced closed Quadrature formula included (Trapezoidal ,Simpson's1/3 ,and Simpson's 3/8 rules) to find the approximation of 1st order Fredholm Integro Differential Equation of the 2nd kind.

By benefiting from previous studies and comparing them with the results obtaind from the illustrative examples in table 1 and 2 the following is shown :-

1) A numerical results of the model problems showed that the proposed methods was computationally efficient

2) all methods gave good results, and when comparing between the methods, it was found that Simpson's 3/8 gave better accuracy results. 3) In general ,as the number of N nodes increase ,the error terms is decreased in all used method see[11,12]

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