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Various Applications of the Number Theory

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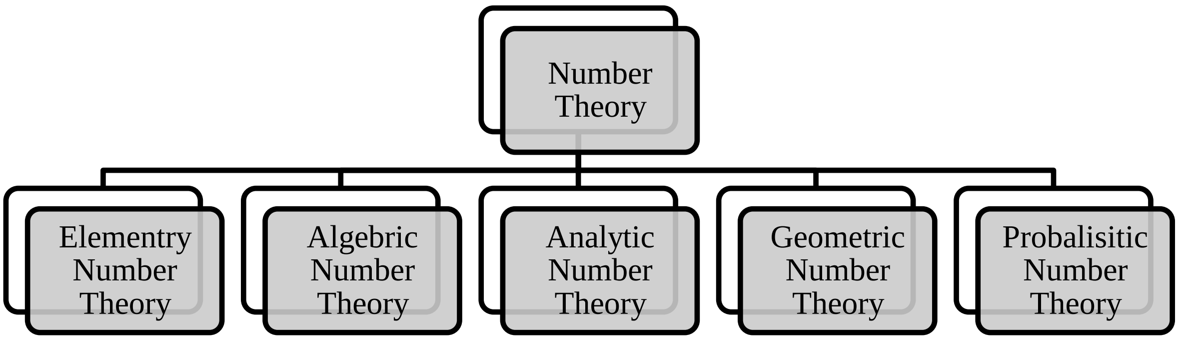
Abstract - The number theory is a branch of mathematics which is primarily dedicated to the study of integers. The number theory, as such, is less applied in engineering compared to calculus, geometry, etc. The problem was that it could not be used directly in any application. But, the number theory, combined with the computational power of modern computers, gives interesting solutions to real-life problems. It has many uses in various fields such as cryptography, computing, numerical analysis and so on. Here, we focus on the applications of the number theory about engineering challenges.

Keywords : Elementary number theory, patterns, proof, Analytic.

Introduction- Number theory, known as the queen of mathematics is the branch of mathematics that concerns about the positive integers 1, 2, 3, 4, 5 which are often called natural numbers and their appealing properties. From antiquity, these natural numbers classified as odd numbers, even numbers, square numbers, prime numbers, Fibonacci numbers, triangular numbers, etc. Due to the dense of unsolved problems, number theory plays a significant role in athematics. The recent classification of number theory depending upon the tools used to address the related problems is shown below



The research on integers in a scientific way is truly credited to Greeks. Later, a big revolution on this theory happened due to the arrival of the famous book "Elements" by Euclid in which the mathematics itself is depicted with precise proofs. There exist only a few kinds of literature discussing on the applications of number theory in engineering to the best of authors knowledge. So, the objective of present work is to perform a critical review on the existing practices related to the number theory applications in engineering.

Number Theory Problems

In this section I illustrate the elegance associated with number theory based argument and the mystery attached to some number theory related problems. The connection to power is in the presumed appeal: in other words, people are attracted to beauty and elegance, and people are also attracted to mystery, especially if either appears accessible and within reach.

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Consider for example a well-known proof for irrationality of the square root of 2.
To prove that the $\sqrt{2}$ is not a rational number we assume that it is rational and therefore can be represented as

$$\frac{a}{b}$$

where a and b are whole numbers, $b \neq 0$, and a fraction was chosen in its reduced form , that is, $(a, b) = 1$.

(1) $\sqrt{2} = \frac{a}{b}$

From this assumption, by raising both sides of the equality to the second power we get

(2) $2 = \frac{a^2}{b^2}$ or
(3) $2b^2 = a^2$

This means that a^2 is an even number and therefore a must be an even number.
Representing a as $2k$, we get
(4) $2b^2 = (2k)^2$ or
(5) $b2= 2k2$

Therefore b must be an even number. However, this claim contradicts our original assumption that the fraction

$$\frac{a}{b}$$

was chosen in its reduced form and completes the proof.
Interestingly, a contradiction can be claimed much earlier, considering (3). The expression on the left of (3) has an odd number of factors in its prime decomposition, while the number of prime factors of the expression on the right side is even. This observation contradicts the Fundamental Theorem of Arithmetic, which guarantees the uniqueness of prime decomposition.
This argument is powerful in its elegance, but assumes a much deeper knowledge. It is not surprising that power of insight often depends on prior knowledge or understanding that is employed.

As research has repeatedly demonstrated, the Fundamental Theorem of Arithmetic is often not employed by learners in problem solving situations. For example, in Zazkis & Campbell (1996b) we presented a group of pre-service elementary school teachers with a number ($K = 16\ 199$) represented as a product of two relatively large primes ($K = 97 \times 167$) and asked them to determine whether K was divisible by 3, 5, 11 or 13. The participants were familiar with the Fundamental Theorem of Arithmetic from their coursework, namely, they could quote it and demonstrate it by choosing a different route in factoring a number and arriving at the same list of prime factors. However, while the divisibility of K is apparent based on its prime decomposition and the answer is immediate based on the Fundamental Theorem of Arithmetic, the majority of participants did not use this argument. Rather, they reached their conclusion by calculating the value of K and then either performing division or attending to the familiar divisibility rules. Some participants explicitly claimed that "because a number is divisible by two primes does not mean that it is not divisible by other primes". As such, explicit consideration of each case was deemed essential. Although a correct conclusion was reached, the power of elegance was lost.

Developing Structure

Devlin (1994) provides an account of how the views on "what is mathematics" changed through the centuries from "the study of number and shape" to "the science of patterns". The 'patterns' in this science can be those of shape, space, number, change, motion, etc. Though 'pattern' implies structure and logical relationship, research has shown that even relationships that the "mathematically inclined" take for granted are often not obvious for learners. Consider, for example, the relationship between multiplication and division. By definition, if $a \times b = c$ then $c \div b = a$. In my early research (Zazkis & Campbell, 1996a) the following question was posed in a clinical interview to pre-service elementary school teachers:

$M = 3^3 \times 5^2 \times 7$. Is M divisible by 7?

Unexpectedly, many participants could reach their conclusion only after finding the value of M by multiplication, then dividing by 7 and observing that the result was a whole number. Similar results were reported by Ferrari (2002).

Developing proofs

Consider the following statements or theorems:

(1) If a divides b and b divides c then a divides c
(2) For any positive integer n, if n^2 is a multiple of 3 then n is a multiple of 3.
(3) If p and q are any two odd numbers, $(p + q) \times (p - q)$ is always a multiple of 4.
(4) Multiples of 5 are closed under addition.

These are examples of propositions in elementary number theory. However, each one of these statements was used in research on proof. (1) was used by Martin & Harel (1989) in investigating proof schemes of preservice elementary school teachers. (2) was used by Selden & Selden (2003) in investigating the ability of undergraduate students to determine validity of presented proofs. (3) was used by Healy & Hoyles (2000) in their investigation of proof conceptions in algebra of students ages 14-15. Lastly, (4) was used by Gholamazad, Liljedahl & Zazkis (2003) in designing a framework for analysing potential pitfalls in short proofs written by elementary school teachers.

The unifying theme in these studies is that the issue under investigation is proof – approaches to its creation and judging its validity – while the content employed is the one of number theory. This is not surprising, since number theory provides "paradigmatic exposure to mathematical thinking" (Mason, 2006), and as such it can be used both to develop mathematical thinking and to investigate it.

With respect to proofs, number theory provides a variety of propositions that require very short proofs and are most appropriate for learning the notion and structure of a rigorous argument by novices and generalists. In school, students are usually introduced to the notion of proof in their study of Euclidian geometry. I believe it would be more helpful to introduce students to proofs via number theory related propositions. In fact, acknowledging students' difficulties with proof many North American Universities developed "Introduction to proof" courses for mathematic majors. In these courses a significant amount of content is typically drawn from number theory. Perhaps, an earlier start in utilizing this area while developing appropriate skills in mathematical reasoning and argumentation would be advantageous.

In addition to practicing short proofs, number theory provides a wonderful opportunity to exemplify a variety of proof strategies. The typical introduction of "proof by contradiction" is with number theory theorems: irrationality of $\sqrt{2}$ and infinity of primes. Further, in investigating proof schemes related to mathematical induction Harel (2002) considered propositions in number theory. Rowland (2002) introduced the pedagogical idea of "proof by generic example" using theorems in number theory, specifically Wilson's theorem and The Chinese Remainder theorem.

Conclusion- Various applications of the number theory were mentioned in detail. The significant contribution of number theory in recent years is in the area of cryptography, and hence computer science engineering was noted initially. The importance of famous series and sequences in almost every field of engineering was observed. It is seen that applications of number theory were not directly in some applications; with the number theory being fundamental, it acted as the driving force in approaching the solution. The versatility of the applications was also recognized. Further research and development of the theory will pave the way for more uses of number theory to both pure as well as applied/engineering mathematics. Elementary number theory tasks are rich and accessible area with which to *enhance and probe* [my italics] students' mathematical sense making". As such, the utility of number theory has been argued for both teaching and learning mathematics and conducting research in mathematics education. This utility is for students as well as for teachers and researchers. Furthermore, for students, additional utility of experiencing number theory lies in developing an appreciation for the beauty and power of mathematics. That is where the roles of Servant and Queen overlap and blur.

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