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New Sets on Local functions of Different Perfect sets in Micro Ideal Topological Spaces

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ABSTRACT

A perfect set is one of the characterizations for compatible ideals in an ideal topological space. In this paper, we introduce D_1^{*M}, D_2^{*M} and $D^{*M}-Sets$ in a brand new ideal space called Micro ideal topological spaces, which give the local functions of certain perfect sets in the space and study their properties. Also we obtain a generalized topology via ideals using $D^{*M}-Sets$ which is finer than micro topology μ and also μ^{*M}.

Key words : Micro topological spaces, Micro ideal topological spaces, $D^{*M}-Sets$, Perfect sets.

I. INTRODUCTION

By a space (X,τ), we mean a topological space X with a topology τ defined on X on which no separation axioms are assumed unless otherwise explicitly stated. For a given point x, the system of open neighborhood of x is denoted by $N(x)=\{U\in\tau : x\in U\}$. A non-empty collection of subsets of X is said to be an ideal on X, if it satisfies the following two conditions (i) If $A\in I$ and $B\subseteq A$, then $B\in I$, (ii) If $A\in I$ and $B\in I$, then $A\cup B\in I$. An ideal topological space (X,τ,I) means a topological space $(X,\tau,)$ with an ideal I defined on X. For any subset A of X, $A^*(I,\tau)=\{x\in X / A\cap U\notin I\}$ for every $U\in N(x)$ is called the local function of A with respect to I and τ. If there is no ambiguity, we will write $A^*(I)$ or simply A^* for $A^*(I,\tau)$. Also $cl^*(A)=A\cup A^*$ defines the Kuratowski closure operator for the topology $\tau^*(I)$ (or simply τ^*) which is finer than τ. An ideal I on (X,τ) is said to be codense ideal if and only if $\tau\cap I=\{\phi\}$.

The contribution of Hamlett and Jankovic [6] in ideal topological spaces initiated the generalization of some important properties in general topology via ideals. The properties like decomposition of continuity, separation axioms, connectedness, compactness and resolvability have been generalized using the concept of ideals in topological spaces.

The concept of nano topology was first introduced by M. Lellis Thivagar et. al. [10] , which is defined in terms of lower, upper approximations and the boundary region of a subset of a universe. The notion of approximations and boundary region of a set was proposed by Z. Pawlak in order to introduce the concept of rough set theory. M. Parimala et. al. introduced the concept of nano ideal topological spaces. In 2016, M. Lellis Thivagar and V. Sutha Devi [11] introduced some new sort of operators in nano ideal topological spaces. The set of elements of $(U, \tau_R(X), I)$ that satisfies $A \subseteq n\mathrm{int}(A_n^*)$ is called the set of Nano ideal open sets.

In 2019, S.Chandrasekar [3] introduced the concept of micro topology which is an extension of nano topology. In a nano topological space, for any $\mu \notin \tau_R(X)$, the collection $\mu_R(X) = \{N \cup (N' \cap \mu) : N, N' \in \tau_R(X)\}$ is called the micro topology on U. The triplet $(U, \tau_R(X), \mu_R(X))$ is called the micro topological space. The elements of $\mu_R(X)$ are called micro open sets and their complements are micro closed sets. We have already introduced and studied the basic properties of Micro topological spaces together with an ideal which is denoted by $(U, \tau_R(X), \mu_R(X), I)$. For our convenience we denote the local function as A^{*M} and the closure operator as $cl^{*M}(A)$.

A set A is said to be $*$ – perfect if $A^* = A$ in an ideal topological space. R.Manoharan and P.Thangavelu [12] introduced the following sets. A set A of U is said to be L^* – perfect if $A - A^{*M} \in I$, R^* – perfect if $A^{*M} - A \in I$, C^* – perfect if A is both L^* – perfect and R^* – perfect. Here we tried to introduce the sets which are the local functions of the above said perfect sets. Also we developed a topology using the sets.

II. D_1^{*M}, D_2^{*M} AND $D^{*M} - SETS$

Definition: 2.1 A subset E of an Micro ideal topological space (MI-space) $(U, \tau_R(X), \mu_R(X), I)$ is said to be

(i) μ^{*M} - closed if $E^{*M} \subseteq E$.

(ii) $*^M$ – dense in itself if $E \subseteq E^{*M}$.

(iii) MI- open if $E \subseteq M\,\mathrm{int}(E^{*M})$.

(iv) MI-dense if $E^{*M} = U$.

(v) $*^M$ – perfect if $E = E^{*M}$.

(vi) L^{*M} – perfect if $E - E^{*M} \in I$.

(vii) R^{*M} – perfect if $E^{*M} - E \in I$.

(viii) C^{*M} – perfect if E is both L^{*M} – perfect and R^{*M} – perfect.

Definition: 2.2 Let $(U, \tau_R(X), \mu_R(X), I)$ be an MI-space. A subset E of U is said to be

(i) $D_1^{*M} - Set$ if $(E - E^{*M})^{*M} \in I$

(ii) $D_2^{*M} - Set$ if $(E^{*M} - E)^{*M} \in I$

(iii) $D^{*M} - Set$ if E is both $D_1^{*M} - Set$ and $D_2^{*M} - Set$

Remark: 2.3 $D_1^{*M} - Set = \wp(U)$, because $E - E^{*M} \in I$, for all subsets of U and $(E - E^{*M})^{*M} = \phi \in I$.

Theorem: 2.4 Every $*^M$ – dense-in-itself set is $D_1^{*M} - Set$.

Proof. Let E be a $*^M$ – dense-in-itself set. Then $E \subseteq E^{*M}$. Then $E - E^{*M} = \phi$. $(E - E^{*M})^{*M} = \phi \in I$. Therefore E is $D_1^{*M} - Set$.

The following example shows that the converse of theorem-2.4 is not true.

Example: 2.5 Let $U = \{p, q, r, s, t\}$, $X = \{p, q, r, t\}$, $R(X) = \{\{p, q, r\}, \{s, t\}\}$, $\tau_R(X) = \{\phi, \{s, t\}\{p, q, r\}, U\}$.

Let $\mu = \{r, s\}$,then $\mu_R(X) = \{\phi, \{r\}, \{s\}, \{r, s\}, \{s, t\}, \{p, q, r\}, \{r, s, t\}, \{p, q, r, s\}, U\}$.

Let $I = \{\phi, \{r\}, \{s, t\}, \{r, s, t\}\}$. Then $D_1^{*M} - Set = \wp(U)$,

$$D_2^{*M} - Set = \left\{\begin{matrix} \phi, \{r\}, \{t\}, \{p, q\}, \{r, t\}, \{s, t\}, \\ \{p, q, r\}, \{p, q, t\}, \{r, s, t\}, \\ \{p, q, r, t\}, \{p, q, s, t\}, U \end{matrix}\right\} = D^{*M} - Set.$$

$*^M$ – dense-in-itself set $= \left\{\begin{matrix} \phi, \{p\}, \{q\}, \{s\}, \{t\}, \{p, q\}, \{p, s\}\{p, t\}, \\ \{q, s\}, \{q, t\}, \{p, q, s\}, \{p, q, t\} \end{matrix}\right\}$. The subset $\{p, q, r, t\}$ is $D_1^{*M} - Set$ but not $*^M$ – dense-in-itself set.

Theorem: 2.6 Every μ^{*M} - closed set is $D^{*M} - Set$.

Proof. Let E be a μ^{*M} - closed set. Then $E^{*M} \subseteq E$. Then $E^{*M} - E = \phi$. $(E^{*M} - E)^{*M} = \phi \in I$. Therefore E is $D_2^{*M} - Set$. By remark-2.3, E is $D_1^{*M} - Set$ and hence $D^{*M} - Set$.

The following example shows that the converse of theorem-2.6 is not true.

Example: 2.7 $U=\{1,2,3,4,5\}$, $U/R=\{\{1,2\},\{3\},\{4,5\}\}$, $X=\{1,2,3\}$, $\tau_R(X)=\{\phi,U,\{1,2,3\}\}$. Let $\mu=\{4\}$, then $\mu_R(X)=\{\phi,\{4\},\{1,2,3\},\{1,2,3,4\},U\}$. Let $I=\{\phi,\{1\},\{1,5\}\}$.

μ^{*M} – closed set $=\{\phi,\{1\},\{5\},\{1,5\},\{4,5\},\{1,4,5\},U\}$.

$D^{*M}-Set=\{\phi,\{1\},\{5\},\{1,5\},\{4,5\},\{2,3,4\},\{1,4,5\},\{2,3,4,5\},U\}$

$\{2,3,4\}$ is $D^{*M}-Set$ but not μ^{*M} – closed set.

Theorem: 2.8 Every MI-closed set is $D_1^{*M}-Set$.

Proof. The proof follows from remark-2.3.

The following example shows that the converse of theorem-2.8 is not true.

Example: 2.9 From example-2.5, MI-closed set $=\{\{p,q,r,t\},U\}$. The set $\{r,s,t\}$ is $D_1^{*M}-Set$ but not MI-closed set.

Theorem:2.10 Every $*^M$ – perfect set is $D^{*M}-Set$.

Proof. Let E be a $*^M$ – perfect set. Then $E=E^{*M}$. Then E is $D_1^{*M}-Set$ and $D_2^{*M}-Set$ and hence $D^{*M}-Set$.

Remark: 2.11 The converse of the above theorem need not be true, because any non-empty member of an ideal is not a $*^M$ – perfect set. That is, $E\neq E^{*M}$, $\forall E\in I$.

The following example shows that the converse of theorem 2.10 is not true.

Example: 2.12 From example-2.5, $*^M$ – perfect set $=\{\phi,\{t\},\{p,q\},\{p,q,t\}\}$. The subset $\{r,s,t\}$ is $D^{*M}-Set$ but not $*^M$ – perfect set.

Theorem: 2.13 Every member of an ideal is a $D^{*M}-Set$.

Proof. Let E be a member of an ideal, $E\in I$. $E^{*M}=\phi$. Hence $\left(E-E^{*M}\right)^{*M}=E^{*M}=\phi\in I$ and $\left(E^{*M}-E\right)^{*M}=\phi\in I$. Therefore E is both $D_1^{*M}-Set$ and $D_2^{*M}-Set$ and hence $D^{*M}-Set$.

Theorem: 2.14 Let U be an MI-space and $E\subseteq U$.

(i) A set E of U is L^{*M} – perfect iff E is $D_1^{*M}-Set$.

(ii) Every R^{*M} – perfect set is $D_2^{*M}-Set$.

(iii) Every C^{*M} - perfect set is $D^{*M}-Set$.

Proof. (i) The result is true from remark-2.3.

(ii) Let E be R^{*M} – perfect

$\Rightarrow E^{*M}-E\in I\Rightarrow\left(E^{*M}-E\right)^{*M}=\phi\in I$

$\Rightarrow$ E is $D_2^{*M}-Set$.

(iii) Let E be C^{*M} -perfect

$\Rightarrow E-E^{*M}\in I$ and $E^{*M}-E\in I$

$\Rightarrow\left(E-E^{*M}\right)^{*M}=\left(E^{*M}-E\right)^{*M}=\phi\in I$

$\Rightarrow$ E is $D^{*M}-Set$.

Theorem:2.15 Let U be an MI-space and $E\subseteq U$. If E is $D^{*M}-Set$, then $E\Delta E^{*M}\in I$.

Proof. E is $D^{*M}-Set$ $\Rightarrow$ E is C^{*M} - perfect

$\Rightarrow$ $E-E^{*M}\in I$ and $E^{*M}-E\in I$

$\Rightarrow\left(E-E^{*M}\right)\cup\left(E^{*M}-E\right)\in I\Rightarrow E\Delta E^{*M}\in I$.

Theorem: 2.16 In an MI-space U,

(i) U and ϕ are $D^{*M}-Set$.

(ii) Every $\mu-$ closed set is $D^{*M}-Set$.

(iii) $Mcl(E),E^{*M},Mcl^*(E)$ are $D^{*M}-Sets$.

Proof. (i) Obviously, ϕ is $D^{*M}-Set$. Always $U^{*M}\subseteq U$. Therefore $\left(U^{*M}-U\right)^{*M}=\phi^{*M}=\phi\in I$. And by remark:2.3, U is $D^{*M}-Set$.

(ii) Let E be a $\mu-$ closed set. Then $Mcl(E)=E$. Since μ^{*M} is finer than μ , then $Mcl^*(E)\subseteq Mcl(E)=E\Rightarrow E^{*M}\subseteq E\Rightarrow E^{*M}-E=\phi\Rightarrow\left(E^{*M}-E\right)^{*M}=\phi\in I$. Hence E is $D_2^{*M}-Set$ and from remark-2.3, E is $D_1^{*M}-Set$ and hence it is $D^{*M}-Set$.

(iii) Since μ^{*M} is finer than μ , then from theorem: 2.6, $Mcl(E),E^{*M},Mcl^*(E)$ are $D^{*M}-Sets$.

The following example shows that the converse of theorem: 2.16(ii) is not true.

Example: 2.17 From example-2.5 , $\mu-$ closed set $=\{\phi,\{t\},\{p,q\},\{s,t\},\{p,q,t\},\{p,q,s,t\},\{p,q,r,t\},U\}$. The subset $\{r\}$ is $D^{*M}-Set$ but not $\mu-$ closed set.

Theorem: 2.18 Let U be an MI-space and $E_1,E_2\subseteq U$. If E_1 and E_2 are $D_1^{*M}-Set$, then

(i) $E_1 \cup E_2$ is $D_1^{*M} - Set$.

(ii) $E_1 \cap E_2$ is $D_1^{*M} - Set$.

Proof. Since E_1 and E_2 are $D_1^{*M} - Sets$, $\left(E_1 - E_1{}^{*M}\right)^{*M} \in I$ and $\left(E_2 - E_2{}^{*M}\right)^{*M} \in I$. By finite additive property of ideals $\left(E_1 - E_1{}^{*M}\right)^{*M} \cup \left(E_2 - E_2{}^{*M}\right)^{*M} \in I$.

(i) Consider

$$(E_1 \cup E_2) - (E_1 \cup E_2)^{*M} = (E_1 \cup E_2) - \left(E_1{}^{*M} \cup E_2{}^{*M}\right)$$
$$\subseteq \left(E_1 - E_1{}^{*M}\right) \cup \left(E_2 - E_2{}^{*M}\right)$$
$$\left((E_1 \cup E_2) - (E_1 \cup E_2)^{*M}\right)^{*M}$$
$$\subseteq \left(\left(E_1 - E_1{}^{*M}\right) \cup \left(E_2 - E_2{}^{*M}\right)\right)^{*M}$$
$$= \left(E_1 - E_1{}^{*M}\right)^{*M} \cup \left(E_2 - E_2{}^{*M}\right)^{*M}.$$

Then by heredity property, $\left((E_1 \cup E_2) - (E_1 \cup E_2)^{*M}\right)^{*M} \in I$. Therefore $E_1 \cup E_2$ is $D_1^{*M} - Set$.

(ii) Consider

$$(E_1 \cap E_2) - (E_1 \cap E_2)^{*M} = (E_1 \cap E_2) - \left(E_1{}^{*M} \cap E_2{}^{*M}\right)$$
$$\subseteq \left(E_1 - E_1{}^{*M}\right) \cap \left(E_2 - E_2{}^{*M}\right) \subseteq \left(E_1 - E_1{}^{*M}\right) \cup \left(E_2 - E_2{}^{*M}\right) \therefore$$
$$\left((E_1 \cap E_2) - (E_1 \cap E_2)^{*M}\right)^{*M} \subseteq \left(\left(E_1 - E_1{}^{*M}\right) \cup \left(E_2 - E_2{}^{*M}\right)\right)^{*M}$$
$$= \left(E_1 - E_1{}^{*M}\right)^{*M} \cup \left(E_2 - E_2{}^{*M}\right)^{*M}.$$

By heredity property $\left((E_1 \cap E_2) - (E_1 \cap E_2)^{*M}\right)^{*M} \in I$. Hence $E_1 \cap E_2$ is $D_1^{*M} - Set$.

Theorem: 2.19 Let U be an MI-space and $E_1, E_2 \subseteq U$. If E_1 and E_2 are $D_2^{*M} - Set$, then

(i) $E_1 \cup E_2$ is $D_2^{*M} - Set$.

(ii) $E_1 \cap E_2$ is $D_2^{*M} - Set$.

Proof. Since E_1 and E_2 are $D_2^{*M} - Set$, $\left(E_1{}^{*M} - E_1\right)^{*M} \in I$ and $\left(E_2{}^{*M} - E_2\right)^{*M} \in I$. By finite additive property of ideals $\left(E_1{}^{*M} - E_1\right)^{*M} \cup \left(E_2{}^{*M} - E_2\right)^{*M} \in I$.

(i) Consider

$$(E_1 \cup E_2)^{*M} - (E_1 \cup E_2) = \left(E_1{}^{*M} \cup E_2{}^{*M}\right) - (E_1 \cup E_2)$$
$$\subseteq \left(E_1{}^{*M} - E_1\right) \cup \left(E_2{}^{*M} - E_2\right)$$
$$\left((E_1 \cup E_2)^{*M} - (E_1 \cup E_2)\right)^{*M}$$
$$\subseteq \left(\left(E_1{}^{*M} - E_1\right) \cup \left(E_2{}^{*M} - E_2\right)\right)^{*M}$$
$$= \left(E_1{}^{*M} - E_1\right)^{*M} \cup \left(E_2{}^{*M} - E_2\right)^{*M}$$

Then by heredity property, $\left((E_1 \cup E_2)^{*M} - (E_1 \cup E_2)\right)^{*M} \in I$. Therefore $E_1 \cup E_2$ is $D_2^{*M} - Set$.

(ii) Consider

$$(E_1 \cap E_2)^{*M} - (E_1 \cap E_2)$$
$$= \left(E_1{}^{*M} \cap E_2{}^{*M}\right) - (E_1 \cap E_2)$$
$$\subseteq \left(E_1{}^{*M} - E_1\right) \cap \left(E_2{}^{*M} - E_2\right)$$
$$\subseteq \left(E_1{}^{*M} - E_1\right) \cup \left(E_2{}^{*M} - E_2\right)$$
$$\left((E_1 \cap E_2)^{*M} - (E_1 \cap E_2)\right)^{*M}$$
$$\therefore \subseteq \left(\left(E_1{}^{*M} - E_1\right) \cup \left(E_2{}^{*M} - E_2\right)\right)^{*M}$$
$$= \left(E_1{}^{*M} - E_1\right)^{*M} \cup \left(E_2{}^{*M} - E_2\right)^{*M}$$

By heredity property $\left((E_1 \cap E_2)^{*M} - (E_1 \cap E_2)\right)^{*M} \in I$. Hence $E_1 \cap E_2$ is $D_2^{*M} - Set$.

Remark: 2.20 From theorems 2.16, 2.18, 2.19 and from a fact that arbitrary union of closed sets need not be closed, we can prove that

(i) finite union and intersection of $D_1^{*M} - Sets$ is $D_1^{*M} - Set$.

(ii) finite union and intersection of $D_2^{*M} - Sets$ is $D_2^{*M} - Set$.

(iii) finite union and intersection of $D^{*M} - Sets$ is $D^{*M} - Set$.

Theorem: 2.21 Let U be an MI-space and $E \subseteq U$.

(i) If E is L^{*M} – perfect, then $E - E^{*M}$ is $D^{*M} - Set$.

(ii) If E is R^{*M} – perfect, then $E^{*M} - E$ is $D^{*M} - Set$.

(iii) If E is C^{*M} – perfect, then $E \Delta E^{*M}$ is $D^{*M} - Set$.

Proof. (i) E is $L^{*M}-$ perfect, then $E-E^{*M}\in I$. This implies $\left(E-E^{*M}\right)^{*M}=\phi\in I$. Now consider

$$\left(\left(E-E^{*M}\right)^{*M}-\left(E-E^{*M}\right)\right)^{*M}$$
$$=\left(\phi-\left(E-E^{*M}\right)\right)^{*M}$$
$$=\phi^{*M}=\phi\in I.$$

$E-E^{*M}$ is $D_2^{*M}-Set$. And $E-E^{*M}$ is a subset of U. Then by Remark: 2.3, $E-E^{*M}$ is $D^{*M}-Set$.

Similarly we can prove (ii)

(iii) E is $C^{*M}-$ perfect, then $E-E^{*M}\in I$ and $E^{*M}-E\in I$. This gives $\left(E-E^{*M}\right)^{*M}=\phi\in I$ and $\left(E^{*M}-E\right)^{*M}=\phi\in I$.

Then $E\Delta E^{*M}=\left(E-E^{*M}\right)\cup\left(E^{*M}-E\right)\in I$.

$$\left(E\Delta E^{*M}\right)^{*M}=\left(\left(E-E^{*M}\right)\cup\left(E^{*M}-E\right)\right)^{*M}=\phi.$$

$$\left(E\Delta E^{*M}\right)-\left(E\Delta E^{*M}\right)^{*M}$$
$$=\left(\left(E-E^{*M}\right)\cup\left(E^{*M}-E\right)\right)-\left(\left(E-E^{*M}\right)\cup\left(E^{*M}-E\right)\right)^{*M}$$
$$=\left(\left(E-E^{*M}\right)\cup\left(E^{*M}-E\right)\right)\in I.$$

$$\left(E\Delta E^{*M}\right)^{*M}-\left(E\Delta E^{*M}\right)$$
$$=\left(\left(E-E^{*M}\right)\cup\left(E^{*M}-E\right)\right)^{*M}-\left(\left(E-E^{*M}\right)\cup\left(E^{*M}-E\right)\right)$$
$$=\phi\in I.$$

$\therefore E\Delta E^{*M}$ is $D^{*M}-Set$.

Theorem: 2.22 In an MI-space U,

(i) Every MI-dense set is $D_1^{*M}-Set$.

(ii) Every MI- open set is $D_1^{*M}-Set$.

Proof. Since the above sets are $*^{M}-$ dense-in-itself, by theorem-2.4, these sets are $D_1^{*M}-Set$.

The following example shows that the converses of the above theorem are not true.

Example: 2.23 Let $U=\{1,2,3,4\}$, $X=\{2,3,4\}$, $U/R(X)=\{\{1,2\},\{2,3\},\{1,3\},\{3,4\},\{1,4\}\}$, $\tau_R(X)=\{\phi,\{1\},\{2,3,4\},U\}$. Let $\mu=\{2,4\}$, then $\mu_R(X)=\{\phi,\{1\},\{2,4\},\{1,2,4\},\{2,3,4\},U\}$ also let $I=\{\phi\}$.

(i) $\{3,4\}$ is $D_1^{*M}-Set$ but not MI-dense set.

(ii) $\{1,2\}$ is $D_1^{*M}-Set$ but not MI-open set.

Remark: 2.24 In ideal spaces usually $E\subset F$ implies $E^{*M}\subset F^{*M}$. In some cases $E\subset F\Rightarrow E^{*M}=F^{*M}$.

Example: 2.25 Let $U=\{1,2,3,4\}$, $U/R(X)=\{\{1,2\},\{2,3\},\{1,3\},\{3,4\},\{1,4\}\}$, $X=\{2,3,4\}$, $\tau_R(X)=\{\phi,\{1\},\{2,3,4\},U\}$. Let $\mu=\{2,4\}$, then $\mu_R(X)=\{\phi,\{1\},\{2,4\},\{1,2,4\},\{2,3,4\},U\}$ also let $I=\{\phi,\{1\}\}$. For $E=\{2\}$ and $F=\{2,3\}$, $E^{*M}=F^{*M}=\{2,3,4\}$. Here $E\subset F$ and $E^{*M}=F^{*M}$.

Theorem: 2.26 Let U be an MI-space. Let E and F be two subsets of U such that $E\subseteq F$ and $E^{*M}=F^{*M}$, then

(i) F is $D_2^{*M}-Set$, if E is $D_2^{*M}-Set$.

(ii) E is $D_1^{*M}-Set$, if F is $D_1^{*M}-Set$.

Proof. (i) Let E be $D_2^{*M}-Set$, $\left(E^{*M}-E\right)^{*M}\in I$.

Now $F^{*M}-F=E^{*M}-F\subseteq E^{*M}-E$.

Then $\left(F^{*M}-F\right)^{*M}\subseteq\left(E^{*M}-E\right)^{*M}\in I$.

Then F is $D_2^{*M}-Set$.

(ii) Let F be $D_1^{*M}-Set$, $\left(F-F^{*M}\right)^{*M}\in I$.

Now $E-E^{*M}=E-F^{*M}\subseteq F-F^{*M}$.

Then $\left(E-E^{*M}\right)^{*M}\subseteq\left(F-F^{*M}\right)^{*M}\in I$.

Then E is $D_1^{*M}-Set$.

Theorem: 2.27 Let E be a subset of an MI-space U such that E is $L^{*M}-$ perfect set and $E\cap E^{*M}$ is $R^{*M}-$ perfect set, then E is $D_2^{*M}-Set$ and $E\cap E^{*M}$ is $D_1^{*M}-Set$.

Proof. Since E is $L^{*M}-$ perfect set, $E-E^{*M}\in I$. For every $J\in I$, $(E\cup J)^{*M}=E^{*M}=(E-J)^{*M}$. Therefore $\left(E\cup\left(E-E^{*M}\right)\right)^{*M}=E^{*M}=\left(E-\left(E-E^{*M}\right)\right)^{*M}$. This implies $E^{*M}=\left(E\cap E^{*M}\right)^{*M}$. Therefore we have $E\cap E^{*M}\subseteq E$ with $\left(E\cap E^{*M}\right)^{*M}=E^{*M}$. And $E\cap E^{*M}$ is $R^{*M}-$ perfect, then it is $D_2^{*M}-Set$. Also since E is $L^{*M}-$ perfect, it is $D_1^{*M}-Set$. By theorem-2.26, E is $D_2^{*M}-Set$ since $E\cap E^{*M}$ is $D_2^{*M}-Set$ and $E\cap E^{*M}$ is $D_1^{*M}-Set$ since E is $D_1^{*M}-Set$.

Theorem: 2.28 Let E be a subset of an MI-space U such that E is $R^{*M}-$ perfect set and E^{*M} is $L^{*M}-$ perfect set, then $E\cap E^{*M}$ is $D_1^{*M}-Set$.

Proof. Since E is R^{*M} – perfect set, $E^{*M}-E\in I$. For every $J\in I$, $(E\cup J)^{*M}=E^{*M}=(E-J)^{*M}$. Therefore $\left(E^{*M}\cup\left(E^{*M}-E\right)\right)^{*M}=\left(E^{*M}\right)^{*M}=\left(E^{*M}-\left(E^{*M}-E\right)\right)^{*M}$. This implies $\left(E^{*M}\right)^{*M}=\left(E\cap E^{*M}\right)^{*M}$. Therefore we have $E\cap E^{*M}\subseteq E^{*M}$ with $\left(E\cap E^{*M}\right)^{*M}=\left(E^{*M}\right)^{*M}$. And E^{*M} is L^{*M} – perfect, then it is $D_1^{*M}-Set$. By theorem-2.26, $E\cap E^{*M}$ is $D_1^{*M}-Set$ since E^{*M} is $D_1^{*M}-Set$.

Theorem: 2.29 If U is an MI-space with U being finite, then the collection of $D^{*M}-Sets$ is a topology which is finer than the topologies of μ^{*M} – closed sets and μ – closed sets.

Proof. By theorem- 2.16 (i), ϕ and U are $D^{*M}-Set$. By remark – 2.20 , finite union of $D^{*M}-Sets$ is $D^{*M}-Set$ and finite intersection of $D^{*M}-Sets$ is $D^{*M}-Set$. Hence the collection of $D^{*M}-Sets$ is a topology if U is finite. By theorems – 2.6 and 2.16(ii), μ^{*M} – closed sets and μ – closed sets are $D^{*M}-Sets$. Hence the topology of $D^{*M}-Sets$ is finer than the topologies of μ^{*M} – closed sets and μ – closed sets.

Theorem: 2.30 In an MI-space U, { μ^{*M} – closed sets} ∪ I $\subseteq\left\{D^{*M}-Sets\right\}$.

Proof. The proof follows from theorem- 2.6 and 2.12. The following example shows that { μ^{*M} – closed sets} ∪ I $\neq D^{*M}-Sets$.

Example: 2.31 $U=\{1,2,3,4,5\}$, $U/R=\{\{1,2\},\{3\},\{4,5\}\}$, $X=\{1,2,3\}$, $\tau_R(X)=\{\phi,U,\{1,2,3\}\}$. Let $\mu=\{4\}$, then $\mu_R(X)=\{\phi,\{4\},\{1,2,3\},\{1,2,3,4\},U\}$. Let $I=\{\phi,\{1\},\{1,5\}\}$.
μ^{*M} – closed set $=\{\phi,\{1\},\{5\},\{1,5\},\{4,5\},\{1,4,5\},U\}$.
$D^{*M}-Set=\{\phi,\{1\},\{5\},\{1,5\},\{4,5\},\{2,3,4\},\{1,4,5\},\{2,3,4,5\},U\}$.
Clearly { μ^{*M} – closed sets} ∪ I $\subseteq\left\{D^{*M}-Sets\right\}$.

Theorem: 2.32 Let U be an MI- space and $E\subseteq U$. The set E is $D_2^{*M}-Set$ if $F\subseteq\left(E^{*M}-E\right)^{*M}$ in U implies that $F\in I$.

Proof. Assume that E is a $D_2^{*M}-Set$. Then $\left(E^{*M}E\right)^{*M}\in I$. By heredity property $F\subseteq\left(E^{*M}-E\right)^{*M}\in I$.

Theorem: 2.33 Let U be an MI- space and $E\subseteq U$. The set E is $D_1^{*M}-Set$ if $F\subseteq\left(E-E^{*M}\right)^{*M}$ in U implies that $F\in I$.

Proof. Assume that E is a $D_1^{*M}-Set$. Then $\left(E-E^{*M}\right)^{*M}\in I$. By heredity property $F\subseteq\left(E-E^{*M}\right)^{*M}\in I$.

Theorem: 2.34 Let $(U,\tau_R(X),\mu_R(X))$ be an MI-space and $E\subseteq U$. Let I_1 and I_2 be two ideals on U with $I_1\subseteq I_2$. Then E is $D_2^{*M}-Set$ with respect to I_2 if it is $D_2^{*M}-Set$ with respect to I_1.

Proof. Since $I_1\subseteq I_2$, $E^{*M}(I_2)\subseteq E^{*M}(I_1)$. Let E be $D_2^{*M}-Set$ with respect to I_1 . Then $\left(E^{*M}(I_1)-E\right)^{*M}\in I_1$. Also
$E^{*M}(I_2)-E\subseteq E^{*M}(I_1)-E$
$\Rightarrow\left(E^{*M}(I_2)-E\right)^{*M}\subseteq\left(E^{*M}(I_1)-E\right)^{*M}$
Hence by heredity property of ideals, $\left(E^{*M}(I_2)-E\right)^{*M}\in I_1\subseteq I_2$. Therefore E is $D_2^{*M}-Set$ with respect to I_2.

III. D^{*M} – TOPOLOGY

By theorem-2.29, we observe that the collection of D^{*M} – sets satisfies the condition of being a basis for some topology and it will be called as $D_C{}^{*M}(\mu,I)$.

We define $D^{*M}(\mu,I)=\left\{E\subseteq U/U-E\in D_C{}^{*M}(\mu,I)\right\}$ on a nonempty set U. Clearly $D^{*M}(\mu,I)$ is a topology if the set U is finite. The members of this collection will be called D^{*M} – open sets. We call this set as D^{*M} – topology. The complement of D^{*M} – open sets are called D^{*M} – closed sets.

Definition: 3.1 Let E be a subset of an MI-space U. The union of D^{*M} – open sets contained in E is called the D^{*M} – interior of E, denoted by $D^{*M}-\mathrm{int}(E)$ and the intersection of D^{*M} – closed sets containing E is called the D^{*M} – closure of E, denoted by $D^{*M}-cl(E)$.

Theorem: 3.2 In an MI-space, a μ^{*M} – open set is an D^{*M} – open set.

Proof. Let E be a μ^{*M} – open set. Then U-E is a μ^{*M} – closed set. This implies that U-E is a D^{*M} – closed set. Hence E is a D^{*M} – open set.

Corollary: 3.3 The topology $D^{*M}(\mu, I)$ on a finite set U is finer than the topology $\mu^{*M}(\mu, I)$.

Remark: 3.4 Since every $\mu -$ open set is D^{*M} – open set, we have $M\,\text{int}(E) \subseteq M\,\text{int}^*(E) \subseteq D^{*M} - \text{int}(E)$.

Theorem: 3.5 Let E and F be subsets of an MI-space U with U being finite. Then the following properties hold.

(i) E is D^{*M} – open if and only if $E = D^{*M} - \text{int}(E)$.

(ii) $D^{*M} - \text{int}(D^{*M} - \text{int}(E)) = D^{*M} - \text{int}(E)$.

(iii) If $E \subseteq F$, then $D^{*M} - \text{int}(E) \subseteq D^{*M} - \text{int}(F)$.

(iv) E is D^{*M} – closed if and only if $E = D^{*M} - cl(E)$.

(v) $D^{*M} - cl(D^{*M} - cl(E)) = D^{*M} - cl(E)$.

(vi) If $E \subseteq F$, then $D^{*M} - cl(E) \subseteq D^{*M} - cl(F)$.

(vii) $D^{*M} - \text{int}(U - E) = U - D^{*M} - cl(E)$.

(viii) $D^{*M} - cl(U - E) = U - D^{*M} - \text{int}(E)$.

Proof. The proof follows from the definition -3.1.

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