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Freely Propagation of Conversing Cylindrical Shock Waves in A Dusty

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ABSTRACT

Neglecting the effect of overtaking disturbances, the cylindrical shock waves in a dusty gas has been studied by Chester- Chisnell- Whitham (CCW) Method. The variation of flow variables with distance in perturbed medium are obtained. Dusty gas i.e., mixture of small solid particles and gas, is assumed to be uniform. The results obtained here are compared with earlier results.

I. INTRODUCTION

The study of shock propagation in a mixture of gas and finely solid particles is of great interest in many area of research, especially in context of star formation. The propagation of strong shock waves in non-uniform medium has been studied by Christer and Helliwell(1), Verma(2) and many other. Hayers (13), Ray and Bhowmick (6), Verma and Vishwakarma (3) have studied the propagation of plane shock wave in a medium having exponentially increasing density distribution. Recently Vishwakarma (9) generalized Ray and Bhowmick (6) solution in gas to the mixture of gas and small solid particles. In a very recent communication, Yadav et al (2005), studied the Spherical shock in a dusty gas.

The aim of the present paper is to extend CCW (4,7,12) method to the case of two phase flow of a mixture of small solid particles and gas. It is assumed that solid particles are uniform disturbed in the gas. The density of solid particles is very high and the volume occupied by them is very-very small. The mixture as a whole is considered to be uniform. Neglecting the effect of overtaking disturbances on the propagation of shock, analytical relations for shock velocity and shock strength have been obtained. The expressions for the pressure, the flow velocity behind the shock have been derived. The variation of flow variables with propagation distance (r), mass concentration of solid particles in the mixture (Z) and G, the ratio of the density of solid particles to the density of gas are computed and discussed through tables.

It is found that flow variables decrease as shock advances in the dusty gas. As mass concentration of solid particles (Z) increases, flow variables decreases in G, very small variables in flow variables is observed.

BOUNDARY CONDITIONS AND BASIC EQUATIONS :

The fundamental equations for one dimensional and unsteady flow of a mixture of gas and small solid particles can be written as –

$$\frac{\partial u}{\partial t} + \frac{u\,\partial u}{\partial r} + \frac{1}{\rho} \cdot \frac{\partial \rho}{\partial r} + = 0 \qquad \text{........... (1)}$$

$$\frac{\partial \rho}{\partial t} + u\,\frac{\partial P}{\partial r} + \rho\,\frac{\partial u}{\partial r} + \frac{i\,\rho\,u}{r} + = 0 \qquad \text{........... (2)}$$

$$\frac{\partial U_m}{\partial t} + u\,\frac{\partial U_m}{\partial r} - \frac{P}{P^2}\left(\frac{\partial \rho}{\partial t} + u\frac{\partial \rho}{\partial r}\right) = 0 \qquad \text{........... (3)}$$

Where, I = 0, 1 and 2 correspond to plane Cylindrical or Spherical Symmetry,

ρ is the Density of Mixture

U the flow velocity

P the pressure of mixture

U_m the internal energy per unit mass of the mixture

r the distance and

t the time

The equation of state of the mixture of gas and small solid particles can be written as (Pat et al [10])

$$P = \frac{(1-kp)}{1-Z}\rho R^* T \qquad \text{............(4)}$$

Where R^* is the gas constant, T the temperature, k_p the mass concentration of solid particles and Z the volume fraction of solid particles in the mixture.

The relation between k_p and Z is given by

$$k_p = \frac{Z\rho_{sp}}{\rho}$$

Where, ρ_{sp} is the species density of solid particles. In equilibrium flow, k_p is a constant in the whole flow field.

The jump conditions across the shock wave are as :

$$U_2 = (1 - \beta)U \qquad \text{............(5)}$$

$$\rho_2 = \frac{\rho_1}{\beta} \qquad \text{............(6)}$$

$$P_2 = (1 - \beta)\rho_1 U^2 \qquad \text{............(7)}$$

$$Z_2 = \frac{Z_1}{\beta} \qquad \text{............(8)}$$

Here, suffixes "1" and "2" refer to the values just ahead and just behind of the shock , U =dR/dt is the shock velocity , and R the distance of the shock front from the plane, the line or the point of symmetry, the quantity b is given by –

$$\beta = \frac{\Gamma + 2Z_1 - 1}{\Gamma + 1}$$

The initial volume fraction of the particles Z1 is, in general not a constant. But the volume occupied by the solid particles is very small because the density of the solid particles is much larger than that of the gas (Miura and Glass[11]) hence Z_1 may be assumed as a small constant. The expression for Z_1 is (Naidu et al [12]).

$$Z1 = \frac{k_p}{G\,(1-k_p) + k_p}$$

Where $G = \rho_{sp}/\rho_{gI}$, the ratio of the density of solid particles to the density of gas. Values of Z1 for some typical values of k_p and G are given in table 1

TABLE-1

Value of Z1 for some typical value of kp and G

k_p	G	Z_1
0.1	50	0.00222
	100	0.00111

	200	0.00056
	500	0.00022
0.2	50	0.00498
	100	0.00249
	200	0.00125
	500	0.00049
0.4	50	0.01316
	100	0.00662
	200	0.00332
	500	0.00133
0.6	50	0.02912
	100	0.01470
	200	0.00744
	500	0.00299

ANALYTICAL RELATION FOR SHOCK STRENGTH AND SHOCK VELOCITY- For cylindrical diverging shock waves, the characteristic form of system of equations (1) – (3) i.e. the form in which equation contains derivatives in only one direction in (r, t) plane, is

$$dP + \rho a du + \rho a^2 \frac{2u}{u-a} \cdot \frac{dr}{r} = 0 \qquad \text{.....(9)}$$

Now substituting shock conditions into this relation, we get, after simplification.

$$\frac{dU}{U} + \frac{1}{[\left(3-2Z+2\sqrt{\frac{\beta(1-Z)}{(1-\beta)}}\right)*\left(\sqrt{\frac{(1-Z)(1-\beta)}{\beta}}-1\right)]}\frac{dr}{r} = 0 \qquad \text{.....(10)}$$

Which gives the relation for shock velocity

$$U = kr^{-1}/[\left(3-2Z+2\sqrt{\frac{\beta(1-Z)}{(1-\beta)}}\right)*\left(\sqrt{\frac{(1-Z)(1-\beta)}{\beta}}-1\right)] \qquad \text{.....(11)}$$

Where, k is constant of integration.

Expression for shock strength can be written as

$$U = \frac{k}{\sqrt{(p_o/\rho_o)(1-Z)}} r^{-1/}[\left(3-2Z+2\sqrt{\frac{\beta(1-Z)}{(1-\beta)}}\right)*\left(\sqrt{\frac{(1-Z)(1-\beta)}{\beta}}-1\right)]$$

i.e.
$$\frac{U}{a_o} = k^I(1-Z)^{1/2} r^{-1/}[\left(3-2Z+2\sqrt{\frac{\beta(1-Z)}{(1-\beta)}}\right)*\left(\sqrt{\frac{(1-Z)(1-\beta)}{\beta}}-1\right)] \qquad \text{.....(12)}$$

Where, $a_o = \sqrt{(p_o/\rho_o)(1-Z)}$

and $k' = k\sqrt{(p_o/\rho_o)}$

RESULT AND DISCUSSION : Expression (11) and (12) representing shock velocity and shock strength, show their dependence on r, k_p, g and Z. Expression (12) is used for computation. Initially, taking U/a_o = 9.49 at r = 5 for k_p = 0.1, G= 50, Z = 0.00222, variation of shck strength has been computed and presented in the table (II). It is found that shock strength decreases as shock advances in the mixture of gas and solid particles. Shocking strength profile with k_p and G is shown in the table (V). As mass concentration of solid particles kp increases, shock strength decreases, where as it increases with specific density of solid particles G.

Finally, the expressions for the non-dimensional pressure and flow velocity behind the shock are :

$$\frac{p}{p_o} = k'^{1}\frac{(1-\beta)}{(1-Z)}(1\text{-}Z)\, r^{-1/}\left[\left(3-2Z+2\sqrt{\frac{\beta\,(1-Z)}{(1-\beta)}}\right) * \left(\sqrt{\frac{(1-Z)(1-\beta)}{\beta}}-1\right)\right] \quad .....(13)$$

$$\frac{u}{p_o} = (1\text{-}\beta)k'\sqrt{(1-Z)}\, r^{-1/}\left[\left(3-2Z+2\sqrt{\frac{\beta\,(1-Z)}{(1-\beta)}}\right) * \left(\sqrt{\frac{(1-Z)(1-\beta)}{\beta}}-1\right)\right] \quad .....(14)$$

The profiles of pressure and fluid velocity are given in the table (III) & (IV). It is concluded that both pressure and fluid velocity decrease with propagation distance r. These variables decrease with increase in k_p and increase with G. (r.f. Table). Increase in G means increase in the specific density of solid particles. Therefore an increase in specific density of solid particles leads to an increase in shock strength as well as flow variables. Similar results are reported by Vishwakarma [3] using, similarly solutions of the problem for non-uniform medium.

Thus, the concentration of solid particles in gas has considerable effect on the motion of shock as well as on the flow variables.

TABLE-II. Variation of Shock strength with fraction of solid particles in the mixture (Z) and the propagation distance (r)

R	U/a_o $Z_1 = 0.0022$	$Z_2 = 0.0011$	$Z_3 = 0.0056$	$Z_4 = 0.00022$
5	9.4978	9.5062	9.5127	9.5024
6	9.0566	9.0647	9.0637	9.0632
7	8.6158	8.6243	8.6264	8.6275
8	8.3256	8.3425	8.3266	8.3525
9	8.0827	8.0526	9.0934	8.0965
10	7.8553	7.8635	7.9125	7.8798

TABLE-III. Variation of pressure with strength with volume fraction of solid particles in the mixture (Z) and the propagation distance (r)

R	p/p_o $Z_1 = 0.0022$	$Z_2 = 0.0011$	$Z_3 = 0.0056$	$Z_4 = 0.00022$
5	85.8143	85.8759	85.9445	85.9525
6	78.3474	78.4213	78.3540	78.3013
7	71.4283	71.4825	71.6831	71.4752
8	67.4331	68.6014	67.5362	66.7956
9	62.6873	63.7546	62.7413	62.5334
10	60.1465	60.2765	60.3215	60.2413

TABLE-IV. Variation of particles strength with volume fraction of solid particles in the mixture (Z) and the propagation distance (r)

R	u/a_o $Z_1 = 0.0022$	$Z_2 = 0.0011$	$Z_3 = 0.0056$	$Z_4 = 0.00022$
5	9.1239	9.1214	9.1132	9.1364
6	8.5643	8.5621	8.5645	8.5643
7	8.2513	8.1989	8.1372	8.1926
8	8.0017	8.0125	8.0347	8.1732
9	7.6541	7.6714	8.6939	8.6852
10	7.4557	7.5832	8.6435	8.5943

TABLE-V. Variation of shock strength with mass concentration of solid particles (k_p) and Ratio of density of solid particles to the density of gas (G)

G	U/a_o $k_p = 0.1$	$k_p = 0.2$	$k_p = 0.4$	$k_p = 0.6$
50	9.5632	9.4691	9.4315	9.2931
100	9.6031	9.4753	9.5021	9.4203
200	9.6743	9.5152	9.5065	9.4715

TABLE-VI. Variation of pressure with mass concentration of solid particles (k_p) and Ratio of density of solid particles to the density of gas (G)

G	p/p_o $k_p = 0.1$	$k_p = 0.2$	$k_p = 0.4$	$k_p = 0.6$
50	85.9241	85.7721	85.4602	84.7923
100	85.9637	85.7812	85.6585	85.4362
200	85.9739	85.9013	85.7678	85.6853

TABLE-VII. Variation of particle velocity with mass concentration of solid particles (k_p) and Ratio of density of solid particles to the density of gas (G)

G	u/a_o $k_p = 0.1$	$k_p = 0.2$	$k_p = 0.4$	$k_p = 0.6$
50	9.0349	9.0061	8.9531	8.8417
100	9.0428	9.0231	8.9945	8.9315
200	9.0432	9.0316	9.0179	8.9891

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