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Spherical Shock Propagating in Non- Uniform Atmosphere Having Constant Angular Velocity

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ABSTRACT : The aim of the present Paper is to study the strong spherical shock propagation in a medium having constant solid body rotation. have studied the propagation of shock waves in non-uniform medium With CCW method. have investigated the influence of the initial density distributions ρo ∝ evr, relations for shock velocity and shock strength in absence as well as in presence of overtaking disturbances have been derived flow variables.

I. Introduction

Introduction- Being an important phenomenon of present data research, the for Information and propagation of shock waves is are received considerable attention of many workers Using different techniques, many authors have studied the propagation of shock waves in uniform and non-uniform medium Whitham (Characteristic method). Chaturani (Similarity method, Kumar and Prakash (CCW method, without effect of overtaking disturbances) leave investigated the influence of the initial density distributions.

The rotation of the fluid introduces two effects, viz. (i) initial density distribution and (ii) coriolis force, with the application of modified similarity method. Kumar et.al.(1978) have obtained numerical solutions including both of these effect Kumar and Chaturani(1980) used characteristics lt method to study the propagation of spherical shock wave through a rotating has and explained the behavior of shock variation with the advancement of shock most of the above mentioned researcher include the effect in the motion of shock waves Significance and importance of overtaking disturbances in the propagation of shock waves has been given by Yousaf (1985) and Yadav (1992), Tripathi (1995) used CCW method to study the propagation of weak cylindrical shock in a rotating gas and included the effect of overtaking disturbances on the shock motion.

Basic Equation

The equations governing the flow of gas enclosed by the shock front are

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial r} + \frac{1}{\rho}\frac{\partial p}{\partial r} - \frac{v^2}{r} = 0 \tag{1-a}$$

$$\left(\frac{\partial}{\partial t} + u\frac{\partial}{\partial r}\right)(vr) = 0 \tag{1-b}$$

$$\frac{\partial \rho}{\partial t} + u\frac{\partial \rho}{\partial r} + \rho\left(\frac{\partial u}{\partial r} + \frac{\alpha u}{r}\right) = 0 \tag{1-c}$$

$$\left(\frac{\partial}{\partial t} + u\frac{\partial}{\partial t}\right)(p\rho^{-\gamma}) = 0 \tag{1-d}$$

where, r is the radial co-ordinate, u (r, t), p (r, t) and p(r, t)are respectively, the particle velocity, pressure and density at distance r from the origin at time t and γ is the specific heat ratio of the gas while v is the radial component of velocity[α=0,1,2 for plane, cylindrical and spherical symmetries respectively]

Boundary Condition- If, the boundary condition reduced by for strong shock

$$p = \frac{2\rho_0}{\gamma+1}U^2$$

$$u = \frac{2U}{\gamma+1}$$

$$\rho = \rho_0\left(\frac{\gamma+1}{\gamma-1}\right) \tag{2}$$

$$v = v_0$$

and $$a = sU\left(\frac{\gamma-1}{\gamma+1}\right), \quad \text{where,} \quad s = \left(\frac{2\gamma}{\gamma-1}\right)^{\frac{1}{2}}$$

Theory- The characteristic form of the system of the basic equation (1.a to 1-d) is,

$$dp + \rho a du + \frac{\alpha\rho a^2 u}{(u+a)}\frac{dr}{r} - \frac{\rho a v^2}{(u+a)}\frac{dr}{r} = 0 \tag{3}$$

The equilibrium of the gas in assumed to specified by the conditions $\frac{\partial}{\partial t} = 0 = u$ and $p = p_0, v = v_0 = r\,\Omega_0$ as the consequence of hydrostatic equilibrium prevailing in front of the shock

$$\frac{1}{\rho_0}\frac{dp_0}{dr} - \frac{v_0^2}{r} = 0 \quad \Rightarrow \quad \therefore dp_0 = \rho_0 v_0^2 dr / r \tag{4}$$

For initial density disturbance $\rho_0=\rho' e^{\nu r}$, equation (4) reduces to,

$$dp_0 = \rho' e^{\nu r} v_0^2 dr / r \tag{5}$$

$$p_0 = K + \frac{\rho' \Omega_0^2 e^{\nu r}}{\nu^2}(\nu r - 1) \tag{6}$$

where, K is the constant of integration here a_0 is taken as constant sound velocity in unperturbed medium.

$$\because \quad a_0^2 = \gamma p_0/\rho_0$$

$$\therefore \quad a_0^2 = \left[\frac{\gamma K}{\rho'} e^{-\nu r} + \frac{\gamma \Omega_0^2}{\nu^2}(\nu r - 1)\right]$$

Strong Shock in absence of overtaking disturbance (SSAOD)

Substituting condition (2) in equation (3) and simplifying we get,

SHOCK VELOCITY

$$U^2 = r^{-\alpha A_\gamma} e^{-B_\gamma r}\left(K + C_\gamma \Omega_0^2 f_1(r,\gamma)\right) \tag{7}$$

where, $f_1(r,\gamma) = \int r^{\alpha A_\gamma + 1} e^{B_\gamma r} dr$

$$A\gamma = \frac{4\gamma}{(2+s)\left[2 + \{2\gamma(\gamma-1)\}^{1/2}\right]}, \qquad B_\gamma = \frac{2\nu}{2+s}$$

$$C\gamma = \frac{(\gamma+1)^2 s}{(2+s)\left[2 + \{2\gamma(\gamma-1)\}^{1/2}\right]}, \qquad s = \left(\frac{2\gamma}{\gamma-1}\right)^{1/2}$$

SHOCK STRENGTH

$$\left(\frac{U}{a_0}\right)^2 = \left[r^{-\alpha A\gamma} e^{-B_\gamma r}\left\{K + C_\gamma \Omega_0^2 f_1(r,\gamma)\right\} \Big/ \left\{\frac{\gamma K}{\rho'} e^{-\nu r} + \frac{\gamma \Omega_0^2}{\nu^2}(\nu r - 1)\right\}\right] \tag{8}$$

Strong Shock in propagation on presence of overtaking disturbance (SSPOD)

For overtaking disturbances, we have taken differential equation

$$dp - \rho a du + \frac{\alpha \rho a^2 u}{(u-a)}\frac{dr}{r} + \frac{\rho a v^2}{(u-a)}\frac{dr}{r} = 0 \qquad (9)$$

With the help of (2) and equation (9), We have

$$U^2 = r^{-\gamma D_\gamma} e^{-E_\gamma r}\left\{K - F_\gamma \Omega_0^2 f_2(r,\gamma)\right\} \qquad (10)$$

where, $f_2(r,\gamma) = \int r^{\alpha D_\gamma + 1} e^{E_\gamma r} dr$, $\quad E_\gamma = \frac{2}{(2-s)}$

$$D_\gamma = \frac{4\gamma}{(2-s)\left[2 - \{2\gamma(\gamma-1)\}^{1/2}\right]}, \qquad F_\gamma = \frac{(\gamma+1)^2 s}{\left[2 - \{2\gamma(\gamma-1)\}^{1/2}\right]}$$

Now from the equation (2)

$$u = \frac{2U}{\gamma+1}, \quad \therefore du_+ = \frac{2dU}{\gamma+1}$$

$$\therefore dU = \frac{\gamma+1}{2} du_+$$

Substituting these values in equation (7) and simplifying, we get

$$du_+ = \frac{2}{\gamma+1}\frac{1}{2U}\frac{d}{dr}\left[r^{-\alpha A_\gamma} e^{-B_\gamma r}\left(K + C_\gamma \Omega_0^2 f_1(r,\gamma)\right)\right]$$

For C_ disturbance generated by the shock, the fluid velocity increment may be expressed as

$$u_- = \frac{2U}{\gamma+1}$$

$$du_- = \frac{2dU}{\gamma+1}$$

Similarly, velocity increment for overtaking disturbances can be obtained

$$du_- = \frac{2}{\gamma+1}\frac{1}{2U}\frac{d}{dr}\left[r^{-\alpha D_\gamma} e^{-E_\gamma r}\left(K - F_\gamma \Omega_0^2 f_2(r,\gamma)\right)\right]$$

For exponential and logarithmic density distribution in presence of overtaking disturbances the fluid velocity increment, will be

$$du_+ + du_- = \frac{2}{\gamma+1} dU^* \tag{11}$$

Substituting respective values and simplifying, we get

SHOCK VELOCITY

$$U^{*2} = \Big[\left(1 + r^{-\alpha A_\gamma} e^{-B_\gamma r} + r^{-\alpha D_\gamma} e^{-E_\gamma r}\right)K + C_\gamma \Omega_0^2 r^{-\alpha A_\gamma} e^{-B_\gamma r} f_1(r,\gamma) - F_\gamma \Omega_0^2 r^{-\alpha D_\gamma} e^{-E_\gamma r} f_2(r,\gamma)\Big] \tag{12}$$

Therefore, the expression for shock strength in presence of overtaking disturbances, can be written as

$$\left(\frac{U^*}{a_0}\right)^2 = \Big[\Big\{\left(1 + r^{-\alpha A_\gamma} e^{-B_\gamma r} + r^{-\alpha D_\gamma} e^{-E_\gamma r}\right)K + C_\gamma \Omega_0^2 r^{-\alpha A_\gamma} e^{-B_\gamma r} f_1(r,\gamma) - F_\gamma \Omega_0^2 r^{-\alpha D_\gamma} e^{-E_\gamma r} f_2(r,\gamma)\Big\} \Big/ \left\{\frac{\gamma K}{\rho'} e^{-\nu r} + \frac{\gamma \Omega_0^2}{\nu^2}(\nu r - 1)\right\}\Big] \tag{13}$$

Result & Discussion- The expressions (7 & 12) is obtained for the shock velocity for freely propagating strong spherical shock in a rotating gas and under the influence of overtaking disturbances. The expressions (8 & 13) is the corresponding relations for shock strength. Initially, taking U/a_0=15 at r=0.31for γ =1.4, υ = 0.55 and Ω_0= 0.5 the flow variables are computed and their profiles are discussed. Variation of shock velocity with propagation distance r, specific heat ratio γ, density parameter υ and angular velocity Ω_0. are shown in {Figure (1 , 2 & 4)}. Shock velocity decreases with γ as well as with propagation distance r {Fig..(1& 2)},where as it increases with Ω_0 (Fig. 3) . The effect of overtaking disturbances is to enhance the shock velocity.

Similarly, variation and percentage improvement in shock strength with r, γ, υ and Ω_0 are obtained and presented in figures (5 to 8) Shock strength in both the cases (FP and EOD) decreases with r,γ and υ {figures(5to 7)}, whereas, it increases with angular velocity Ω_0 {figure(8)}.

Expression for the pressure and particle velocity immediately behind the shock for freely propagating shock (p, u) and under the influence of overtaking disturbances (p*,u*) are

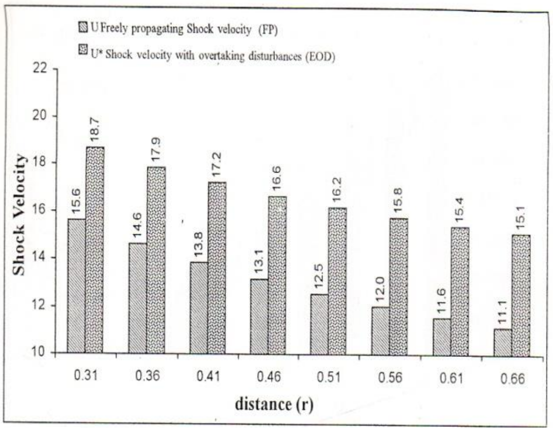
$$P = \frac{2\rho' \exp(\nu r)}{(\gamma+1)} r^{-\alpha A_\gamma} e^{-B_\gamma r}\left\{K + C_\gamma \Omega_0^2 f_1(r,\gamma)\right\}$$

$$u = \frac{2}{(\gamma+1)}\left[r^{-\alpha A_\gamma} e^{-B_\gamma r}\left\{K + C_\gamma \Omega_0^2 f_1(r,\gamma)\right\}\right]^{1/2}$$

and

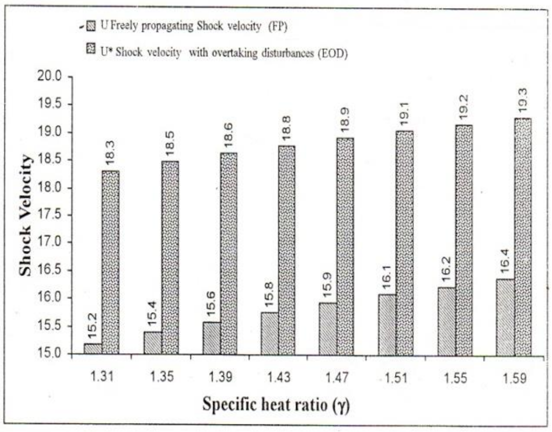
$$P^* = \frac{2}{(\gamma+1)}\rho' \exp(\nu r)\Big[\left(1 + r^{-\alpha A_\gamma} e^{-B_\gamma r} + r^{-\alpha D_\gamma} e^{-E_\gamma r}\right)K + C_\gamma \Omega_0^2 r^{-\alpha A_\gamma} e^{-B_\gamma r} f_1(r,\gamma) - F_\gamma \Omega_0^2 r^{-\alpha D_\gamma} e^{-E_\gamma r} f_2(r,\gamma)\Big]$$

$$u^* = \frac{2}{(\gamma+1)}\Big[\left(1 + r^{-\alpha A_\gamma} e^{-B_\gamma r} + r^{-\alpha D_\gamma} e^{-E_\gamma r}\right)K + C_\gamma \Omega_0^2 r^{-\alpha A_\gamma} e^{-B_\gamma r} f_1(r,\gamma) - F_\gamma \Omega_0^2 r^{-\alpha D_\gamma} e^{-E_\gamma r} f_2(r,\gamma)\Big]^{1/2}$$



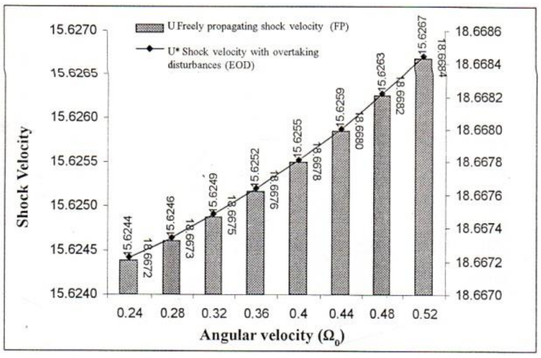
Variation of shock velocity with propagation distance (r) showing the effect of overtaking disturbances for initial density distribution $\rho_0 \propto e^{\nu r}$ for strong shock.

Fig.(1)

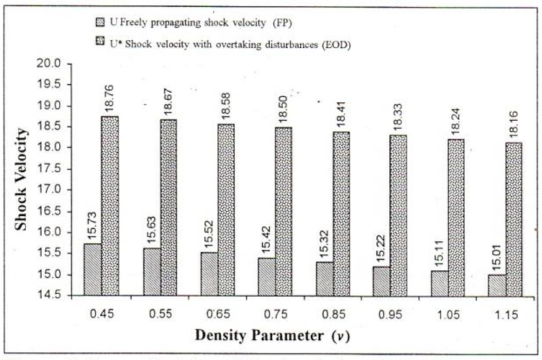


Variation of shock velocity with specific heat ratio (γ) showing the effect of overtaking disturbances for initial density distribution $\rho_0 \propto e^{\nu r}$ for strong shock.

Fig.(2)

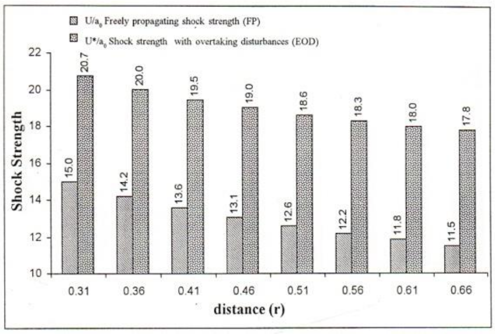


Variation of shock velocity with angular velocity (Ω_0) showing the effect of overtaking disturbances for initial density distribution $\rho_0 \propto e^{\nu r}$ for strong shock.



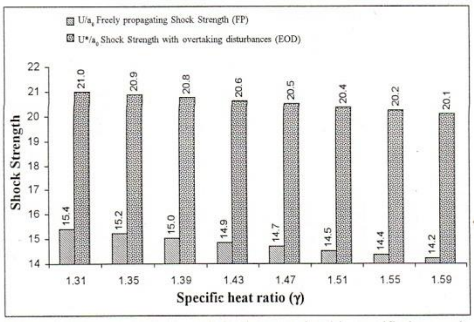
Variation of Shock velocity with density parameter (ν) showing the effect of overtaking disturbances for initial density distribution $\rho_0 \propto e^{\nu r}$ for Strong shock.

Fig.(3)



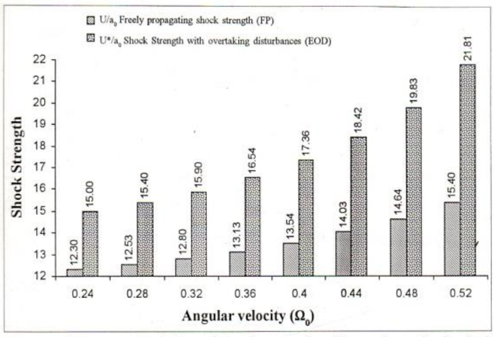
Variation of shock strength with propagation distance (r) showing the effect of overtaking disturbances for initial density distribution $\rho_0 \propto e^{\nu r}$ for strong shock.

Fig.(4)



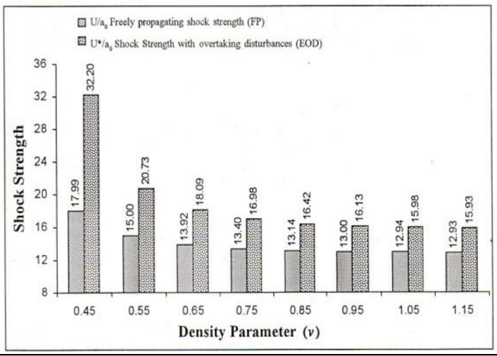
Variation of shock strength with specific heat ratio (γ) showing the effect of overtaking disturbances for initial density distribution $\rho_0 \propto e^{\nu r}$ for strong shock.

Fig.(5)



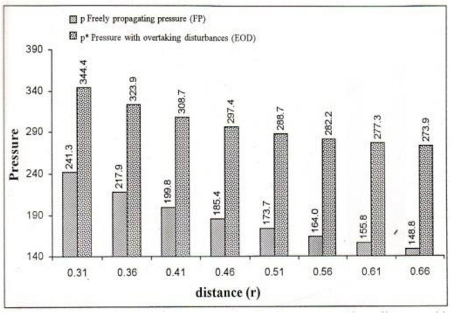
Variation of shock strength with angular velocity (Ω_0) showing the effect of overtaking disturbances for initial density distribution $\rho_0 \propto e^{\nu r}$ for strong shock.

Fig.(6)



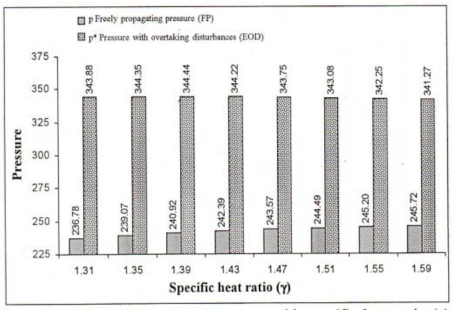
Variation of Shock strength with density parameter (ν) showing the effect of overtaking disturbances for initial density distribution $\rho_0 \propto e^{\nu r}$ for strong shock.

Fig.(7)



Variation of pressure with propagation distance (r) showing the effect of overtaking disturbances for initial density distribution $\rho_0 \propto e^{\nu r}$ for strong shock.

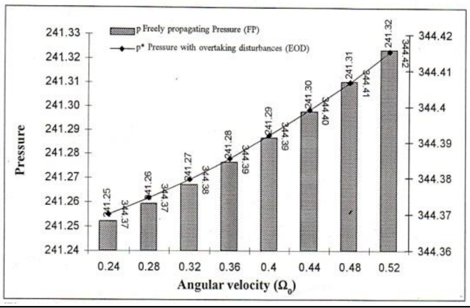
Fig.(8)



Variation of pressure with specific heat ratio (γ) showing the effect of overtaking disturbances for initial density distribution $\rho_0 \propto e^{\nu r}$ for strong shock.

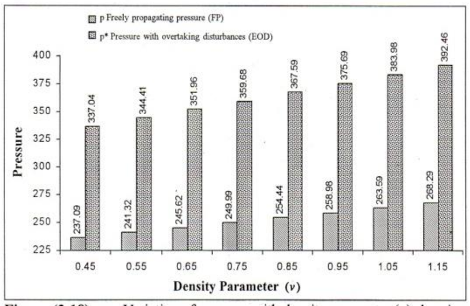
Fig.(9)

Fig.(10)



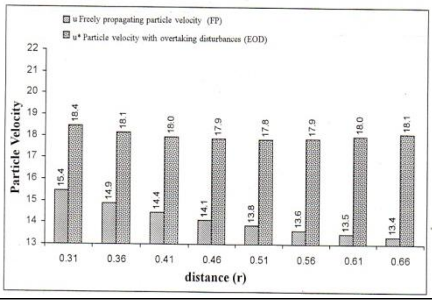
Variation of pressure with angular velocity (Ω_0) showing the effect of overtaking disturbances for initial density distribution $\rho_0 \propto e^{\nu r}$ for strong shock.

Fig.(11)



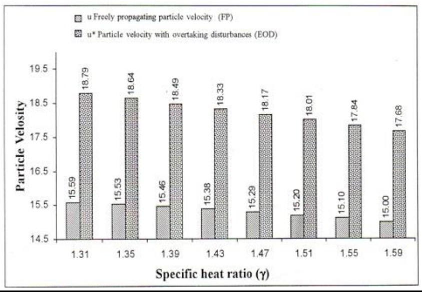
Variation of pressure with density parameter (ν) showing the effect of overtaking disturbances for initial density distribution $\rho_0 \propto e^{\nu r}$ for strong shock.

Fig.(12)



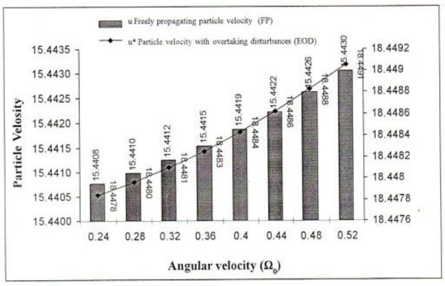
Variation of particle velocity with propagation distance (r) showing the effect of overtaking disturbances for initial density distribution $\rho_0 \propto e^{\nu r}$ for strong shock.

Fig.(13)



Variation of particle velocity with specific heat ratio (γ) showing the effect of overtaking disturbances for initial density distribution $\rho_0 \propto e^{\nu r}$ for strong shock.

Fig.(14)



Variation of particle velocity with angular velocity (Ω_0) showing the effect of overtaking disturbances for initial density distribution $\rho_0 \propto e^{\nu r}$ for strong shock.

Fig.(15)

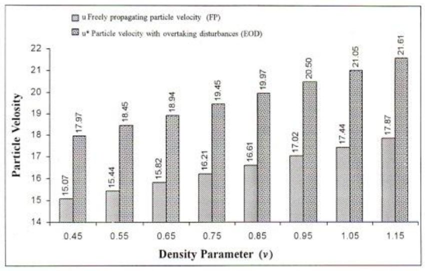


Figure (2.25) : Variation of particle velocity with density parameter (ν) showing the effect of overtaking disturbances for initial density distribution $\rho_0 \propto e^{\nu r}$ for strong shock.

Variation of particle velocity with density parameter (ν) showing the effect of overtaking disturbances for initial density distribution $\rho_0 \propto e^{\nu r}$ for strong shock.

Fig.(16)

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