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Verifications of Harmonic Mean labeling in Degree Splitting Graphs

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ABSTRACT

A graph G = (V,E) with p vertices and q edges is called a Harmonic mean graph if it is possible to label the vertices x ε V with distinct labels from 1, 2, …, q+1 in such a way that when each edge e = uv is labeled with f(e = uv), then the edge labels are distinct. In this case f is called Harmonic mean labeling of G. In this paper we investigate Harmonic mean labeling of degree splitting graph of some standard graphs.
Keywords : Harmonic Mean, Splitting Graph, Vertices, Edges

I. INTRODUCTION

We begin with simple, finite and undirected graph G = (V, E) with p vertices and q edges. The vertex set is denoted by V(G) and the edge set is denoted by E(G). we follow Harry for standard terminology and notations.

Splitting graph S(G) was introduced by E. Sampathkumar and Walikar.
For each vertex v of a graph G, takes a new vertex v'. we have to Join v' to all vertices of G adjacent to v. The graph thus obtained is called the splitting graph of G. it is denoted by S(G) Degree splitting graph DS(G) and mean labeling for Degree splitting graph was introduced by R. Ponraj and S. Somasundaram . we discuss some theorem of Harmonic mean labeling for Degree splitting graph and we investigate Harmonic mean labeling for Degree splitting graphs and we give the definition and example of Degree splitting graph.

1.1. Degree splitting graphs

Definition 1.1.1. Let G = (V, E) be a graph with V = $S_1 \cup S_2 \cup \ldots \cup S_t \cup$ T, where each S_i is a set of vertices having atleast two vertices and having the same degree and T = V - $\cup S_i$. The Degree Splitting graph of G denoted by DS(G) is obtained from G by adding vertices $w_1, w_2, \ldots, w_t$ and joining w_i to each vertex of S_i $(1 \le i \le t)$.

Example 1.1.2. A graph G and its Degree splitting graph DS(G) are given in figures 1.1 and 1.2 respectively.

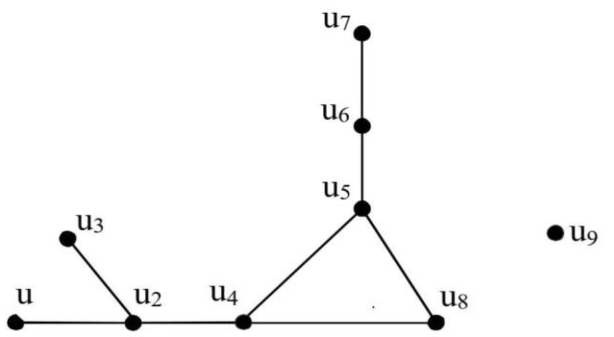

Figure 1.1 G

Here $S_1 = \{u_1, u_3, u_7\}$, $S_2 = \{u_2, u_4, u_5\}$, $S_3 = \{u_6, u_8\}$. $T = \{u_9\}$.

1.2. Harmonic mean labeling of degree splitting graphs

Theorem 1.2.1. nDS(P_3) is a Harmonic mean graph.

Proof. The graph DS(P_3) is shown in figure 1.3.

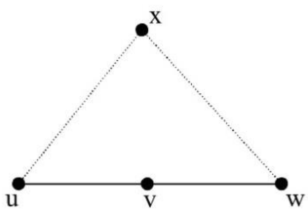

Figure 1.2 DS(P_3)

Let G = nDS(P_3).

Let the vertex set of G be $V = V_1 \cup V_2 \cup \ldots \cup V_n$, where $V_i = \{u_i, v_i, w_i, x_i / 1 \le i \le n\}$ and Set of G be $E = \{ u_iv_i, v_iw_i, u_ix_i, w_ix_i / 1 \le i \le n\}$.

Define a function f: $V(G) \rightarrow \{1, 2, \ldots, q+1\}$ by

$f(u_i) = 4i - 3$, $1 \le i \le n$;

$f(v_i) = 4i - 2$, $1 \le i \le n$;

$f(w_i) = 4i - 1$, $1 \le i \le n$;

$f(x_i) = 4i$, $1 \le i \le n$.

Then the edges are labeled with $f(u_iv_i) = 4i - 3$, $1 \le i \le n$;

$f(v_iw_i) = 4i - 1$, $1 \le i \le n$;

$f(u_ix_i) = 4i - 2$, $1 \le i \le n$;

$f(w_ix_i) = 4i$, $1 \le i \le n$.

Thus f provides a Harmonic mean labeling for G.

Example 1.2.2. A Harmonic mean labeling of 4DS(P_3) is given in figure 1.4.

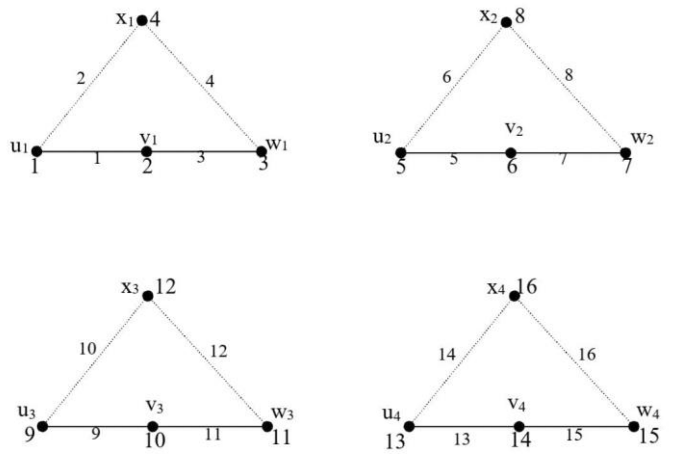

Figure 1.3 4DS(P_3)

Theorem 1.2.3. nDS($P_3 \odot K_1$) is a Harmonic mean graph.

Proof. DS($P_3 \odot K_1$) is given in figure 1.5.

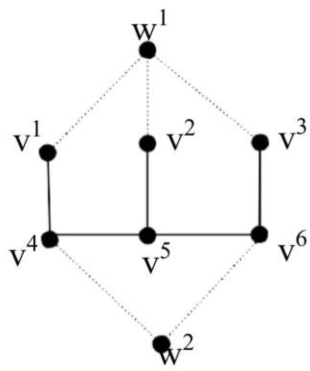

Figure 1.4 DS($P_3 \odot K_1$)

Let G = nDS($P_3 \odot K_1$).

Let the vertex set of G be $V = V_1 \cup V_2 \cup \ldots \cup V_n$, where $V_i = \{v_i^1, v_i^2, v_i^3, v_i^4, v_i^5, v_i^6, w_i^1, w_i^2 / 1 \le i \le n\}$ is the vertex set of i^th^ copy set of G be DS($P_3 \odot K_1$).

The edge set is $E = \{ v_i^1v_i^2, v_i^2v_i^5, v_i^3v_i^6, v_i^4v_i^5, v_i^5v_i^6, v_i^1w_i^1, v_i^2w_i^1, v_i^3w_i^1, v_i^4w_i^2, v_i^6w_i^2\}$.

Define a function f: $V(G) \rightarrow \{1, 2, \ldots, q+1\}$ by

$f(v_i^1) = 10i - 3$, $1 \le i \le n$;

$f(v_i^2) = 10i - 2$, $1 \le i \le n$;

$f(v_i^3) = 10i - 1$, $1 \le i \le n$;

$f(v_i^4) = 10i - 8, 1 \leq i \leq n.$
$f(v_i^5) = 10i - 5, 1 \leq i \leq n.$
$f(v_i^6) = 10i - 6, 1 \leq i \leq n.$
$f(w_i^1) = 10i, 1 \leq i \leq n.$
$f(w_i^2) = 10i - 9, 1 \leq i \leq n.$

Then the edge are labeled with

$f(v_i^1 v_i^2) = 10i - 6, 1 \leq i \leq n.$
$f(v_i^2 v_i^5) = 10i - 3, 1 \leq i \leq n.$
$f(v_i^3 v_i^6) = 10i - 4, 1 \leq i \leq n.$
$f(, v_i^4 v_i^5) = 10i - 7, 1 \leq i \leq n.$
$f(v_i^5 v_i^6) = 10i - 5, 1 \leq i \leq n.$
$f(v_i^1 w_i^1) = 10i - 2, 1 \leq i \leq n.$
$f(v_i^2 w_i^1) = 10i - 1, 1 \leq i \leq n.$
$f(v_i^3 w_i^1) = 10i, 1 \leq i \leq n.$
$f(v_i^4 w_i^2) = 10i - 9, 1 \leq i \leq n.$
$f(v_i^6 w_i^2) = 10i - 8, 1 \leq i \leq n.$

By the above labeling patterns, nDS($P_3 \odot K_1$) is a Harmonic mean graph.

Example 1.2.4. The Harmonic mean labeling of 4DS($P_3 \odot K_1$) is shown in the following figure

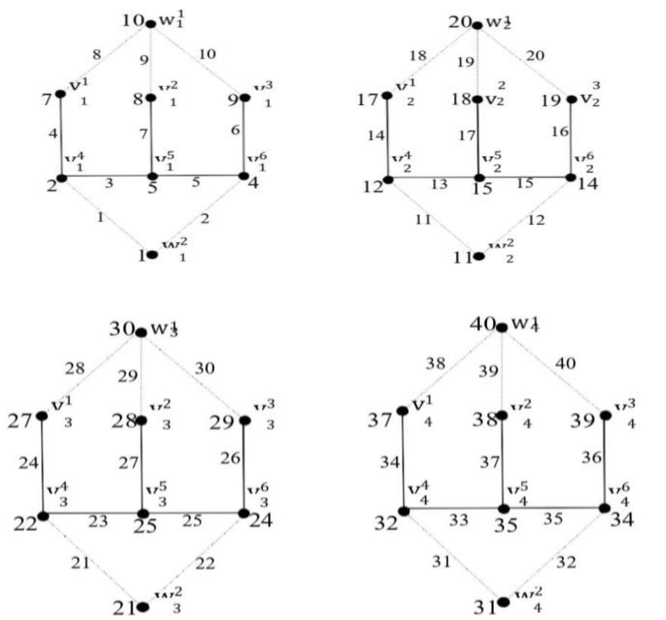

Figure 1.5 4DS ($P_3 \odot K_1$)

Theorem 1.2.5. nDS($K_{1,3}$) is a Harmonic mean graph.

Proof. DS($K_{1,3}$) is given in figure 1.7.

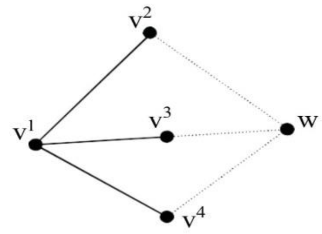

Figure 1.6 DS($K_{1,3}$)

Let G = nDS($K_{1,3}$). Let $V_i = \{v_i^1, v_i^2, v_i^3, v_i^4, w_i^1\}$ be the vertex set and $E_i = \{ v_i^1 v_i^2 , v_i^1 v_i^3 , v_i^1 v_i^3, v_i^1 v_i^4 , v_i^2 w_i, v_i^3 w_i, v_i^4 w_i\}$ be the edge set of of i^th copy of is DS($K_{1,3}$), $1 \leq i \leq n$. Define a function f: V(G) → {1, 2 , …, q+1} by

$f(v_1^1) = 3, f(v_i^1) = 6i - 4, 2 \leq i \leq n;$
$f(v_i^2) = 6i - 5, 1 \leq i \leq n;$
$f(v_i^3) = 6i - 1, 1 \leq i \leq n;$
$f(v_1^4) = 4; f(v_i^4) = 6i - 3, 2 \leq i \leq n;$
$f(w_i) = 6i , 1 \leq i \leq n;$

Then the edges are labeled with

$f(v_i^1 v_i^2) = 6i - 5, 1 \leq i \leq n;$
$f(v_i^1 v_i^3) = 6i - 2, 1 \leq i \leq n;$
$f(v_1^1 v_1^4) = 3 ; f(v_i^1 v_i^4) = 6i - 4, 2 \leq i \leq n;$
$f(v_1^2 w_1) = 2; f(v_i^2 w_i) = 6i - 3, 2 \leq i \leq n;$
$f(v_i^3 w_i) = 6i, 1 \leq i \leq n;$
$f(v_i^4 w_i) = 6i - 1, 1 \leq i \leq n;$

In the view of the above labeling pattern, f is a Harmonic mean labeling.

Hence nDS($K_{1,3}$) is a Harmonic mean graph.

Example 1.2.6. A Harmonic mean labeling of 4DS($K_{1,3}$) is given in figure

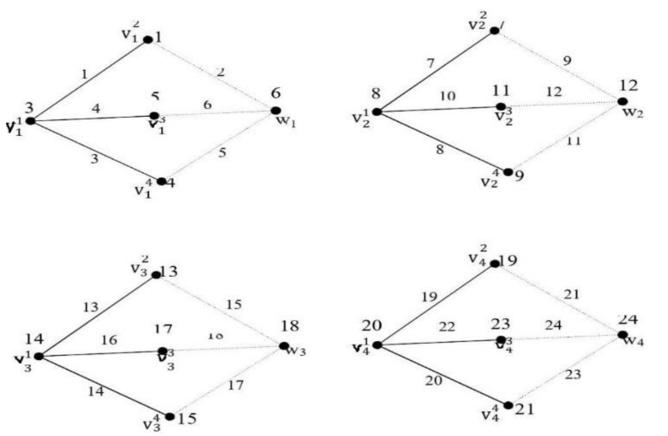

Figure 1.7 4DS($K_{1,3}$)

II. Conclusion

Since all degree splitting graphs are not Harmonic mean graphs, it is very interesting to investigate degree splitting graphs which are Harmonic mean graphs. In this paper, we proved that degree splitting graph of some standared graphs and n copies of some graphs are Harmonic mean graphs.

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