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Modulo Three Harmonic Mean Labeling of Acyclic Graphs

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ABSTRACT

In this paper we give some definition and theorem of Modulo three harmonic mean labeling of graphs. A graph is said to be Modulo three harmonic mean labeling graph if there is a function φ from the vertex set of G {1,3,4,6,…, 3q-2, 3q} with φ is one to one and φ induces a bijective φ* from the edge of G to {1,4,7, …, 3q-2} where φ* (e= uv) $\lceil \frac{2\varphi(u)\varphi(v)}{\varphi(u)+\varphi(v)} \rceil$ or $\lfloor \frac{2\varphi(u)\varphi(v)}{\varphi(u)+\varphi(v)} \rfloor$ and the function φ is called as Modulo three mean labeling of G. Furthermore we define modulo three harmonic labeling of some trees.

Keywords : Labeling Techniques, Harmonic Mean

I. INTRODUCTION

All graphs considered here are simple, finite, connected and directed. We follow the basic notations and terminologies of graph theory. Given a graph G, the symbol V(G) and E(G) denote the vertex and edge set of the graph G. Let G= G (p,q) be a graph with p= |V(G)| vertices and q= |E(G)| edges. A graph labeling is the assignment of integers to the vertices or edges or both, subject to a certain conditions.

V. Swaminathan et al., introduced the concept of one modulo three graceful labeling.

S. Somasundram and R. Ponraj introduced mean labeling of graphs and further studied.

P. Jeyanthi and A. Maheswari introduced the concept one modulo three mean labeling of graphs. V.Maheswari et al.,developed relaxed mean labeling for tree related graphs.

Definition 2.1.1:

A graph is said to be modulo three harmonic mean labeling graph if there is a function φ from the vertex set of G {1,3,4,6,…, 3q-2, 3q} with φ is one to one and φ induces a bijective φ* from the edge of G to {1,4,7, …, 3q-2} where φ* (e= uv) $\lceil \frac{2\varphi(u)\varphi(v)}{\varphi(u)+\varphi(v)} \rceil$ or $\lfloor \frac{2\varphi(u)\varphi(v)}{\varphi(u)+\varphi(v)} \rfloor$ and the function φ is called as one modulo three mean labeling of G.

Example 2.1.2 Modulo three harmonic mean labeling of a graph G is shown in figure 2.1.

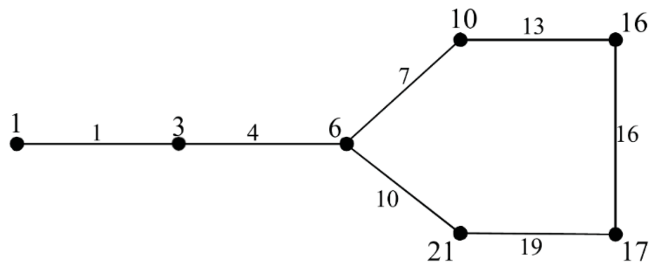

Figure 2.1

Remark 2.1.3. If G is modulo three harmonic mean graph, then 1 must be a label of one of the vertices of G, since an edge must get the label 1.

Theorem 2.1.4. If G is modulo three harmonic mean graph, then G has atleast one vertex of degree one and an end vertex gets label 1.

Proof: Suppose G is modulo three harmonic mean graphs. Then by remark 2.1.3, 1 must be the label of one of the vertices of G. If the vertex u gets the label 1, then any edge incident with u gets the label 1 or 2, since $1 \leq \frac{2m}{m+1} < 2$, where m is the label of a vertex adjacent to u. But 2 is not in the set {1, 3, 4, … , 3q – 2, 3q} and hence any edge incident with u must have label 1. Since φ^* is a bijection from an edge set G to {1, 3, 4, … , 3q – 2, 3q}, only one edge incident with u. Hence the degree of u is 1.

2.2. Modulo three harmonic mean labeling of some trees

In this section, we investigate modulo three harmonic mean labeling of some classes of trees like path, star, comb etc.

Theorem 2.2.1. Any path P_n is modulo three harmonic mean graph.

Proof. Let P_n be the path $u_1 u_2 \ldots u_n$. Define a function $\varphi : V(P_n) \rightarrow \{1, 3, 4, 6, \ldots, 3q-2, 3q\}$

$$\varphi(u_1) = 1, \varphi(u_i) = 3(i-1), 2 \leq i \leq n.$$

Then φ induces a bijection $\varphi^* : E(P_n) \rightarrow \{1, 4, \ldots, 3q-2\}$, where

$$\varphi(u_i u_{i+1}) = 3i-2, 1 \leq i \leq n-1.$$

Therefore, φ is a one modulo three harmonic mean graph.

Example2.2.2. Modulo three harmonic mean labeling of P_8 is shown in figure2.2.

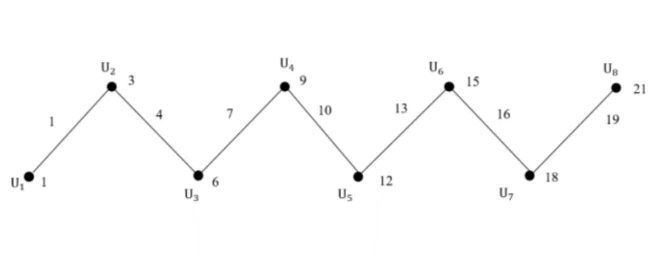

Figure 2.2. P_8

Theorem 2.2.3. $P_n \odot K_1$ is modulo three harmonic mean graph.

Proof. Let $u_1 u_2 \ldots u_n$ be the path P_n. Join vertex v_i with u_i, $1 \leq i \leq n$. Then the resultant graph is $P_n \odot K_1$. Define a function $\varphi : V(P_n \odot K_1) \rightarrow \{1, 3, 4, 6, \ldots, 3q-2, 3q\}$ by

$$\varphi(u_1) = 3, \varphi(u_i) = 6i-3 \text{ for all even } i \text{ and } i \leq n;$$
$$\varphi(u_i) = 6(i-1), \text{ for all odd } i, i \neq 1 \text{ and } i \leq n;$$
$$\varphi(v_1) = 1, \varphi(v_i) = 6(i-1) \text{ for all even } i \text{ and } i \leq n;$$
$$\varphi(v_i) = 6i-3, \text{ for all odd } i, i \neq 1 \text{ and } i \leq n;$$

Then φ induces a bijective function $\varphi^* : E(P_n \odot K_1) \rightarrow \{1, 4, \ldots, 3q-2\}$, where

$$\varphi^*(u_i u_{i+1}) = 6i-2, 1 \leq i \leq n-1.$$

$$\varphi^*(u_i v_i) = 6i-5, 1 \leq i \leq n-1.$$

Thus the edges get the distinct labels 1, 4, …, 3q – 2. Therefore, φ is one modulo three harmonic mean labeling. Hence $P_n \odot K_1$ is modulo three harmonic mean graph.

Example 2.2.4. modulo three harmonic mean labeling of $P_6 \odot K_1$ and $P_7 \odot K_1$ are shown in figure 2.3 and 2.4 respectively

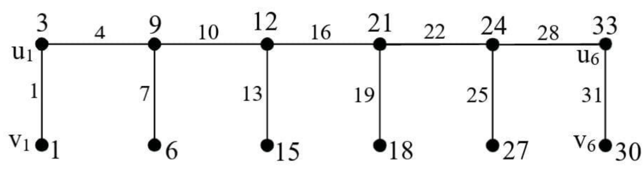

Figure 2.3

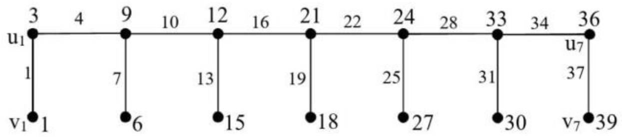

Figure 2.4

Theorem 2.2.5. A graph obtained by attaching the central vertex of $K_{1,2}$ at each pendent vertex of a comb $P_n \odot K_1$ is modulo three harmonic mean graph.

Proof. Let P_n be the path $u_1u_2\ldots u_n$. Let v_i be a vertex adjacent to u_i, $1 \leq i \leq n$. The resultant graph is $P_n \odot K_1$. Let x_i, w_i, y_i be the vertices of i^{th} copy of $K_{1,2}$ with w_i is the central vertex. Identify the vertex w_i with v_i, $1 \leq i \leq n$, we get the required graph G whose edge set is

$E = \{u_iu_{i+1}/1 \leq i \leq n\} \cup \{ u_iv_i, v_ix_i, v_iy_i / 1 \leq i \leq n \}$.

Define a function $\varphi : V(P_n \odot K_1) \rightarrow \{1, 3, 4, 6, \ldots, 3q - 2, 3q\}$ by

$\varphi(u_1) = 7$, $\varphi(u_i) = 12(i-1)+6$, $2 \leq i \leq 3$; $\varphi(u_i) = 12(i-1)+4$, $4 \leq i \leq n$;

$\varphi(v_1) = 6$, $\varphi(v_2) = 19$; $\varphi(v_i) = 12i-3$, $3 \leq i \leq 4$; $\varphi(v_i) = 12i-3$, $5 \leq i \leq n$;

$\varphi(x_1) = 1$, $\varphi(x_2) = 21$; $\varphi(x_3) = 21$; $\varphi(x_4) = 31$;

$\varphi(x_i) = 12(i-1) - 6$, $5 \leq i \leq n$;

$\varphi(y_1) = 3$; $\varphi(y_2) = 13$; $\varphi(y_i) = 12(i-1)$, $3 \leq i \leq n$;

Then φ induces a bijective functio $\varphi^* : E(G) \rightarrow \{1, 4, \ldots, 3q-2\}$, where

$\varphi^*(u_iv_i) = 12i - 5$, $1 \leq i \leq n$;

$\varphi^*(u_iu_{i+1}) = 12i - 2$, $1 \leq i \leq n-1$;

$\varphi^*(v_ix_i) = 12(i-1) + 1$, $1 \leq i \leq n$;

$\varphi^*(v_iy_i) = 12(i-1) + 4$, $1 \leq i \leq n$.

In the view of the above labeling pattern, φ provides modulo three harmonic mean labeling for G. Hence G is one modulo three harmonic mean graph.

Example 2.2.6. modulo three harmonic mean labeling for G when n = 7 is given in figure 2.5

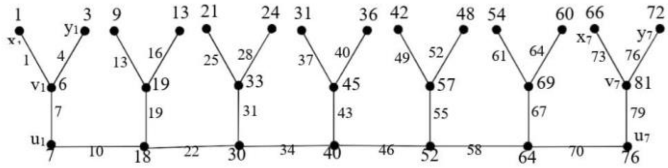

Figure 2.5

II. Conclusion

In this chapter, we used number theoretical approach to label the edges. In this labeling techniques edges get labels from 1, 4, …., 3q – 2. Therefore it is very interesting to investigate graphs which have one modulo three harmonic mean labeling.

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