© 2016 IJSRSET | Volume 2 | Issue 5 | Print ISSN: 2395-1990 | Online ISSN : 2394-4099
Themed Section: Science

The Study of Strong Spherical Shock Wave Through A Rotating Gas Having Distance Dependent Angular Velocity

Anil Kumar
Department of Physics, Bareilly College, Bareilly, Uttar Pradesh, India

ABSTRACT

The effect of variable rotation on the propagation of strong spherical shock wave in non uniform medium has been investigated expression for shock strength and shock velocity have been derived for two case (i) when shock is not effected by disturbances behind the shock and (ii) when the shock is effected by the overtaking disturbances in presence of a rotating gas with variable angular velocity.

Keywords - Study, Strong, Spherical, Shock, Wave, Through, Rotating, Gas, Distance, Dependent, Angular, Velocity.

I. INTRODUCTION

The field of shock waves has tremendous potential for the researcher. Therefore, it has been receiving considerable attention of many workers, for example, Sedov (1959), Taylor (1950), Sakurai (1953), Chester (1954), Chinsnell (1958), Whitham (1958), Prasad (1990), Yadav (1992) etc. and many other. Considering the effect of overtaking disturbances on the propagation of strong diverging shock in non-uniform medium, the temperature variation behind shock front has been computed by Yadav et.al. (2001) have studied the propagation of spherical converging strong shock in uniform medium and obtained the change in entropy and temperature of medium due to propagation of spherical converging strong shock wave Yadav and Gangwar (2002) as studied the propagation of spherical converging strong shock in non-uniform medium. Recently, Yadav and Gangwar (2003) have studied freely propagation of strong spherical diverging shock in non-uniform medium. Very recently, neglecting the effect of rotation of the medium, Yadav and Singh (2004), have studied the propagation of strong spherical and cylindrical shock wave in the non-uniform medium for density distribution (i) $\rho_0=\rho' r^w$, $\rho_0=\rho' e^{vr}$ and (iii) $\rho_0=\rho'$ log r, where ρ' and w are the constants. The effect of variable rotation on the propagation of spherical strong shock waves in non-uniform medium has been investigated in this paper. Assuming an initial angular velocity $\Omega_0 =\Omega' r^{\lambda}$ and initial density $\rho_0=\rho' r^w$, where Ω' , ρ', w and λ are constants, analytical expression for shock velocity and shock strength have been derived for two cases viz. (i) when shock is not affected by the disturbances behind the shock, and (ii) when the shock in affected by the overtaking disturbance.

The relations for flow variables (pressure and particle velocity) are obtained and computed. The flow variables so obtained are discussed through figures. Results obtained are compared with those obtained for constant rotation.

II. BASIC EQUATION

The equations governing the flow of gas enclosed by the shock front are

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial r} + \frac{1}{\rho}\frac{\partial p}{\partial r} - \frac{v^2}{r} = 0 \qquad ...(3.1)$$

$$\left(\frac{\partial}{\partial t} - u\frac{\partial}{\partial r}\right)(vr) = 0 \qquad ...(3.2)$$

$$\left(\frac{\partial}{\partial t} + u\frac{\partial}{\partial r}\right)\rho + \rho\left(\frac{\partial u}{\partial r} + \frac{\alpha u}{r}\right) = 0 \qquad ...(3.3)$$

$$\left(\frac{\partial}{\partial t} + u\frac{\partial}{\partial r}\right)\left(p\rho^{-\gamma}\right) = 0 \qquad ...(3.4)$$

where, r is the radial co-ordinate u (r, t), p (r, t) and ρ(r, t) are respectively the particle velocity, pressure and density at distance r from the origin at time t and γ is the specific heat ratio of the gas while v is the radial component of velocity.

III. BOUNDRY CONDITIONS

Let p_0 and ρ_0 denote undisturbed values of pressure and density in front of the shock wave and u_1, p_1 and ρ_1 be the values of respective quantities at any point immediately after the passage of shock, then the well-know Rankine-Hugoniet conditions permit as to express u_1, p_1 and ρ_1 in terms of the undisturbed values of these quantities by means of the following equations.

$$p_1 = \rho_0 a_0^2\left[\frac{2M^2}{(\gamma+1)} - \frac{(\gamma-1)}{\gamma(\gamma+1)}\right]$$

$$\rho_1 = \rho_0\left[\frac{(\gamma+1)M^2}{(\gamma-1)M^2+2}\right] \qquad ...(3.5)$$

$$u_1 = \frac{2a_0}{\gamma+1}\left[M - \frac{1}{M}\right]$$

where, $M=U/a_0$, U being the shock velocity, a_0 is the sound velocity $\left(\sqrt{\gamma p_0/\rho_0}\right)$ in undisturbed medium. M is the mach number.

FOR STRONG SHOCK WAVE

$U >> a_0$ under this condition the boundary conditions reduces to,

$$p = \frac{2\rho_0}{\gamma + 1} U^2, \quad u = \frac{2U}{\gamma + 1}, \quad \rho = \rho_0 \left(\frac{\gamma + 1}{\gamma - 1} \right), \quad v = v_0 \qquad \ldots(6)$$

and $$a = sU \left(\frac{\gamma - 1}{\gamma + 1} \right), \quad \text{where, } s = \left(\frac{2\gamma}{\gamma - 1} \right)^{\frac{1}{2}} \qquad (\ldots 7)$$

THEORY

The characteristic form of the system of the basic equations (1) and (3) is,

$$dp + \rho a du + \frac{\alpha \rho a^2 u}{(u + a)} \frac{dr}{r} - \frac{\rho a v^2}{(u + a)} \frac{dr}{r} = 0 \qquad \ldots(8)$$

The equilibrium of the gas is assumed to specified by the conditions $\frac{\partial}{\partial t} = 0 = u$ and $p = p_0$, $v = v_0 = r\,\Omega_0$, as the consequence of hydrostatic equilibrium prevailing in front of the shock, equation (1) gives

$$\frac{1}{\rho_0} \frac{dp_0}{dr} - \frac{v_0^2}{r} = 0 \quad \Rightarrow \quad \therefore \quad dp_0 = \rho_0 v_0^2 dr / r \qquad \ldots(9)$$

Assuming the initial density distribution $\rho_0 = \rho' r^w$, and $\Omega_0 = \Omega' r^\lambda$, Ω'=constant, equation (9), reduces to

$$dp_0 = \rho' \Omega'^2 r^{w+2\lambda+1} dr \qquad \ldots(10)$$

Integrating,

$$p_0 = K + \frac{\rho' \Omega'^2 r^{w+2\lambda+2}}{(w + 2\lambda + 2)} \qquad \ldots(11)$$

where, K is the constant of integration

Therefore,

$$\because \quad a_0^2 = \gamma p_0 / \rho_0$$

$$\therefore \quad a_0^2 = \left[\frac{\gamma K}{\rho'} r^{-w} + \frac{\gamma \Omega'^2 r^{2\lambda+2}}{(w + 2\lambda + 2)} \right]$$

Strong shock in absence of overtaking disturbances

Substituting condition (6) into equation (8) and simplifying, we get,

SHOCK VELOCITY

$$U^2 = r^{-(A_\gamma+\alpha B_\gamma)}\left[K + C_\gamma \frac{r^{A_\gamma+\alpha B_\gamma+2\lambda+2}}{(A_\gamma+\alpha B_\gamma+2\lambda+2)}\right] \quad \text{...(12)}$$

where $A_\gamma = \frac{2w}{(2+s)}$, $B_\gamma = \frac{4\gamma}{(2+s)\left[2+\{2\gamma(\gamma-1)\}^{1/2}\right]}$

$$C_\gamma = \frac{(\gamma+1)^2 s\Omega'^2}{(2+s)\left[2+\{2\gamma(\gamma-1)\}^{1/2}\right]}, \quad s = \left(\frac{2\gamma}{\gamma-1}\right)^{1/2}$$

SHOCK STRENGTH

$$(U/a_0)^2 \quad r^{-(A_\gamma+\alpha B_\gamma)}\left[K + C_\gamma \frac{r^{A_\gamma+\alpha B_\gamma+2\lambda+2}}{(A_\gamma+\alpha B_\gamma+2\lambda+2)}\right] \Big/ \left(\frac{\gamma K}{\rho'}r^{-w} + \frac{\gamma\Omega'^2 r^{2\lambda+2}}{w+2\lambda+2}\right) \quad \text{...(13)}$$

Strong shock in propagation in presence of overtaking disturbances

For overtaking disturbances, we have taken differential equation

$$dp - \rho a du + \frac{\alpha\rho a^2 u}{(u-a)}\frac{dr}{r} + \frac{\rho a v^2}{(u-a)}\frac{dr}{r} = 0 \quad \text{...(14)}$$

with the help of (6) and (14), we have

$$U^2 = r^{-(D_\gamma+\alpha E_\gamma)}\left[K - \frac{F_\gamma r^{D_\gamma+\alpha E_\gamma+2\lambda+2}}{(D_\gamma+\alpha E_\gamma+2\lambda+2)}\right] \quad \text{...(15)}$$

where,

$$D\gamma = \frac{2w}{(2-s)}, \qquad E\gamma = \frac{4\gamma}{(2-s)\left[2-\{2\gamma(\gamma-1)\}^{1/2}\right]}$$

$$F\gamma = \frac{(\gamma-1)^2 s\Omega'^2}{\left[2-\{2\gamma(\gamma-1)\}^{1/2}\right](2-S)}$$

Now from equation (6), we get

$$u = \frac{2U}{\gamma+1} \Rightarrow du_+ = \frac{2dU}{\gamma+1}$$

$$\therefore dU = \frac{\gamma+1}{2} du_+ \qquad \ldots(16)$$

Substituting these values in equation (15) and simplifying, we get

$$du_+ = \frac{2}{\gamma+1}\frac{1}{2U}\frac{d}{dr}\left[r^{-(A_\gamma+\alpha B_\gamma)}\left\{K + \frac{C_\gamma r^{A_\gamma+\alpha B_\gamma+2\lambda+2}}{\left(A_\gamma+\alpha B_\gamma+2\lambda+2\right)}\right\}\right]$$

For overtaking disturbances the fluid velocity increment may be expressed as

$$u_- = \frac{2U}{\gamma+1} \Rightarrow du_- = \frac{2dU}{\gamma+1}$$

Thus, velocity increment for overtaking disturbances can be written as

$$du_- = \frac{2}{\gamma+1}\frac{1}{2U}\frac{d}{dr}\left[r^{-(D_\gamma+\alpha E_\gamma)}\left\{K - \frac{F_\gamma r^{D_\gamma+\alpha E_\gamma+2\lambda+2}}{D_\gamma+\alpha E_\gamma+2\lambda+2}\right\}\right]$$

In presence of overtaking disturbances, the fluid velocity increment will be

$$du_+ + du_- = \frac{2}{\gamma+1} dU \qquad \ldots(17)$$

Substituting values of du_+, du and simplifying, we get modified

IV. SHOCK VELOCITY

$$U^{*2} = \left[\left\{1 + r^{-(A_\gamma + \alpha B_\gamma)} + r^{-(D_\gamma + \alpha E_\gamma)}\right\}K + \frac{C_\gamma r^{2\lambda+2}}{(A_\gamma + \alpha B_\gamma + 2\lambda + 2)} - \frac{F_\gamma r^{2\lambda-2}}{(D_\gamma + \alpha E_\gamma + 2\lambda + 2)}\right] \quad ...(18)$$

Therefore, the expression of the shock strength in presence of overtaking disturbances, can be written as

$$\left(\frac{U^*}{a_0}\right)^2 = \left[\left\{1 + r^{-(A_\gamma + \alpha B_\gamma)} + r^{-(D_\gamma + \alpha E_\gamma)}\right\}K + \frac{C_\gamma r^{2\lambda+2}}{(A_\gamma + \alpha B_\gamma + 2\lambda + 2)} - \frac{F_\gamma r^{2\lambda+2}}{(D_\gamma + \alpha E_\gamma + 2\lambda + 2)}\right] \Big/ \left\{\frac{\gamma K}{\rho'} r^{-w} + \frac{\gamma \Omega' r^{2\lambda+2}}{w + 2\lambda + 2}\right\} \quad ...[5] \quad ...(19)$$

V. RESULT AND DISCUSSION

The expressions for shock velocity (12) & (18) and shock strength (13) and (19) are derived for the strong spherical shock propagation in non-uniform medium having distance depended angular velocity ($\Omega \propto r^\lambda$). Initially, taking $U/a_0 = 9.5$ at $r = 0.5$ for $\gamma = 1.27$, $\Omega' = 0.2$, $\lambda = 2.5$ and $w = 0.35$, profiled of the shock velocity, and shock strength are obtainedrepresented graphically (Fig.-1-10). It is observed that shock velocity continuously decreases with propagation distance r (Fig.-1) whereas it is increases with γ (Fig.2) , w (Fig.-3) and λ (Fig.-4). Freely propagating sgock velocity increases with angular velocty Ω' (Fig.-5) whereas it decreases continuously when effect of overtaking disturbance is taking into account.

In case of contant initial angular velocity, shock velocity for freely as well as EOD increases with agular velocity $\Omega 0$, Shock Strength decreases with r (Fig.-6). λ (Fig.-09) and Ω' (Fig.-10) and increases with γ (Fig.-7), Freely propagating shock strength increases with w (Fig-8) and decreases continuously when effect of overtaking disturbances are considered.

Finally, expressions for the pressure and the particle velocity immediately behind the shock are obtained as . These flow variables are numerically computed and presented in graph (Fig.-11-20).

$$p = \frac{2\rho' r^w}{(\gamma+1)} r^{-(A_\gamma + \alpha B_\gamma)}\left[K + C_\gamma \frac{r^{A_\gamma + \alpha B_\gamma + 2\lambda + 2}}{(A_\gamma + \alpha B_\gamma + 2\lambda + 2)}\right] \quad ...(3.26)$$

$$u = \frac{2}{(\gamma+1)}\left[r^{-(A_\gamma + \alpha B_\gamma)}\left\{K + C_\gamma \frac{r^{A_\gamma + \alpha B_\gamma + 2\lambda + 2}}{(A_\gamma + \alpha B_\gamma + 2\lambda + 2)}\right\}\right]^{1/2}$$

and

 …(20)

$$p^* = \frac{2\rho' r^{w}}{(\gamma+1)}\left[\left\{1 + r^{-(A_\gamma+\alpha B_\gamma)} + r^{-(D_\gamma+\alpha E_\gamma)}\right\}K + \frac{C_\gamma r^{2\lambda+2}}{(A_\gamma+\alpha B_\gamma+2\lambda+2)} - \frac{F_\gamma r^{2\lambda-2}}{(D_\gamma+\alpha E_\gamma+2\lambda+2)}\right] \quad ...(3.27)$$

$$u^* = \frac{2}{(\gamma+1)}\left[\left\{1 + r^{-(A_\gamma+\alpha B_\gamma)} + r^{-(D_\gamma+\alpha E_\gamma)}\right\}K + \frac{C_\gamma r^{2\lambda+2}}{(A_\gamma+\alpha B_\gamma+2\lambda+2)} - \frac{F_\gamma r^{2\lambda+2}}{(D_\gamma+\alpha E_\gamma+2\lambda+2)}\right]^{1/2} \quad ...(3.28)$$

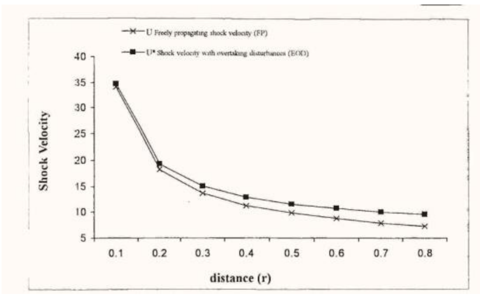


Fig.-1 Variation of shock velocity with propagation distance (r) showing the effect of overtaking disturbances for initial density distribution $\rho_0 \propto r^w$ for strong shock.

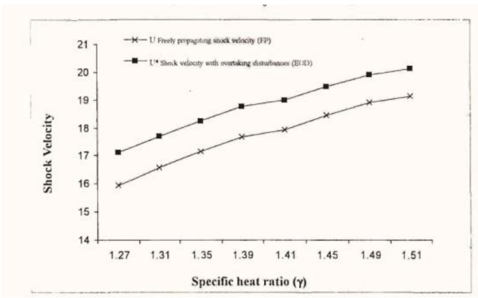


Fig.-2 Variation of shock velocity with specific heat ratio (γ) showing the effect of overtaking disturbances for initial density distribution $\rho_0 \propto r^w$ for strong shock.

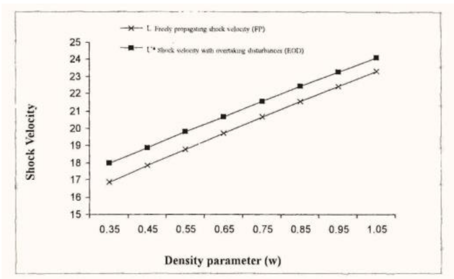


Fig.-3 Variation of shock velocity with density parameter (w) showing the effect of overtaking disturbances for initial density distribution $\rho_0 \propto r^w$ for strong shock.

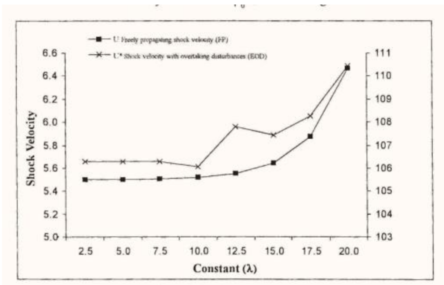
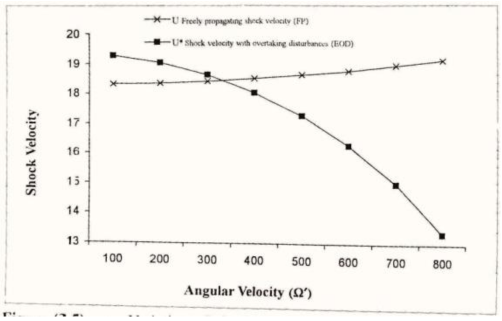


Fig.-4 Variation of shock velocity with constant (λ) showing the effect of overtaking disturbances for initial density distribution $\rho_0 \propto r^w$ for strong shock.



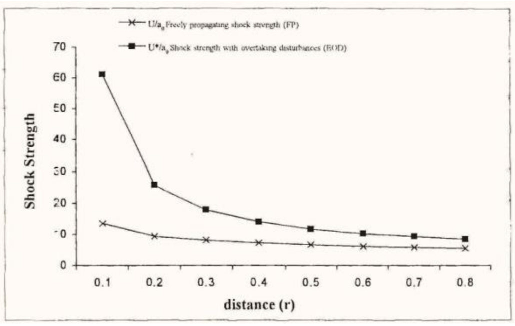


Fig.-5 Variation of shock velocity with angular velocity (Ω') showing the effect of overtaking disturbances for initial density distribution $\rho_0 \propto r^w$ for strong shock.

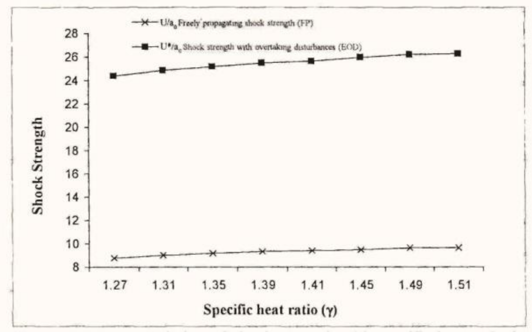


Fig.-7 Variation of shock strength with specific heat ratio (γ) showing the effect of overtaking disturbances for initial density distribution $\rho_0 \propto r^w$ for strong shock.

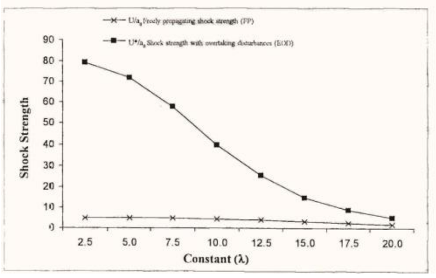


Fig.-9 Variation of shock strength with donstant (λ) showing the effect of overtaking disturbances for initial density distribution $\rho_0 \propto r^w$ for strong shock.

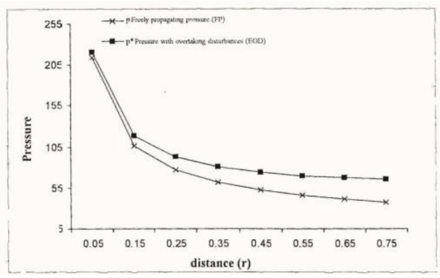


Fig.-11 Variation of shock strength with propagation distance (r) showing the effect of overtaking disturbances for initial density distribution $\rho_0 \propto r^w$ for strong shock.

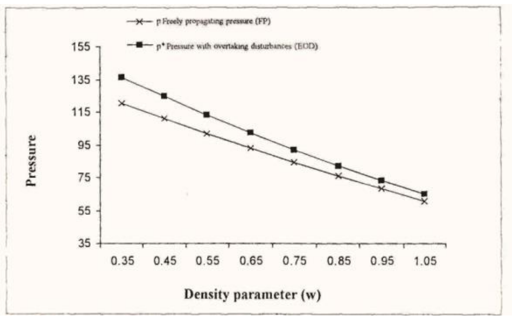


Fig.-13 Variation of shock strength with density parameter (w) showing the effect of overtaking disturbances for initial density distribution $\rho_0 \propto r^w$ for strong shock.

Fig.-6 Variation of shock strength with propagation distance (r) showing the effect of overtaking disturbances for initial density distribution $\rho_0 \propto r^w$ for strong shock.

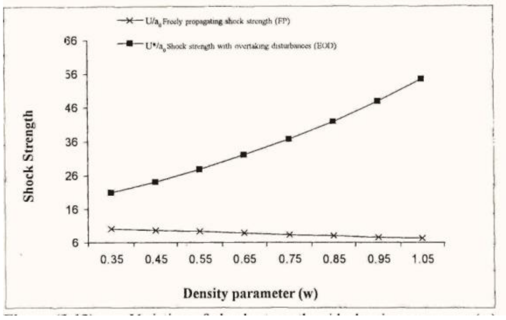


Fig.-8 Variation of shock strength with density parameter (w) showing the effect of overtaking disturbances for initial density distribution $\rho_0 \propto r^w$ for strong shock.

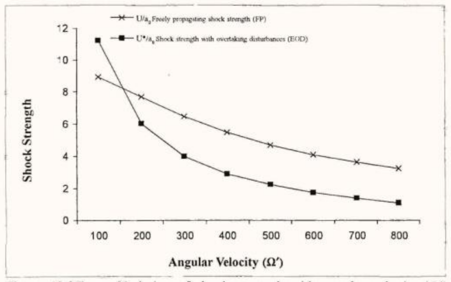


Fig.-10 Variation of shock strength with angular velocity (Ω') showing the effect of overtaking disturbances for initial density distribution $\rho_0 \propto r^w$ for strong shock.

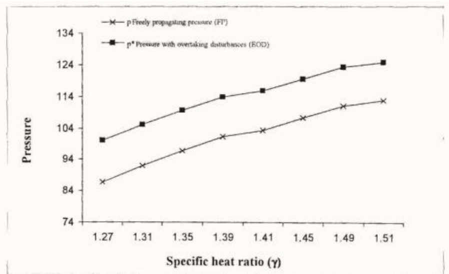


Fig.-12 Variation of shock strength with specific heat ratio (γ) showing the effect of overtaking disturbances for initial density distribution $\rho_0 \propto r^w$ for strong shock.

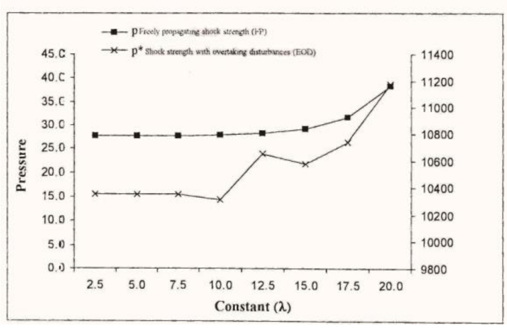


Fig.-14 Variation of pressure with constant (λ) showing the effect of overtaking disturbances for initial density distribution $\rho_0 \propto r^w$ for strong shock.

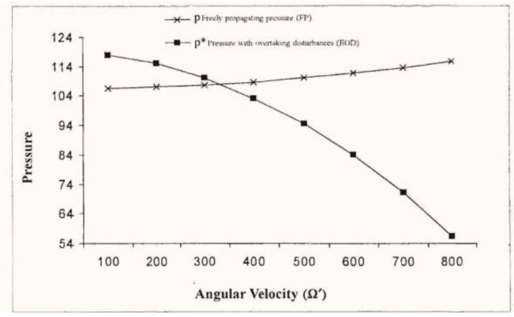


Fig.-15 Variation of pressure with angular velocity (Ω′) showing the effect of overtaking disturbances for initial density distribution $\rho_0 \propto r^w$ for strong shock.

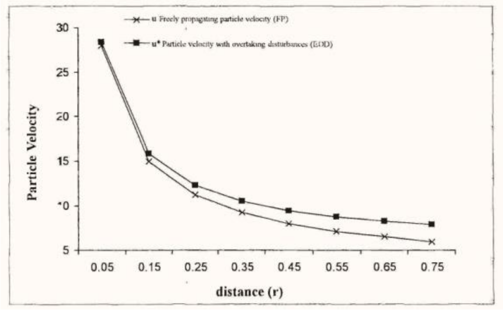


Fig.-16 Variation of particle velocity with propagation distance (r) showing the effect of overtaking disturbances for initial density distribution $\rho_0 \propto r^w$ for strong shock.

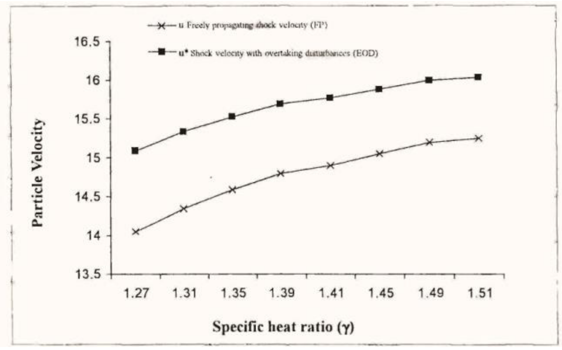


Fig.-17 Variation of particle velocity with specific heat ratio (γ) showing the effect of overtaking disturbances for initial density distribution $\rho_0 \propto r^w$ for strong shock.

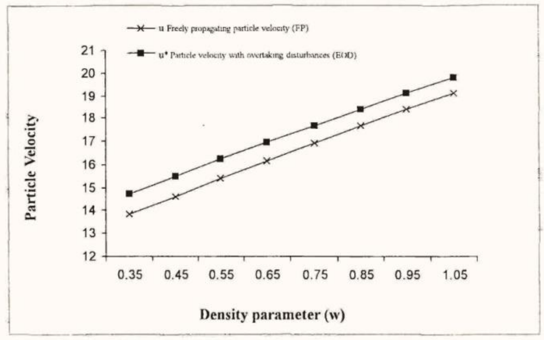


Fig.-18 Variation of particle velocity with density parameter (w) showing the effect of overtaking disturbances for initial density distribution $\rho_0 \propto r^w$ for strong shock.

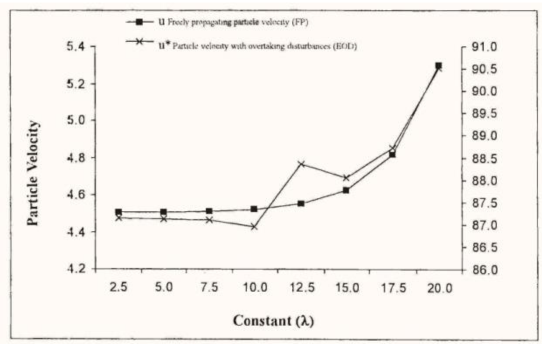


Fig.-19 Variation of particle velocity with constant (λ) showing the effect of overtaking disturbances for initial density distribution $\rho_0 \propto r^w$ for strong shock.

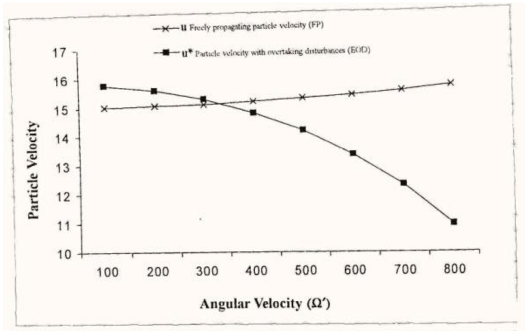


Fig.-20 Variation of particle velocity with angular velocity (Ω′) showing the effect of overtaking disturbances for initial density distribution $\rho_0 \propto r^w$ for strong shock.

VI. REFERENCES

[1]. Taylor. G.I. : (1950) Proc. Roy Soc. (London), 201 A, 159

[2]. Sakurai, A. : (1953) J. Physics Soc. (Japan) 8, 662

[3]. Chester, W. : (1954) Philos. Mag., 45(7), 123

[4]. Whitham, G.B. : (1958) J. Fluid Mach. 4, 337]

[5]. Sedov, L.V. : (1959) Similarity and Dimensional Methods in Mechanics, Academic Press, New York

[6]. Prasad, P. and Sriniwas,R. : (1990) Proc. Ind. Academic Science, 100, 93

[7]. Yadav, R.P. : (1992) Mod. Meas Cont. B. 46, (4)

[8]. Yadav, at.al. : (2001) Indian Journal of Theoretical Physics, 49 (3), 231

[9]. Yadav,R.P. & Gangwar, P.K. : (2002) 47th Conference (International) ISTAM.

[10]. Yadav & Gangwar : (2003) J. Nat. Phy.Science 17 (2), 109

[11]. Yadav,R.P. & Singh, A. : (2004) J. Science Physics Science 16(2), 251