

Simulation Studies of Three-Way Unbalanced Design on Fixed, Random, and Mixed Model

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ABSTRACT

Analysis of Variance (ANOVA) is a statistical technique used to compare means from various samples. Generally, a balanced design is used in ANOVA, but in some conditions, an unbalanced design can happen when the sample size is different in each treatment. This design will have the calculation of the F-test is different from usual for fixed, random, and mixed models. In this research, a simulation study will be carried out to see the differences in the results of the F-test decision in a three-way ANOVA with an unbalanced design based on a fixed, random, and mixed model. Simulation data is generated based on several scenarios, small sample size and large sample size, $e \sim \text{Normal}(0,1)$ and $e \sim \text{Gamma}(2,3)$, and applied to 8 models, that combine fixed effects and random effects in a 3-factor design. The simulation shows that sample size, error distribution, and the used model can affect F-test decisions. Designs with large sample sizes and $e \sim \text{Normal}(0,1)$ produce more significant F-test decisions than small sample sizes and $e \sim \text{Gamma}(2,3)$, and model 1 or the fixed model has more significant F-test decisions than other models in each scenario.

Keywords: ANOVA, F-Test, Simulation, Unbalanced Design

I. INTRODUCTION

Analysis of Variance (ANOVA) is a statistical technique used to compare means from various samples [1]. The model in ANOVA is assumed to be approximated by a linear combination of the components effect corresponding to each experimental factor, which consists of a fixed model, a random model, and a mixed model. According to [2], a fixed

model if all levels are fixed, a random model if all levels are chosen randomly, and a mixed model if several factors are chosen as fixed and chosen randomly. ANOVA involves determining the sum of squares of each component in the model, degrees of freedom, and determining the appropriate F-test. The existence of a fixed model, random model, and mixed model indicates that there are differences in the ANOVA results, which are differences in F-test decisions. The

F-test ratio is formed by dividing the corresponding expected mean squared, the greater the F-test ratio, the greater the probability of rejecting the null hypothesis [3]. Besides using the F-test ratio, in identifying the significance of hypothesis testing of the main factors and interactions effect, one can also use p-value [4]. According to [5], the hypothesis test results will be significant when the p-value is smaller than alpha or when the F-value is greater than the critical value. The lower the p-value, the smaller the chance that the null hypothesis is true [6].

R.A. Fisher developed factorial ANOVA for use on data sets with the same number of observations from each factor, called a balanced design [7]. However, in some situations, an unbalanced design can occur when the number of observations for each factor is different, or the number of sample sizes for each treatment is different. The unbalanced design can affect determining the F-test ratio because it has a more complex calculation considering the amount of data in each cell.

In addition to the model and the number of observations, the distribution of the data distribution also influences the F-test results. According to [8], one of the assumptions of ANOVA is that the data is normally distributed. When this assumption is violated, it will result in an F-test decision with more type I errors. In this research, a simulation will be carried out to determine the differences in the results of the F-test decisions in a three-way ANOVA with an unbalanced design, which will be compiled using three scenarios: the model scenario, the number of sample size, and the error distribution scenario.

II. METHODS AND MATERIAL

A simulation study was used in this research using an unbalanced three-factor ANOVA design. Each factor has three levels with different sample sizes. Simulation data is generated using RStudio with predetermined

scenarios. These scenarios are the model scenario, the number of sample sizes, and the error distribution. In the error distribution scenario, using $e \sim \text{Normal}(0,1)$ and $e \sim \text{Gamma}(2,3)$.

Model Restrictions

If a three-way design is used with fixed model assumptions, then $\alpha_i, \beta_j, \gamma_k, (\alpha\beta)_{ij}, (\alpha\gamma)_{ik}, (\beta\gamma)_{jk}$, and $(\alpha\beta\gamma)_{ijk}$ are constant with the following restrictions [9]:

$$\begin{aligned} \sum_{i=1}^a \alpha_i &= \sum_{j=1}^b \beta_j = \sum_{k=1}^c \gamma_k = 0, \\ \sum_{i=1}^a (\alpha\beta)_{ij} &= \sum_{j=1}^b (\alpha\beta)_{ij} = \sum_{i=1}^a (\alpha\gamma)_{ik} = \\ &= \sum_{k=1}^c (\alpha\gamma)_{ik} \\ &= \sum_{j=1}^b (\beta\gamma)_{jk} = \sum_{k=1}^c (\beta\gamma)_{jk} = 0, \\ \sum_{i=1}^a (\alpha\beta\gamma)_{ijk} &= \sum_{j=1}^b (\alpha\beta\gamma)_{ijk} = \sum_{k=1}^c (\alpha\beta\gamma)_{ijk} = 0 \end{aligned}$$

In the random model, $\alpha_i, \beta_j, \gamma_k, (\alpha\beta)_{ij}, (\alpha\gamma)_{ik}, (\beta\gamma)_{jk}$, and $(\alpha\beta\gamma)_{ijk}$ will have a normal distribution with an expected value of 0 and each variance, $\sigma_\alpha^2, \sigma_\beta^2, \sigma_\gamma^2, \sigma_{\alpha\beta}^2, \sigma_{\alpha\gamma}^2, \sigma_{\beta\gamma}^2$, and $\sigma_{\alpha\beta\gamma}^2$. In mixed models, which are a combination of fixed and random models, assumptions are adjusted to the influence of each factor. According to [3], the interaction between fixed and random factors will be considered random factors.

The model used in this research consists of 8 models, which combine of all 3-factor design models, which can be seen more clearly in Table 1.

Table 1. Description of the model used

Model	Factor A	Factor B	Factor C
Model 1	fixed	fixed	fixed
Model 2	random	random	random
Model 3	fixed	random	random
Model 4	random	fixed	fixed
Model 5	random	fixed	random
Model 6	fixed	random	fixed
Model 7	random	random	fixed
Model 8	fixed	fixed	random

Sample Size

In the sample size scenario, use a small sample size and a large sample size. Research conducted by [10] used two sample sizes in each cell experiment that is, n=30 for large samples and n=10 for small sample sizes. [11] in his research using two designs with n=5 for small samples and n=50 for large samples. A study from [12] used a different sample size for each cell in the experiment, n=3 until n=6. According to [13], only some studies can be conducted well with samples less than 100. Based on these studies, simulation data with a small sample size were used N=120 with different data counts on each cell experiments using n=3 until n=10. In a large sample size, used N=1200 with different counts of data on each cell experiments with n=30 until n=100. The sample size in each cell can be seen in Table 2.

Table 2. Sample size of each cell

	Number of sample size (n)
Small sample size	$n_{111} = 6 ; n_{112} = 6 ; n_{113} = 10 ; n_{121} = 4 ; n_{122} = 6 ; n_{123} = 8 ; n_{131} = 3 ; n_{132} = 4 ; n_{133} = 6 ; n_{211} = 4 ; n_{212} = 5 ; n_{213} = 7 ; n_{221} = 4 ; n_{222} = 5 ; n_{223} = 8 ; n_{231} = 3 ; n_{232} = 4 ; n_{233} = 6 ; n_{311} = 0 ; n_{312} = 0 ; n_{313} = 5 ; n_{321} = 0 ; n_{322} = 3 ; n_{323} = 5 ; n_{331} = 0 ; n_{332} = 3 ; n_{333} = 5$
Large sample size	$n_{111} = 60 ; n_{112} = 60 ; n_{113} = 100 ; n_{121} = 40 ; n_{122} = 60 ; n_{123} = 80 ; n_{131} = 30 ; n_{132} = 40 ; n_{133} = 60 ; n_{211} = 40 ; n_{212} = 50 ; n_{213} = 70 ; n_{221} = 40 ; n_{222} = 50 ; n_{223} = 80 ; n_{231} = 30 ; n_{232} = 40 ; n_{233} = 60 ; n_{311} = 0 ; n_{312} = 0 ; n_{313} = 50 ; n_{321} = 0 ; n_{322} = 30 ; n_{323} = 50 ; n_{331} = 0 ; n_{332} = 30 ; n_{333} = 50$

Simulation Procedure

1. Determine the number of levels on factors A, B, and C, with three levels each.
2. Determine the number of sample sizes for each cell, as seen in Table 2.

3. Generating data using RStudio with $e \sim \text{Normal}(0, 1)$ and $e \sim \text{Gamma}(2, 3)$ based on:
 - a. Model 1: uses fixed model restrictions
 - b. Model 2: generate $\alpha \sim N(0, 1), \beta \sim N(0, 1), \gamma \sim N(0, 1)$
 - c. Model 3: generate $\beta \sim N(0, 1), \gamma \sim N(0, 1)$, with α is fixed
 - d. Model 4: generate $\alpha \sim N(0, 1)$, with β and γ is fixed
 - e. Model 5: generate $\alpha \sim N(0, 1), \gamma \sim N(0, 1)$, with β is fixed
 - f. Model 6: generate $\beta \sim N(0, 1)$, with α and γ is fixed
 - g. Model 7: generate $\alpha \sim N(0, 1), \beta \sim N(0, 1)$, with γ is fixed
 - h. Model 8: generate $\gamma \sim N(0, 1)$, with α and β is fixed
4. Calculating Y_{ijkl} based on simulation data obtained using a three-factor design linear model [14]:

$$Y_{ijkl} = \mu + \alpha_i + \beta_j + \gamma_k + (\alpha\beta)_{ij} + (\alpha\gamma)_{ik} + (\beta\gamma)_{jk} + (\alpha\beta\gamma)_{ijk} + \varepsilon_{ijkl}$$

5. Analyze the simulated data using ANOVA
6. Each simulation scenario will be combined so that there are 32 models. There are 100 replications in every simulation, so that will get 100 decisions on the result of the F-test and visualize the results based on each factor.

Different simulation scenarios are used to see the differences in F-test decision results by looking at the p-value in each scenario with several different designs. P-value < 0,05 means rejecting the null hypothesis, indicating that the factor is significant.

III. RESULTS AND DISCUSSION

Several factors and interactions are tested in the three-way ANOVA: factor A, factor B, factor C, interaction AB, interaction AC, interaction BC, and interaction ABC. Each factor will see the F-test decision by looking at the p-value in each scenario. Based on the simulation, from 100 replications, the number of rejecting the null hypothesis would be seen based on p-value < 0,05. The accumulation results of the F-test in each scenario are presented based on the factors

tested and calculated based on the number of significant F-test.

Regarding the effect of factor A, the differences in the F-test decision for each scenario can be seen in Figure 1.

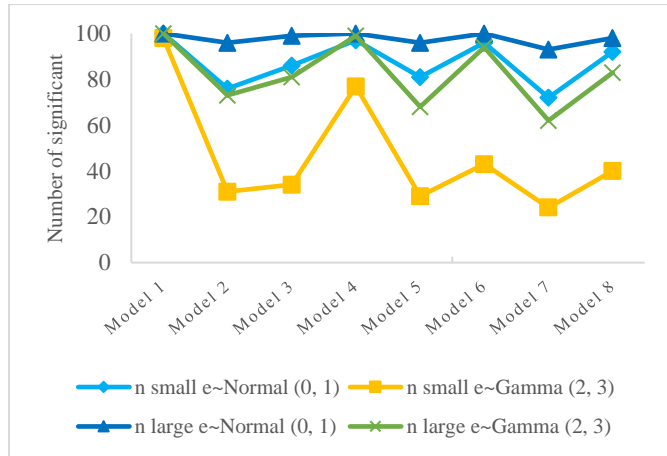


Figure 1: Comparison of F-test decision result based on the effect of factor A

Based on Figure 1, it can be seen that in model 1, model 2, model 3, model 4, model 5, model 6, model 7, and model 8, scenarios with large sample size and $e \sim \text{Normal}(0, 1)$ has the most number of significant or reject the null hypothesis. In scenarios with small sample sizes and $e \sim \text{Normal}(0, 1)$ and scenario with large sample size and $e \sim \text{Gamma}(2, 3)$ has almost the same number of significant or reject the null hypothesis, whereas in the scenario with small sample size and $e \sim \text{Gamma}(2, 3)$ has the fewest number of significant.

In model 1 or fixed model, it can be seen that all of the simulations have significant F-test decisions in each scenario. Model 4, model 6, and model 8 have more significant F-test decisions than model 2, model 3, model 5, and model 7 in the scenario with large sample size and $e \sim \text{Normal}(0, 1)$, small sample size and $e \sim \text{Normal}(0, 1)$, and large sample size and $e \sim \text{Gamma}(2, 3)$. In the scenario with small sample size and $e \sim \text{Gamma}(2, 3)$, model 4 has more significant F-test

decisions than model 2, model 3, model 5, model 6, model 7, and model 8.

Regarding the effect of factor B, the differences in the F-test decision for each scenario can be seen in Figure 2.

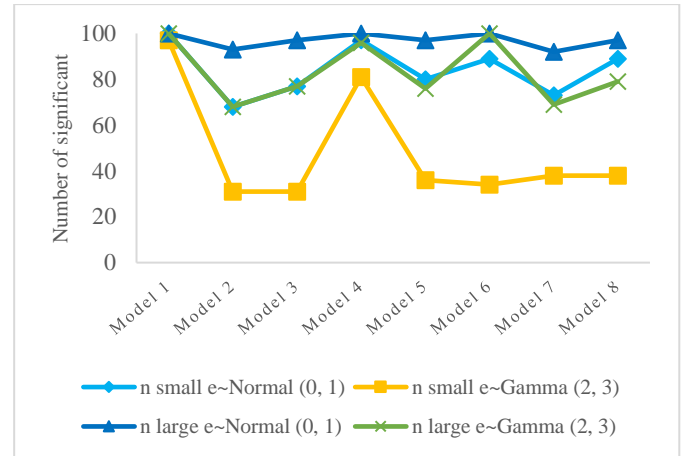


Figure 2: Comparison of F-test decision result based on the effect of factor B

Based on Figure 2, it can be seen that in model 1, model 2, model 3, model 4, model 5, model 6, model 7, and model 8, scenario with large sample size and $e \sim \text{Normal}(0, 1)$ has the most number of significant or reject the null hypothesis, while scenario with the small sample size and $e \sim \text{Gamma}(2, 3)$ has the fewest number of significant or reject the null hypothesis. In model 1, model 2, model 3, model 4, model 5, model 7, and model 8, the scenario with a small sample size and $e \sim \text{Normal}(0, 1)$ and the scenario with large sample size and $e \sim \text{Gamma}(2, 3)$ has almost the same number of significant, whereas, in model 6, the scenario with large sample size and $e \sim \text{Gamma}(2, 3)$ has the number of significant same as the scenario with large sample size and $e \sim \text{Normal}(0, 1)$.

In model 1 or fixed model, it can be seen all of the simulations have significant F-test decisions in each scenario. Model 4, model 6, and model 8 have more significant F-test decisions than model 2, model 3, model 5, and model 7 in the scenario with a large sample size and $e \sim \text{Normal}(0, 1)$, scenario with a small

sample size and $e \sim \text{Normal}(0, 1)$, and scenario with large sample size and $e \sim \text{Gamma}(2, 3)$. In the scenario with a small sample size and $e \sim \text{Gamma}(2, 3)$, model 4 has more significant F-test decisions than model 2, model 3, model 5, model 6, model 7, and model 8.

Regarding the effect of factor C, the differences in the F-test decision for each scenario can be seen in Figure 3.

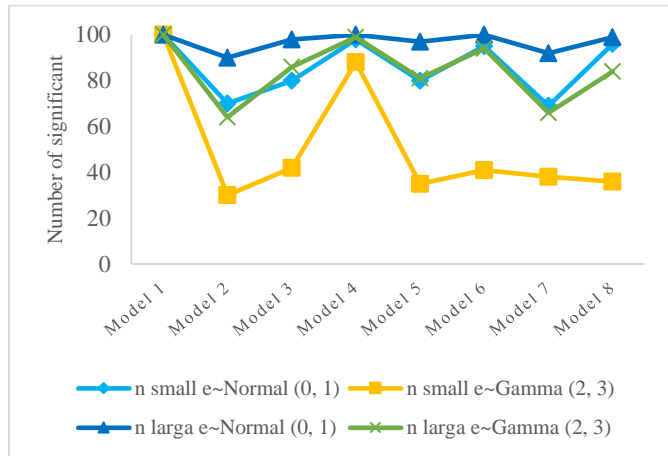


Figure 3: Comparison of F-test decision result based on the effect of factor C

Based on Figure 3, it can be seen that in model 1, model 2, model 3, model 4, model 5, model 6, model 7, and model 8, scenarios with large sample size and $e \sim \text{Normal}(0, 1)$ has the most number of significant or reject the null hypothesis, while in the scenario with small sample size and $e \sim \text{Gamma}(2, 3)$ has the fewest number of significant or reject the null hypothesis. In model 1, model 2, model 3, model 4, model 5, model 6, and model 7, scenarios with small sample sizes and $e \sim \text{Normal}(0, 1)$ and scenarios with large sample size and $e \sim \text{Gamma}(2, 3)$ have almost the same number of reject the null hypothesis, whereas, in model 8, the scenario with large sample size and $e \sim \text{Gamma}(2, 3)$ has the number of reject the null hypothesis that is almost the same as the scenario with large sample size and $e \sim \text{Normal}(0, 1)$.

In model 1 or fixed model, it can be seen that all of the simulations have significant F-test decisions in each

scenario. Model 4, model 6, and model 8 have more significant F-test decisions than model 2, model 3, model 5, and model 7 in the scenario with a large sample size and $e \sim \text{Normal}(0, 1)$, scenario with a small sample size and $e \sim \text{Normal}(0, 1)$, and scenario with a large sample size and $e \sim \text{Gamma}(2, 3)$. In the scenario with a small sample size and $e \sim \text{Gamma}(2, 3)$, model 4 has more significant F-test decisions than model 2, model 3, model 5, model 6, model 7, and model 8.

Regarding the effect of AB interaction, the differences in the F-test decision for each scenario can be seen in Figure 4.

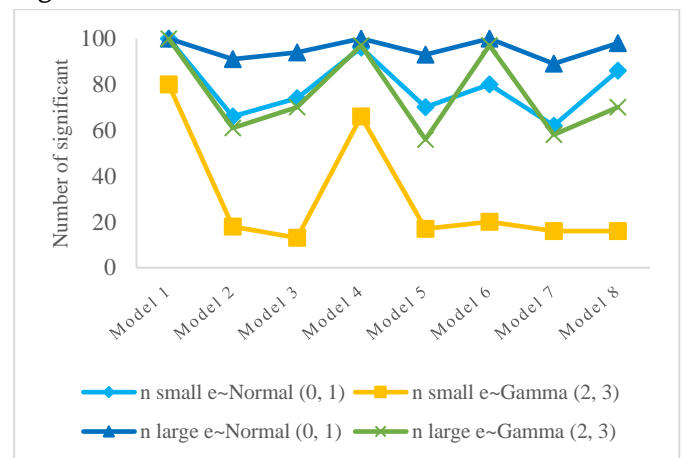


Figure 4: Comparison of F-test decision result based on the interaction effect of AB

Based on Figure 4, it can be seen that in model 1, model 2, model 3, model 4, model 5, model 6, model 7, and model 8, scenarios with large sample size and $e \sim \text{Normal}(0, 1)$ has the most number of significant or reject the null hypothesis, while in the small sample size scenario and $e \sim \text{Gamma}(2, 3)$ has the fewest number of significant or reject the null hypothesis. In model 2, model 3, and model 7, scenarios with small sample size and $e \sim \text{Normal}(0, 1)$ and scenarios with large sample size scenarios and $e \sim \text{Gamma}(2, 3)$ have almost the same number of significant. In model 1 and model 4, the scenario with a large sample size and $e \sim \text{Normal}(0, 1)$, the scenario with a small sample size and $e \sim \text{Normal}(0, 1)$, and the scenario with a large sample size scenarios and $e \sim \text{Gamma}(2, 3)$ have almost

the same number of significant. In model 5 and model 8, the scenario with a small sample size and $e \sim \text{Normal}(0, 1)$ has more significance than the scenario with a large sample size and $e \sim \text{Gamma}(2, 3)$. In contrast, in model 6, the scenario with a large sample size and $e \sim \text{Gamma}(2, 3)$ has the number of significant that is almost the same as the scenario with a large sample size and $e \sim \text{Normal}(0, 1)$.

In model 1, model 4, model 6, and model 8, there are more significant F-test decisions than model 2, model 3, model 5, and model 7 in the scenario with large sample size and $e \sim \text{Normal}(0, 1)$, scenario with small sample size and $e \sim \text{Normal}(0, 1)$, and scenario with large sample size and $e \sim \text{Gamma}(2, 3)$. In the scenario with a small sample size and $e \sim \text{Gamma}(2, 3)$, model 1 and model 4 have more significant F-test decisions than model 2, model 3, model 5, model 6, model 7, and model 8.

Regarding the effect of AC interaction, the differences in the F-test decision for each scenario can be seen in Figure 5.

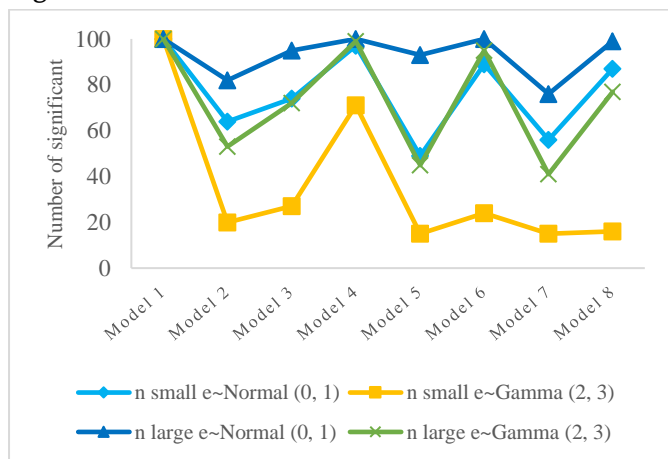


Figure 5: Comparison of F-test decision result based on the effect of AC

Based on Figure 4, it can be seen that in model 1, model 2, model 3, model 4, model 5, model 6, model 7, and model 8, scenarios with large sample size and $e \sim \text{Normal}(0, 1)$ has the most number of significant or reject the null hypothesis, while in the scenario with small sample size and $e \sim \text{Gamma}(2, 3)$ has the fewest

number of significant or reject the null hypothesis. In model 3 and model 5, scenarios with small sample sizes and $e \sim \text{Normal}(0, 1)$ and scenarios with large sample sizes and $e \sim \text{Gamma}(2, 3)$ have almost the same number of significant or reject the null hypothesis. In model 1, model 4, and model 6, the scenario with a large sample size and $e \sim \text{Normal}(0, 1)$, the scenario with small sample size and $e \sim \text{Normal}(0, 1)$, and the scenario with a large sample size and $e \sim \text{Gamma}(2, 3)$ have almost the same number of significant. In contrast, in model 2, model 7, and model 8, the scenario with a small sample size and $e \sim \text{Normal}(0, 1)$ has more significance than the scenario with a large sample size and $e \sim \text{Gamma}(2, 3)$.

In model 1 or fixed model, it can be seen that all of the simulations have significant F-test decisions in each scenario. Model 4, model 6, and model 8 have more significant F-test decisions than model 2, model 3, model 5, and model 7 in the scenario with a large sample size and $e \sim \text{Normal}(0, 1)$, scenario with a small sample size and $e \sim \text{Normal}(0, 1)$, and scenario with large sample size and $e \sim \text{Gamma}(2, 3)$. In the scenario with a small sample size and $e \sim \text{Gamma}(2, 3)$, model 4 has more significant F-test decisions than model 2, model 3, model 5, model 6, model 7, and model 8.

Regarding the effect of BC interaction, the differences in the F-test decision for each scenario can be seen in Figure 6.

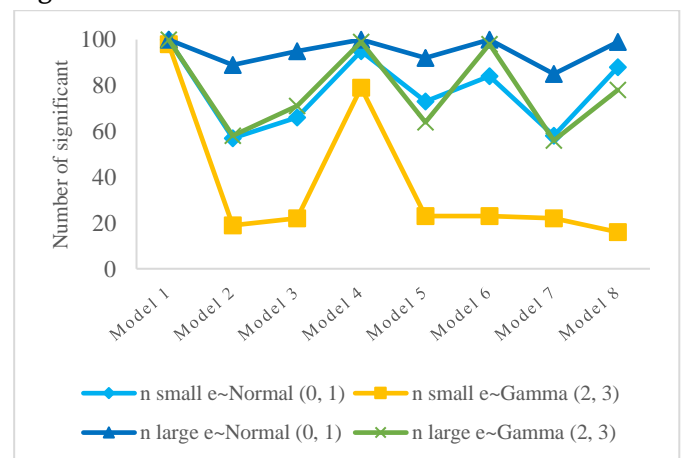


Figure 6: Comparison of F-test decision result based on the interaction effect of BC

Based on Figure 6, it can be seen that in model 1, model 2, model 3, model 4, model 5, model 6, model 7, and model 8, scenarios with large sample size and $e \sim \text{Normal}(0, 1)$ has the most number of significant or reject the null hypothesis, while in the scenario with small sample size and $e \sim \text{Gamma}(2, 3)$ has the fewest number of significant or reject the null hypothesis. In model 1 and model 4, the scenario with a large sample size and $e \sim \text{Normal}(0, 1)$, the scenario with a small sample size and $e \sim \text{Normal}(0, 1)$, and the scenario with a large sample size and $e \sim \text{Gamma}(2, 3)$ have almost the same number of significant or reject the null hypothesis. In model 2, model 3, and model 7, scenarios with small sample sizes and $e \sim \text{Normal}(0, 1)$ and scenarios with large sample sizes and $e \sim \text{Gamma}(2, 3)$ have almost the same number of significance. In model 6, a scenario with a large sample size and $e \sim \text{Gamma}(2, 3)$ has the number of significant or reject the null hypothesis that is almost the same as the scenario with large sample size and $e \sim \text{Normal}(0, 1)$, while model 5 and model 8, the scenario with small sample size and $e \sim \text{Normal}(0, 1)$ have more number of significant or reject the null hypothesis than the scenario with large sample size and $e \sim \text{Gamma}(2, 3)$.

In model 1 or fixed model, it can be seen that all of the simulations have significant F-test decisions in each scenario. Model 4, model 6, and model 8 have more significant F-test decisions than model 2, model 3, model 5, and model 7 in the scenario with a large sample size and $e \sim \text{Normal}(0, 1)$, scenario with a small sample size and $e \sim \text{Normal}(0, 1)$, and scenario with large sample size and $e \sim \text{Gamma}(2, 3)$. In the small sample size and $e \sim \text{Gamma}(2, 3)$, model 4 has more significant F-test decisions than model 2, model 3, model 5, model 6, model 7, and model 8.

Regarding the effect of ABC interaction, the differences in the F-test decision for each scenario can be seen in Figure 7.

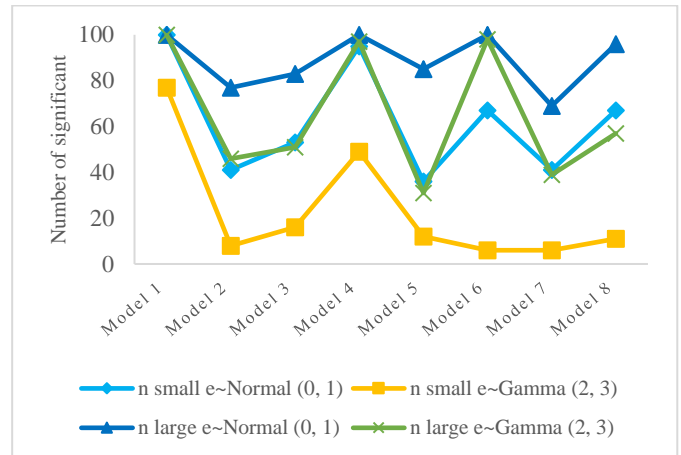


Figure 7: Comparison of F-test decision result based on the effect of ABC

Based on Figure 7, it can be seen that in model 1, model 2, model 3, model 4, model 5, model 6, model 7, and model 8, scenarios with large sample size and $e \sim \text{Normal}(0, 1)$ has the most number of significant or reject the null hypothesis, while in the scenario with small sample size and $e \sim \text{Gamma}(2, 3)$ has the fewest number of significant or reject the null hypothesis. In model 2, model 3, model 5, and model 7, scenarios with small sample sizes and $e \sim \text{Normal}(0, 1)$ and scenarios with large sample sizes and $e \sim \text{Gamma}(2, 3)$ have almost the same number of significance. In model 1 and model 4, the scenario with a large sample size and $e \sim \text{Normal}(0, 1)$, the scenario with a small sample size and $e \sim \text{Normal}(0, 1)$, and the scenario with a large sample size and $e \sim \text{Gamma}(2, 3)$ have almost the same number of significant. In model 8, the scenario with a small sample size and $e \sim \text{Normal}(0, 1)$ has more significance than the scenario with a large sample size and $e \sim \text{Gamma}(2, 3)$. In contrast, in model 6, the scenario with a large sample size and $e \sim \text{Gamma}(2, 3)$ has the number of significance that is almost the same as the scenario with large sample size and $e \sim \text{Normal}(0, 1)$.

In model 1, model 4, model 6, and model 8, there are more significant F-test decisions than in model 2, model 3, model 5, and model 7 in the scenario with large sample size and $e \sim \text{Normal}(0, 1)$, scenario with

small sample size and $e \sim \text{Normal}(0, 1)$, and scenario with large sample size and $e \sim \text{Gamma}(2, 3)$. In the scenario with a small sample size and $e \sim \text{Gamma}(2, 3)$, model 1 and model 4 have more significant F-test decisions than model 2, model 3, model 5, model 6, model 7, and model 8.

Based on the simulation results on factor A, factor B, factor C, interaction AB, interaction AC, interaction BC, and interaction ABC, the number of sample sizes and the distribution of errors can influence the F-test decision. Large sample sizes generally have more significant F-tests than small sample sizes. In error distribution, $e \sim \text{Normal}(0,1)$ has more significant F-test decisions than $e \sim \text{Gamma}(2,3)$. This is related to the ANOVA assumption, which requires the assumption of normality.

Model 1, or fixed model, has more significant F-test decisions than the other models. On average, model 1, model 4, model 6, and model 8 have more number of significant F-test decisions than model 2, model 3, model 5, and model 7. Model 1 and model 4 can produce more significant F-test decisions than the other models in each scenario. Please note that model 4, model 6, and model 8 are models consisting of two fixed factors and one random factor, while model 1 is a fixed model. This follows [15] that the added fixed effects can potentially reject the null hypothesis. However, being careful when rejecting the null hypothesis too often will affect the type I error.

IV.CONCLUSION

In the three-way unbalanced design, the number of sample size, error distribution, and model used can affect F-test decisions. Large sample sizes have more significant F-test decisions than small sample sizes. In error distribution, $e \sim \text{Normal}(0,1)$ has more significant F-test decisions than $e \sim \text{Gamma}(2,3)$. Adding fixed effects to the model could result in significant F-test decisions.

V. REFERENCES

- [1]. S. Hahn and L. Salmaso, "A comparison of different synchronized permutation approaches to testing effects in two-level two-factor unbalanced ANOVA designs," *Stat. Pap.*, vol. 58, no. 1, 2017, doi: 10.1007/s00362-015-0690-2.
- [2]. H. M. Choe, M. Kim, and E. K. Lee, "EMSaov: An R package for the analysis of variance with the expected mean squares and its shiny application," *R J.*, vol. 9, no. 1, 2017, doi: 10.32614/rj-2017-011.
- [3]. M. McFarquhar, "Modeling Group-Level Repeated Measurements of Neuroimaging Data Using the Univariate General Linear Model," *Frontiers in Neuroscience*, vol. 13. 2019, doi: 10.3389/fnins.2019.00352.
- [4]. R. Vijayaragunathan and M. R. Srinivasan, "Bayes factors for comparison of two-way ANOVA models," *J. Stat. Theory Appl.*, vol. 19, no. 4, 2020, doi: 10.2991/jsta.d.201230.001.
- [5]. D. Lakens and A. R. Caldwell, "Simulation-Based Power Analysis for Factorial Analysis of Variance Designs," *Adv. Methods Pract. Psychol. Sci.*, vol. 4, no. 1, 2021, doi: 10.1177/2515245920951503.
- [6]. A. M. R. R. García and J. L. Puga, "Deciding on null hypotheses using p-values or bayesian alternatives: A simulation study," *Psicothema*, vol. 30, no. 1, 2018, doi: 10.7334/psicothema2017.308.
- [7]. C. E. Smith and R. Cribbie, "Factorial ANOVA with unbalanced data: A fresh look at the types of sums of squares," *J. Data Sci.*, vol. 12, no. 3, 2021, doi: 10.6339/jds.201407_12(3).0001.
- [8]. H. J. Keselman, A. R. Othman, and R. R. Wilcox, "Generalized linear model analyses for treatment group equality when data are non-normal," *J. Mod. Appl. Stat. Methods*, vol. 15, no. 1, 2016, doi: 10.22237/jmasm/1462075380.

- [9]. H. Sahai and M. I. Ageel, *The Analysis of Variance*. Boston, MA: Birkhäuser Boston, 2000.
- [10]. J. C. Oliver-Rodríguez and X. T. Wang, "Non-parametric three-way mixed ANOVA with aligned rank tests," *Br. J. Math. Stat. Psychol.*, vol. 68, no. 1, 2015, doi: 10.1111/bmsp.12031.
- [11]. A. V. Frane, "Experiment-Wise Type I Error Control: A Focus on 2×2 Designs," *Adv. Methods Pract. Psychol. Sci.*, vol. 4, no. 1, 2021, doi: 10.1177/2515245920985137.
- [12]. L. W. Xu, F. Q. Yang, A. Abula, and S. Qin, "A parametric bootstrap approach for two-way ANOVA in presence of possible interactions with unequal variances," *J. Multivar. Anal.*, vol. 115, pp. 172–180, Mar. 2013, doi: 10.1016/j.jmva.2012.10.008.
- [13]. M. Brysbaert, "How many participants do we have to include in properly powered experiments? A tutorial of power analysis with reference tables," *Journal of Cognition*, vol. 2, no. 1. 2019, doi: 10.5334/joc.72.
- [14]. D. C. A. S. U. Montgomery, *Design and Analysis of Experiments Ninth Edition*. 2017.
- [15]. E. de Bodt, J. G. Cousin, and R. Roll, "Improved method for detecting acquirer fixed effects," *J. Empir. Financ.*, vol. 50, 2019, doi: 10.1016/j.jempfin.2018.12.003.

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