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Analysis for Form Finding of Three Strut Tensegrity Structure

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ABSTRACT

A tensegrity structure is any structure realized from cables and struts to which a state of prestress is imposed that imparts tension to all cables. The tensegrity concept has long been considered as a basis for lightweight and compact packaging deployable structures. The qualities of tensegrity structures which make the technology attractive for human use are their resilience and their ability to use materials in a very economical way. Their extreme resilience makes tensegrity structures able to withstand large structural shocks like earthquakes.

This paper deals with the design of three strut tensegrity structure where the key step is the determination of its geometrical configuration that is the form-.finding analysis. For general problem Several methods proposed for this step are scrutinized, the analytical method found most suitable, which analyzes the internal force organic to n-strut tensegrity systems to obtain a better understanding of tensegrity and develop generic design equations for self deployable tensegrity systems. A static analysis of the internal forces is conducted on the top and bottom platforms of 3 – strut tensegrity systems, which determine the geometry of the systems and relationship between the internal forces. The results of this analysis are used to design n – strut tensegrity assuming symmetry and patterns consistent with all systems by relating the results to number of struts in the system (n). The theory of elasticity is used to develop generic design equations that calculate the lengths and diameter of the struts and elastic ties required to create a desired geometry and stiffness. Further mathematical model was verified by plotting the results in Cartesian coordinates in MATLAB, Which helped to visualize the designed structure and also to observe the change in an apparent physical model by changing design variable.

Keywords : Tensegrity, Deployable structures, form-finding analysis

1. INTRODUCTION

1.1 Deployable Structures

Deployable structures are used for easy storage and transportation. When required, they are deployed into their service configuration. A well known example is the umbrella. Deployable structures are sometimes known under other names like expandable, extendible, developable and unfurlable structures. Deployable structures have many potential applications both on Earth and in space. In civil engineering, temporary or emergency structures have been used for a long time. A more recent application is retractable roofs of large sports stadia. Erectable structures are versatile and can be compactly stowed, but possess the disadvantage of requiring risky in-space construction. Therefore, deployable structures are the only practical way to construct large, lightweight structures for remote locations in space. Obvious advantages of deployable structures are savings in mass and volume. Another, not easily recognized, benefit is that the structure can better withstand the launch loads in the stowed configuration. In its deployed configuration, the structure is only subjected to the orbital loads, which are considerably lower. An important issue in the design of all deployable space structures is the trade-off between the size of the packaged structure and its precision in the deployed state. Both aspects are usually critical to the mission performance, but are sometimes conflicting requirements.

A significant amount of research has been carried out in the field of deployable structures. Deployable structures are of many types. Some structures can be retracted again after they have been deployed, others rely on stored strain energy for deployment and some structures are stiff during deployment. Structures not depending on stored energy are deployed by external means, e.g. a motor. Most of the structures do not obtain full stiffness until fully deployed while others can immediately sustain loads.

Despite the amount of research into deployable structures, several high-profile failures have occurred in the last two decades. Failures in space are very expensive and extremely difficult to correct. One reason for these failures is an incomplete understanding of the behavior of the structure. Another reason, probably more rare, is that the concept itself is poor.

The actual design procedure that follows involves just a trade-off between packing efficiency, structural stiffness and precision of the deployed structure. The final steps in the construction of a structure involve extensive ground and, possibly, flight testing, which are extremely costly but unavoidable to ensure mission success.

1.2 Tensegrity Structures

The word tensegrity, which is a contraction of tensile integrity, Fuller [1] describes a tensegrity structure as "an assemblage of tension and compression components arranged in a discontinuous compression system. A tensegrity structure consists of multiple members, some of which are solely in tension; the others are in compression. None of the compressional members come in contact with another compressional member. However, the members in tension are different in this respect. Any vertex of the structure can be connected to another point on the structure by tracing a line along tensional members. This is evidence of the structure's tensional continuity.

Tibert A. G. [2] also refers to the definition given by Pugh for tensegrity system as: "A tensegrity system is established when a set of discontinuous compressive components interacts with a set of continuous tensile components to define a stable volume in space. To explain the mechanical principle of tensegrity structures, Pugh uses a balloon analogy. If the enclosed air is at higher pressure than the surrounding air it pushes outwards against the inwards-pulling balloon skin. If the air pressure inside the balloon is increased, the stresses in the skin become greater and the balloon will be harder to deform. In a tensegrity structure the struts have the role of the air and the cables that of the skin. Increasing the forces in the elements of a tensegrity structure will increase its strength and load bearing capacity. Burkhardt R. W. Jr.[3] distinguished tensegrity structures by the way forces are distributed within them. The members of a tensegrity structure are either always in tension or always in compression. Since the compression members do not have to transmit loads over long distances, they are not subjected to the great buckling loads they would be otherwise, and thus they can be made more slender without sacrificing structural integrity.

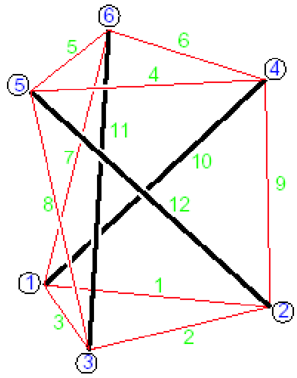

Fig.1 Rotationally symmetric 3 strut tensegrity structure.

The structure shown in fig.1. has a circumscribing radius r, height h and the two parallel equilateral triangles (nodes 1–3 and nodes 4–6) are rotated π/6 with respect to each other.

2. FORM FINDING OF TENSEGRITY STRUCTURES

2.1 Introduction

The state of pre-stress stabilizes the tensegrity structure, i.e., provides first-order stiffness to all infinitesimal mechanisms. A key step in the design of tensegrity structures is the determination of their geometrical configuration, known as form-finding. As stated by Tibert A. G. and Pellegrino S.[4], form-finding methods for tensegrity structures have been investigated by many authors, but the various methods have not been previously classified or linked. Different method are classified as Kinematic and Static methods. In Kinematical Methods the lengths of the cables are kept constant while the lengths of the struts are increased until a maximum is reached. Alternatively, the strut lengths are kept constant while the cable lengths are decreased until they reach a minimum.

2.2 Kinematical Methods

In Kinematical Methods we have Analytical solutions which can be obtained only for very simple structures, e.g., tensegrity prisms where the equilibrium configuration is determined by the relative rotation between the upper and lower regular polygon. In Non-Linear Programming the form-finding of any tensegrity structure is turned into a constrained minimization problem. Starting from a system for which the element connectivity and nodal coordinates are known, one or more struts are elongated, maintaining fixed length ratios, until a configuration is reached in which their length is maximized [5]. In Dynamic Relaxation method a static problem is turned into a fictitious dynamic problem. Convergence is controlled by an appropriate choice of damping coefficients or, alternatively, by a technique called kinetic damping. The motion of the structure is initiated by an increase of the length of the struts, while the cable lengths are held constant.

2.3 Statical Methods

These methods are characterized by a relationship between equilibrium configurations of a structure with given topology and the forces in its members. In Statical methods we have Analytical Solutions in which a general solution of the relative rotation between the upper and lower polygon of tensegrity prisms can be derived. Force method and group representation theory can be used to find analytical solutions to a number of spherical tensegrities and multi-stage tensegrity towers. These methods may become complex when the structure is very complex and does not have symmetry. However, for a tensegrity prism, the method is very suitable. Statical methods are classified in to Force Density, Energy Method and Reduced Coordinates methods.

3. STATIC ANALYSIS

The n-strut tensegrity is studied by rotating the top platform to a certain predicted range. However, it is more accurate to state that the n-strut tensegrity is obtained by rotating the top "platform" on one of several semi-regular Archimedean anti-prisms. The anti-prisms used to form an
n-strut tensegrity are "formed from two parallel congruent regular polygons joined by equilateral triangle".

An n-strut tensegrity is obtained by rotating the top platform of a wire frame anti-prism about the vertical axis while keeping the bottom platform fixed. As the top platform rotates, the length of the legs (members along the side) of the anti-prism will change appropriately. The top platform will continue to rotate about the vertical axis until the system becomes a stable tensegrity. The system becomes a stable tensegrity system once the top platform is rotated π/n radians and remains in stable tensegrity until the top platform is rotated $(n-1)\pi/n$ radians.

The initial rotation of the anti-prism's top platform defines the tensegrity as positive twist or a negative twist. If the top platform is rotated in the positive Z direction (counter clockwise looking down on the anti-prism), the system is a positive twist tensegrity. If the top platform is rotated in the negative Z direction, the system is a negative twist tensegrity. The twist of the tensegrity determines which legs are struts. The initial position of tensegrity, when the top platform is rotated π/n radians, is defined as the minimum limit and the final position at $(n-1)\,\pi/n$ radians is defined as the maximum limit. The angle of rotation of the top platform beyond the minimum limit is defined as alpha α, whereas alpha is equal to zero at the minimum limit position and is equal to $(n-2)\,\pi/n$ at the maximum limit. α is always in the direction of the initial rotation of the anti-prism. Following figure shows the top view of the minimum and maximum limits of a positive twist and negative twist 3-strut tensegrity.

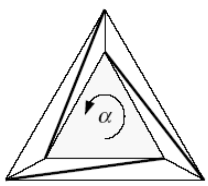

Fig.2
Positive Twist
Minimum Limit

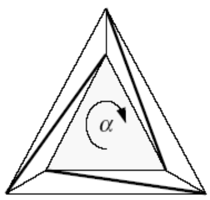

Fig. 3
Negative Twist
Minimum Limit

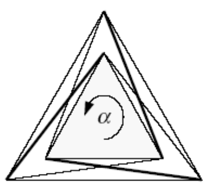

Fig. 4
Positive Twist
Maximum Limit

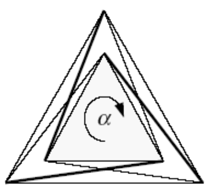

Fig. 5
Negative Twist
Maximum Limit

3.1 3-Strut Tensegrity System

The first step in statically analyzing the 3-strut tensegrity is to develop the vector systems of the internal forces. The rotation of the top platform, or twist angle α, will be positive in the positive Z-direction.

Figure 6. shows a top view of th e 3-strut tensegrity at the minimum limit and the variables used in the static analysis. The lengths of the top and bottom ties are L_a and L_b respectively and the distance between the base and top platform is h . The length of the struts is L_s and the length of the leg ties is L_t. The vectors symbolized by S are direction unit vectors of the members. Only the vectors along the member at AA and DD are needed since those are the two points where static equilibrium will be analyzed.

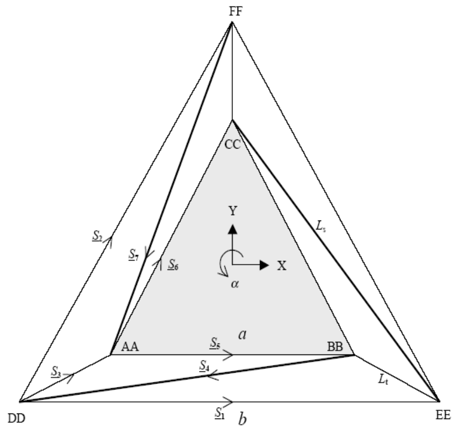

Fig. 6 3-strut tensegrity [minimum limit]

The coordinate system was initially developed at the minimum limit and the coordinates for the joints of the top platform were redeveloped to accommodate changes in α. The coordinates of the joints are used to determine the 3-dimensional vectors of each member.

The coordinates of the joints before the top platform is rotated are

AA $\left(-\frac{1}{2}a,\ -\frac{\sqrt{3}}{6}a,\ h\right)$,

BB $\left(\frac{1}{2}a,\ -\frac{\sqrt{3}}{6}a,\ h\right)$,

CC $\left(0,\ \frac{\sqrt{3}}{3}a,\ h\right)$,

DD $\left(-\frac{1}{2}b,\ -\frac{\sqrt{3}}{6}b,\ 0\right)$,

EE $\left(\frac{1}{2}b,\ -\frac{\sqrt{3}}{6}b,\ 0\right)$,

FF $\left(0,\ \frac{\sqrt{3}}{3}b,\ 0\right)$.

The coordinates of top platform free to rotate based on α are

AA $\left(-\frac{\sqrt{3}}{3}a\cos\left(\frac{\pi}{6}+\alpha\right),\ -\frac{\sqrt{3}}{3}a\sin\left(\frac{\pi}{6}+\alpha\right),\ h\right)$,

BB $\left(\frac{\sqrt{3}}{3}a\sin\left(\frac{\pi}{3}+\alpha\right),\ -\frac{\sqrt{3}}{3}a\cos\left(\frac{\pi}{3}+\alpha\right),\ h\right)$,

CC $\left(-\frac{\sqrt{3}}{3}a\sin(\alpha),\ \frac{\sqrt{3}}{3}a\cos(\alpha),\ h\right)$.

The direction vector of each member is the unit vector along that member.

$$S1=\{1,0,0,\}$$

$$S2=\left\{\frac{1}{2},\frac{\sqrt{3}}{2},0\right\}$$

$$S3=\frac{1}{L_t}\left\{\frac{1}{2}b-\frac{\sqrt{3}}{3}a\cos\left(\frac{\pi}{6}+\alpha\right),\frac{\sqrt{3}}{6}b-\frac{\sqrt{3}}{3}a\sin\left(\frac{\pi}{6}+\alpha\right),h\right\}$$

$$S4=\frac{1}{Ls}\left\{-\frac{1}{2}b-\frac{\sqrt{3}}{3}a\sin\left(\frac{\pi}{3}+\alpha\right),-\frac{\sqrt{3}}{6}b+\frac{\sqrt{3}}{3}a\cos\left(\frac{\pi}{3}+\alpha\right),-h\right\}$$

$$S5=\{\cos\alpha,\sin\alpha,0\}$$

$$S6=\left\{\frac{1}{2}\cos\alpha-\frac{\sqrt{3}}{2}\sin\alpha,\frac{\sqrt{3}}{2}\cos\alpha+\frac{1}{2}\sin\alpha,0\right\}$$

$$S7=\frac{1}{Ls}\left\{-\frac{\sqrt{3}}{3}a\cos\left(\frac{\pi}{6}+\alpha\right),-\frac{\sqrt{3}}{3}b-\frac{\sqrt{3}}{3}a\sin\left(\frac{\pi}{6}+\alpha\right),h\right\}$$

where the lengths of the struts Ls and the length of the leg ties Lt are

$$L_s=\frac{\sqrt{3}}{3}\sqrt{b^2+2ab\sin\left(\frac{\pi}{6}+\alpha\right)+a^2+3h^2}\ , \tag{Eq. (1)}$$

$$L_t=\frac{\sqrt{3}}{3}\sqrt{b^2-2ab\cos(\alpha)+a^2+3h^2}\ . \tag{Eq. (2)}$$

3.2 Static Analysis at Joint DD

With the unit vectors established, the forces can be analyzed at a point.
For static equilibrium,

$$\sum_{i=1}^{n}F_i Si=0 \tag{Eq. (3)}$$

Figure 7 shows the forces acting at joint DD.

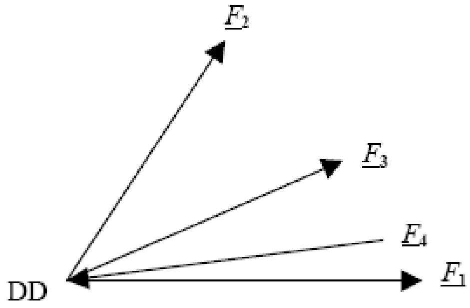

Fig 7: Forces at Joint DD

The forces acting on joint DD are

$F_1 = F_b S_1$, $F_2 = F_b S_2$, $F_3 = F_t S_3$, $F_4 = F_s S_4$. Eq. (4)

Because we are analyzing supported members at their joints, there are no moments acts on any members. Only the axial forces need to be considered. The equation for the sum of the forces at joint DD is

$$\sum_{DD} F_i = F_b S1 + F_b S2 + F_t S3 + F_s S4$$

Eq. (5)

This equation can now be resolved into three scalar equations, the sum of the forces in the X, Y and Z directions.

$$\sum_{DD} F_X = F_b + \frac{1}{2} F_b + \left[\frac{1}{2} b - \frac{\sqrt{3}}{3} a \cos\left(\frac{\pi}{6} + \alpha\right)\right] \frac{F_t}{L_t} - \left[\frac{1}{2} b + \frac{\sqrt{3}}{3} a \sin\left(\frac{\pi}{3} + \alpha\right)\right] \frac{F_s}{L_s} = 0,$$

Eq. (6)

$$\sum_{DD} F_Y = \frac{\sqrt{3}}{2} F_b + \left[\frac{\sqrt{3}}{6} b - \frac{\sqrt{3}}{3} a \sin\left(\frac{\pi}{6} + \alpha\right)\right] \frac{F_t}{L_t} - \left[\frac{\sqrt{3}}{6} b - \frac{\sqrt{3}}{3} a \cos\left(\frac{\pi}{3} + \alpha\right)\right] \frac{F_s}{L_s} = 0,$$

Eq. (7)

$$\sum_{DD} F_Z = \frac{h}{L_t} F_t - \frac{h}{L_s} F_s = 0.$$

Eq. (8)

Solving Eq. (8) for Ft yields

$$F_t = \frac{L_t}{L_s} F_s .$$

Eq. (9)

This equation will be found to be constant throughout all n-strut tensegrity systems because only the leg ties and the struts carry the forces in the Z direction. Thus the relation between the lengths of the leg ties and the length of the struts defines the relation between the internal forces of the leg ties to the internal forces of the struts.

Using equation (9) to substitute for Ft into Eqs. (6) and (7) and simplifying yields

$$\sum_{DD} F_X = \frac{3}{2} F_b - \frac{a}{L_S} F_s \cos(\alpha) = 0,$$

Eq. (10)

$$\sum_{DD} F_Y = \frac{\sqrt{3}}{2} F_b - \frac{a}{L_S} F_s \sin(\alpha) = 0.$$

Eq. (11)

Both equations must be satisfied for the platform tensegrity to be in static equilibrium. Solving Eqs. (10) and (11) for F_b yields

$$F_b = \frac{2a}{3L_s} F_s \cos(\alpha),$$

Eq. (12)

$$F_b = \frac{2\sqrt{3}\,a}{3L_s} F_s \sin(\alpha).$$

Eq. (13)

Setting the right side of Eq. (12) equal to the right side of Eq. (13) and solving for α results in the equation

$$\tan(\alpha) = \frac{\sqrt{3}}{3},$$

Eq. (14)

and yields a value for $\boldsymbol{\alpha}$

$$\alpha = \frac{\pi}{6}, \quad \frac{7\pi}{6}.$$

Eq. (15)

Though there are mathematically two stable solutions for the angles of rotation past the minimum limit, substituting 7π/6 radians for $\boldsymbol{\alpha}$ into the equations of static equilibrium produces a negative force on the leg ties and the struts. Switching the leg ties and struts at this configuration would result in a stable negative twist tensegrity. The initial rotation to the proposed minimum limit of π/n radians is added to determine for the total rotation of the top platform from the Archimedean 3-3 anti-prism. Therefore, there are only two stable configurations of a 3-strut tensegrity system: positive twist and negative twist. Both configurations have the top platform rotated π/2 radians (90°) from the original 3-3 anti-prism.

The sum of the forces at DD is now

$$\sum_{DD} F_X = \frac{3}{2} F_b - \frac{\sqrt{3}\,a}{2\,L_S} F_s = 0,$$

Eq. (16)

$$\sum_{DD} F_Y = \frac{\sqrt{3}}{2} F_b - \frac{a}{2\,L_S} F_s = 0,$$

Eq. (17)

$$\sum_{DD} F_Z = \frac{h}{L_t} F_t - \frac{h}{L_s} F_s = 0,$$

Eq. (18)

where the lengths of the struts and leg ties are

$$L_s = \frac{\sqrt{3}}{3}\sqrt{b^2 + \sqrt{3}\,a\,b + a^2 + 3h^2}\ ,$$

Eq. (19)

$$L_t = \frac{\sqrt{3}}{3}\sqrt{b^2 - \sqrt{3}\,a\,b + a^2 + 3h^2}\ .$$

Eq. (20)

Because the three equations of static equilibrium are linearly dependent, Eqs. (16) and (17) are the same equation. Therefore, the value of each force can only be related to another. This shows, as one member is stressed, the others members are also stressed. Solving Eq. (16) for the forces in the struts and substituting Eq. (9) in to find the forces in the leg ties in terms of the force in the bottom ties yields

$$F_s = \frac{\sqrt{3}\,L_s}{a} F_b,$$

Eq. (21)

$$F_t = \frac{\sqrt{3}\,L_t}{a} F_b\,.$$

Eq. (22)

4. DESIGN OF THREE STRUT TENSEGRITY STRUCTURE

Considering three strut tensegrity system with the following dimensions:

L_a = 25 mm, L_b = 25 mm, h = 65 mm

Material - polyurethane/ABS blend.

σ_y - 35.139 X 10^6 *N/mm²*

E - 12 X 6.89 X 10^6 *N/mm²*

Material for strut – Aluminium Alloy

yield stress σy - 15 X 6.89 X 10^6 N/mm²

L_t is found to be 70.7284 mm;

L_s is found to be 65.4280 mm

Using a factor of safety G = 1.3

Geometric stiffness ratio is found to be 1.4629. Internal force generated against the leg ties of be F = 500 N and finding all the values from derived equations

A_t = 13.0369 mm², K_t = 22.309 N/mm and A_s = 8.0557 mm². A_a and A_b is found to be 3.4846 mm². And hence L_b= 18.4615 mm and L_a = 18.4615 mm.

These calculations will produce a self deployable n-strut tensegrity system with bottom platform edges of length b, top platform edges of length a and height h.The designed system is under no external force and weight. Here in this example we have taken length of a and b same so we get same value for force induced in top and bottom ties, area of top and bottom ties and initial lengths of top and bottom ties.

5. MATLAB RESULT :

A mathematical model is verified by plotting the results in Cartesian coordinate in MATLAB. This helped to verify the correctness and helped also in observing the change in an apparent physical model by changing design variable. Following figure shows the plot for the three and four strut tensegrity structure and top view obtained from the MATLAB result.

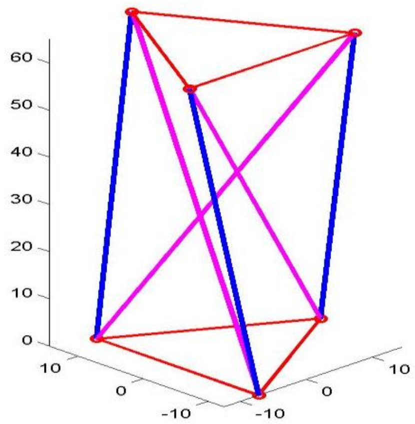

Fig. 8 Plot of 3 strut tensegrity structure.

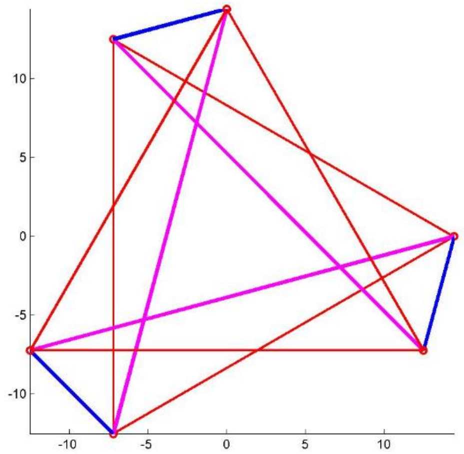

Fig. 9 Top View of 3 strut tensegrity structure.

After changing the design variable the four strut tensegrity structure obtained from the MATLAB also plotted.

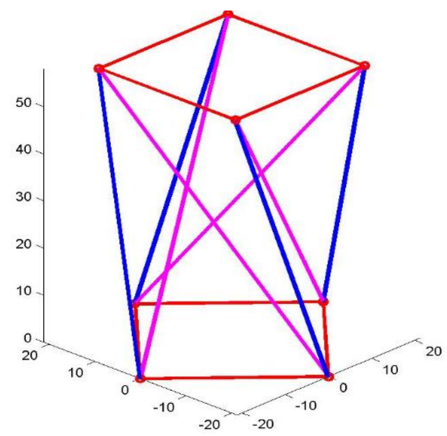

Fig. 10 Plot of 4 strut tensegrity structure.

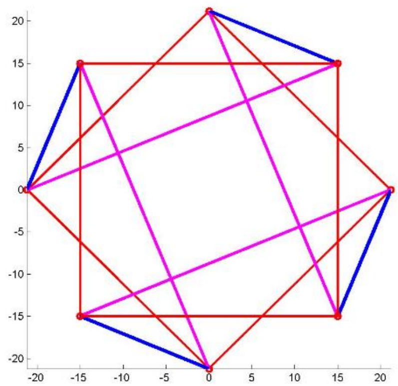

Fig.11 Top View of 4 strut tensegrity structure.

6. CONCLUSION

Various form finding methods were studied and it was found that analytical method is most suitable for n-strut tensegrity prism. Form finding analysis for three strut tensegrity is done. There are only two stable configurations of a 3-strut tensegrity system: positive twist and negative twist. Both configurations have the top platform rotated π/2 radians (90^o) from the original 3-3 anti-prism. The system becomes a stable tensegrity system once the top platform is rotated 60^0 and remains in stable tensegrity until the top platform is rotated 120^0.

The lengths of the struts and leg ties in the design are found as L_t = 70.7284 mm and L_s = 65.4280 mm and the forces induced in bottom and top ties are found as F_b = 102.04 N and F_a = 102.04 N. The initial lengths of bottom and top ties of the system are L_b = 18.4615 mm and L_a = 18.4615 mm. These calculations will produce a self deployable 3-strut tensegrity system with bottom platform edges of length 25mm, top platform edges of length 25mm and height 65mm.

A mathematical model was verified by plotting the results in Cartesian coordinate in MATLAB. This helped to verify the correctness and helped also in observing the change in an apparent physical model by changing design variable.

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